Time Since What? (Re)interpreting the Neolithic Transition in a Malthusian Environment

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Abstract: Differences across societies in the timing of the Neolithic (agricultural) transition are usually interpreted as implying differences in non-growing levels of land productivity at later stages of preindustrial development. In standard Malthusian models, such differences generate variation in steady-state population densities only, and not in living standards. Here I instead interpret the Neolithic transition as the starting point of a *gradual rise* in the *growth rate* of land productivity. Societies with earlier transitions then have somewhat higher living standards, and much higher population densities. This suggests that a non-zero correlation between the time passed since the Neolithic transition and preindustrial per-capita incomes is not itself reason to reject the Malthusian model. Moreover, when allowing for territorial competition between societies, and imposing an upper bound on population growth, the model can account also quantitatively for both variation in per-capita incomes and population densities, and global time trends in the size of empires.

1 Introduction

When measuring how advanced societies were in preindustrial times, one commonly used variable is the time passed since the Neolithic transition (or revolution), i.e., the transition from hunting and gathering to agriculture, originally compiled and analyzed by Putterman and Trainor (2006) and Putterman (2008).[1]

It is well known that the regions of the world which went through the Neolithic transition first – like the Middle East around 8000 BCE – were also forerunners in many other ways, being centres of the first big hierarchical states and empires, where the first cities were located, and where writing developed (see, e.g., Diamond 1997).

As is also well known, these transformations of human society had greater impact on population densities than living standards (see, e.g., Galor and Weil 2000, Lucas 2002). Global variation in per-capita incomes before the Industrial Revolution was relatively modest, compared to modern times. At the same time, early development does show *some* correlation with living standards. From the little data that is available – estimates and guesses by Angus Maddison – an earlier Neolithic transition is associated with higher per-capita income levels in the years 1 CE, 1000 CE, or 1500 CE.

Ashraf and Galor (2011) were the first to examine these data systematically, showing that the positive correlation between time since Neolithic and preindustrial per-capita income levels is not robust to the inclusion of regional dummies, and notably much smaller than the corresponding correlation for population densities.

Here I propose a new theoretical interpretation of the same data. I argue

[1] These data are today widely used. For recent applications, and summaries of the existing literature, see Ashraf and Galor (2011, 2013), Spolaore and Wacziarg (2013), Ashraf and Michalopoulos (2014), and Chanda et al. (2014). Earlier empirical work, using other or related sources and data, include Hibbs and Olsson (2004) and Baker (2008). Diamond (1997) was among the first to formulate and popularize the idea that the Neolithic transition mattered for later development.

that even if we do observe a positive raw correlation between time since Neolithic and preindustrial per-capita incomes, this is not inconsistent with a Malthusian model, if we think of the Neolithic transition as the starting point of a *gradual rise* in the *growth rate* of land productivity, which I argue is plausible.[2] Then societies with earlier transitions indeed have somewhat higher per-capita incomes, but also much larger population densities, qualitatively consistent with the findings of Ashraf and Galor (2011).

While a useful theoretical insight, this simple extension of the Malthusian model is not itself sufficient to account quantitatively for the observed variation in both per-capita incomes and population densities. Setting productivity growth rates to match observed population densities, the implied variation in per-capita incomes is an order of magnitude smaller than in the data (about 0.05 log units in the model, compared to 0.5-1 in the data).

However, when adding two more extensions to the model – territorial competition between societies, and an upper bound on the rate of population growth – and then calibrating the relevant parameters to make the model fit observed time trends in the size of empires, it can account quite well also quantitatively for both variation in per-capita incomes and population densities. Intuitively, when the era of empire building begins around 1000 BCE, the first empires are built by societies with larger initial populations, i.e., the very same ones that underwent early Neolithic transitions. The result is a rise in per-capita income levels which – if population growth rates do not rise too much and too fast – may be sustained for several millennia, thus affecting the per-capita income distribution long after the first societies entered the Neolithic transition.

The rest of this paper is organized as follows. Next Section 2 provides an overview of the facts that I seek to explain. Section 3 sets up a simple

[2] One can debate whether a model allowing for sustained exponential growth in land productivity should at all be called Malthusian. Here I label it such because it has the property that per-capita output converges to a non-growing level. While Thomas Malthus himself might have objected, this seems to be the operational definition of a Malthusian model today.

Malthusian model, first showing the effects on per-capita incomes and population densities from changes in levels of land productivity (Section 3.1) and then the same for productivity growth rates (Section 3.2). Informed by these insights, Section 3.3 considers different model interpretations of a Neolithic transition, proposing that a model where the Neolithic transition constitutes the starting point of a gradual rise in growth rates of land productivity can in principle explain the empirical patterns. However, Section 3.4 shows that this interpretation falls short quantitatively. An extended setting is considered in Section 3.5, allowing for territorial competition and bounded population growth, which generates a better quantitative fit. Section 4 concludes.

2 Background

Figure 1 shows Maddison's measures of per-capita incomes in 1 CE, 1000 CE, and 1500 CE, as well as the (unweighted) averages of these across countries defined by modern borders, plotted against the year of the Neolithic transition. Figure 2 illustrates the corresponding relationship for population densities, and Tables 1 and 2 show the associated correlation coefficients and p-values.[3]

At first glance, countries that transited earlier did have higher per-capita incomes and population densities in each of these three years, and all the relationships are strongly statistically significant. This holds also when using the average of the three years, suggesting that the overall pattern is driven by roughly the same set of countries.

Before proceeding, it is worth noting that the size of the gaps in per-capita income are small compared to those for densities. In that sense, the data seem consistent with a Malthusian model. Moreover, as argued by Ashraf

[3]Most of the data used here are those compiled by Ashraf and Galor (2011), whose sources include Maddison (2003) for per-capita incomes and McEvedy and Jones (1978) for population densities. See also Maddison (2008) for his own interpretation and discussion of these data.

and Galor (2011), the significance of the results for per-capita incomes is less robust than that for population densities, when entering regional dummies. (However, they do not discuss in much detail the theoretical basis for entering those regional controls; below we suggest that an empire dummy might be a more suitable control.) My starting point in this paper is to take the raw correlations at face value, and examine whether, or not, an augmented Malthusian model can generate the type of patterns observed in Figures and 1 and 2.

Figure 1 reveals that the highest per-capita incomes in most years are found in places like modern-day Italy, Iran, Iraq, and Turkey. These all had relatively early Neolithic transitions, and were also centres of various incarnations of the Roman and Muslim empires. Exploring this further, Table 3 reports the correlations between per-capita incomes in these three years and measures of state presence over different periods, 1-500 CE, 1-1000 CE and 1-1500 CE. These state presence indicators are from Bockstette et al. (2002), also used by Chanda and Putterman (2007), and capture the extent to which a country had a state above the tribal level over different time periods, and whether this was a local or foreign power, with higher scores for the former.[4] In other words, they partly measure whether a country was the centre of an empire over the reported period. As seen in Table 3, more state presence is associated with higher per-capita incomes. It also seems like the correlations are often larger when the state presence variable refers to a period close to, or overlapping with, the year in which per-capita incomes are measured.

In Table 4 I regress log per-capita incomes in 1000 CE on state presence 1-1000 CE, and an indicator variable for whether, or not, the country was the centre of an empire around the time, chosen to be China, Iran, Iraq, and Turkey, corresponding approximately to those listed as one of the three largest empires around 600-1000 CE by Taagepera (1978b, Table 2). These

[4] These data start in 1 CE, but work is currently under way by Borcan et al. (2014) to extend them backward in time.

were the Muslim empires, which were assigned by Taagepera to the region of Mesopotamia (Iraq); the Samanid empire, assigned to Iran; the Tang and Sung dynasties of China; and the Byzantine empire, located around what is today Turkey. (A few other empires are listed by Taagepera, in particular in Central Asia, but none of those corresponds closely to any of the 29 countries for which we have per-capita income data in 1000 CE.)

The insight from columns (1)-(3) of Table 4 is that the correlation between time since the Neolithic transition (measured in millennia) and per-capita incomes in 1000 CE is weakened when adding the empire dummy, not always alone but together with state presence. Columns (4)-(6) add some standard geographical controls, used by, e.g., Ashraf and Galor (2011): log land productivity, log absolute latitude, mean distance to nearest coast or river, and percentage of land within 100 km of coast or river. Again, the measured effect of time since the Neolithic transition is weakened when entering empire and state variables as controls. Finally, columns (7)-(9) drop New World countries, which by 1000 CE had been isolated from the territorial struggles associated with the rise and fall of Old World empires and states. The results are similar: when controlling for the presence of empires and states, the correlation between time passed since the Neolithic transition and per-capita incomes in 1000 CE is weakened or absent.

Empire building is a more recent phenomenon than agriculture. Figure 3 illustrates the time trends in the size of empires as reported by Taagepera (1978b, Table 2), for the three largest empires, and the sum of these, from 3000 BCE (the earliest year in the data) to 1500 CE (after which empires expanded into new world regions). Size is here expressed in proportion to the area that empires competed over, proxied by the whole of Eurasia and half of Africa.[5] There is a notable rise in the overall size of empires from around 1000 BCE. In Section 3.5, we shall calibrate the model to make it roughly match these paths, and then see what that may imply in terms of per-capita income gaps and population densities around 1000 CE.

[5] This area equals about 70 million square kilometers.

3 A simple model

Consider a world where there are N different societies (or ethnic groups). In each society agents live in overlapping generations as active adults and passive children. An agent belonging to group $i \in \{1, ..., N\}$ and being adult in period t takes as given her income, $y_{i,t}$, and chooses her number of children, $n_{i,t}$, to maximize

$$U_{i,t} = (1-\gamma)\ln(y_{i,t} - \rho n_{i,t}) + \gamma \ln(n_{i,t}), \tag{1}$$

where ρ is the (goods) cost per child and $y_{i,t} - \rho n_{i,t}$ is the agent's own consumption. Optimal behavior thus implies $n_{i,t} = (\gamma/\rho) y_{i,t}$.

Total output of group i is given by

$$Y_{i,t} = (A_{i,t} L_{i,t})^{\alpha} (P_{i,t})^{1-\alpha}, \tag{2}$$

where $L_{i,t}$ and $A_{i,t}$ denote the size and productivity, respectively, of the land available to society i in period t, and $P_{i,t}$ is the number of adult agents in the same society and period, each supplying one unit of labor. Output is distributed equally among adults, implying

$$y_{i,t} = \left(\frac{A_{i,t} L_{i,t}}{P_{i,t}}\right)^{\alpha}. \tag{3}$$

Agents die after the adult phase of life. Using $n_{i,t} = (\gamma/\rho) y_{i,t}$ and (3), adult population therefore evolves according to

$$P_{i,t+1} = P_{i,t} n_{i,t} = \frac{\gamma}{\rho} (A_{i,t} L_{i,t})^{\alpha} (P_{i,t})^{1-\alpha}. \tag{4}$$

3.1 Variations in levels of land productivity

First let $L_{i,t}$ and $A_{i,t}$ be constant at A_i and L_i. Then the economy converges to a steady-state equilibrium where the size and density of the population are constant. Writing the dynamics in (4) in terms of population density, $D_{i,t} = P_{i,t}/L_i$, gives:

$$D_{i,t+1} = \frac{\gamma}{\rho} A_i^{\alpha} (D_{i,t})^{1-\alpha}. \tag{5}$$

The steady-state density level associated with (5) can be written

$$D_i^* = \left(\frac{\gamma}{\rho}\right)^{\frac{1}{\alpha}} A_i, \tag{6}$$

which depends on A_i (but not L_i).

Similarly, writing the dynamics in (4) in terms of per-capita income in (3), $y_{i,t} = [(A_i L_i)/P_{i,t}]^\alpha$, gives:

$$y_{i,t+1} = \left(\frac{\rho}{\gamma}\right)^{\alpha} (y_{i,t})^{1-\alpha}, \tag{7}$$

and the steady-state density level associated with (7) can be written

$$y_i^* = \frac{\rho}{\gamma}, \tag{8}$$

which does not depend on A_i (or L_i).

Consider thus a world where different societies are endowed with different constant levels of land productivity, A_i, but are otherwise identical. From (6) and (8) follows that there is no steady-state variation in per-capita incomes, but societies with higher A_i have higher population densities.

This is how Ashraf and Galor (2011) motivate why they think that one should expect to observe no correlation between per-capita incomes and time since the Neolithic transition during the Malthusian era of development. In their own words, "variation in the onset of the Neolithic Revolution across the globe is exploited as a proxy for variation in the level of technological advancement during the time period 1–1500 CE." While this is probably the most natural starting point, it is not clear in terms of the model what *event* the Neolithic transition represents.

3.2 Variation in growth rates of land productivity

Next we instead let $A_{i,t}$ grow at a constant rate g_i (still letting land be constant). The joint dynamics for population density and productivity are now described by a simple two-dimensional system:

$$\begin{aligned} D_{i,t+1} = & \ \tfrac{\gamma}{\rho} A_{i,t}^{\alpha} \left(D_{i,t}\right)^{1-\alpha}, \\ A_{i,t+1} = & \quad (1+g_i) A_{i,t}. \end{aligned} \tag{9}$$

The associated dynamics for per-capita incomes become

$$y_{i,t+1} = \left[\frac{\rho(1+g_i)}{\gamma}\right]^{\alpha} \left(y_{i,t}\right)^{1-\alpha}. \tag{10}$$

It can be seen from (9) that population density converges to a balanced growth path where it grows at the same rate as productivity, i.e., rate g_i. Per-capita incomes on the balanced growth path are still non-growing and given by

$$y_i^* = \frac{\rho(1+g_i)}{\gamma}. \tag{11}$$

That is, societies with higher growth rates in land productivity have higher per-capita incomes on a balanced growth path.

Population densities will not be constant on the balanced growth path, but grow at the same rate as land productivity and increase in levels over time. Section A of the Appendix shows that per-capita incomes and population densities in any period $t \geq 0$ can be written

$$y_{i,t} = \left[\frac{\gamma\left(1+g_i\right)}{\rho}\right]^{1-(1-\alpha)^t} \left(y_{i,0}\right)^{(1-\alpha)^t}, \tag{12}$$

and

$$D_{i,t} = \left(\frac{\gamma}{\rho\left(1+g_i\right)}\right)^{\frac{1-(1-\alpha)^t}{\alpha}} \left(\frac{1}{y_{i,0}}\right)^{\frac{(1-\alpha)^t}{\alpha}} A_{i,0}(1+g_i)^t. \tag{13}$$

Note that $(1-\alpha)^t$ approaches zero as t goes to infinity, implying that initial conditions matter less with time.

It can easily be seen (and is shown in Section A of the Appendix) that both densities and per-capita incomes are increasing in g_i (for $t > 1$), as long as $y_i^* > y_{i,0}$. That is, if two societies are initially in a steady state with the same level of per-capita income (for example, the steady state associated with zero growth, $y_{i,0} = \rho/\gamma$), and experience the same increase in productivity growth (a Neolithic transition) but at different dates, then densities and per-capita incomes will be higher in any given later period for the society where growth rates increased earlier.

Moreover, the effects on densities are larger. For sufficiently large t, a society with larger g_i will have much larger $D_{i,t}$ but only mildly larger $y_{i,t}$, because g_i affects $D_{i,t}$ exponentially, and $y_{i,t}$ proportionally.

This version of the Malthusian model thus seems to explain the observed raw correlations shown in Figures 1 and 2, if the comparison is made between societies that have made a Neolithic transition and those that have not.

However, when comparing societies which all experienced a Neolithic transition several millennia ago, it is not clear what would make them look different, since they should be close to a balanced growth path associated with the same productivity growth rate, and thus have roughly the same per-capita incomes.

3.3 Illustration: three interpretations of the Neolithic transition

Figure 4 uses a simple numerical example to illustrate the different outcomes when the Neolithic transition is interpreted as either one of three events: a one-time increase in the *level* of land productivity; a one-time increase in the *growth rate* of land productivity; or the starting point of a *gradual* increase in the growth rate of land productivity. The first two interpretations are those considered in Sections 3.1 and 3.2 above. The four different panels in Figure 4 show the time paths for a few different variables, setting the period in which the Neolithic transition takes place to zero.

A one-time increase in productivity levels translates to a (big) one-period jump in growth rates, a temporary rise in per-capita incomes, and a permanent rise in densities. Across societies transiting at different points in time, there is little variation in terms of densities or per-capita incomes among those who passed the Neolithic long ago. Indeed, among those that passed recently, the relationship is such that those that a more recent Neolithic transition implies higher per-capita incomes, seemingly contrary to the facts.

A one-time increase in productivity growth rates translates to a permanent rise in per-capita incomes, and a transition from constant to growing population densities. Again, there is little variation in outcomes among those which passed the Neolithic long ago, but among those that passed recently a more distant transition is associated with higher per-capita incomes.

Consider finally the case where the Neolithic transition is the beginning of a gradual increase in productivity growth rates. With this interpretation, there is indeed variation in both per-capita incomes and population densities also among those that passed the Neolithic long ago. Assuming the rise is gradual enough, the predictions would thus be qualitatively consistent with the patterns in Figures 1 and 2.

Gradually rising growth rates also seems like a plausible consequence of the Neolithic transition, if we believe that the invention of agriculture resulted in new ways in which agents could innovate and experiment, ways which were not available in a hunter-gatherer environment, like improvements of plants and animals through selection and breeding. Some species could be imported from earlier developers, but distances and local climatic conditions may have made at least some local innovation needed. Moreover, it seems plausible that human societies became better at innovating over time. Since growth rates in land productivity in the long run determine growth rates in population, this is also consistent with observed accelerating post-Neolithic population growth rates worldwide (Kremer 1993).

In what follows, growth rates will increase over time by assumption, but it may not be too hard to generate these patterns endogenously in a richer setting.

However, a quantitative challenge remains, as explored next.

3.4 Gradually increasing productivity growth: a quantitative exercise

When variation in per-capita incomes is driven by variation in growth rates across societies, these growth rates also determine population densities in any given year. In that sense, the model can be quantitatively disciplined by choosing parameters to make it match observed variation in population densities, say in 1000 CE, and then examine how the associated variation in per-capita incomes implied by the model compare with those in the data.

First a little more precise notation is needed. Let the period in which society i enters the Neolithic transition be denoted $\tau_i > 0$. All societies have the same (possibly non-zero) growth rate before the Neolithic, denoted $\underline{g} \geq 0$, after which their growth rates increase linearly by $\Delta > 0$ per period until they reach a maximum level $\overline{g} > \underline{g}$. More precisely, the productivity growth rate of society i between periods t and $t+1$ equals

$$g_{i,t} = \min\left\{\overline{g}, \max\left\{\underline{g}, \underline{g} + \Delta(\overline{g} - \underline{g})(t - \tau_i)\right\}\right\}. \tag{14}$$

In a different setting, with results similar to those presented here, growth rates could fluctuate stochastically between $\overline{g}$ and $\underline{g}$ with a time dependent probability, which rises with time after period τ_i. That is, $g_{i,t}$ could equal $\overline{g}$ with probability $\phi_{i,t}$, and $\underline{g}$ with probability $1 - \phi_{i,t}$, where $\phi_{i,t} = \min\{1, \max\{0, \Delta(t - \tau_i)\}\}$.

Figure 5 shows the cross-sectional outcomes in 1000 CE for a simple numerical example, where growth rates evolve according to (14). The model is simulated for 500 societies ($N = 500$) and 360 periods. The timing of the Neolithic transition is set such that τ_i is uniformly distributed across all 360 periods, roughly corresponding to the data in Figures 2 and 3. We let each period correspond to 30 years, and the first society transit in -8000 (i.e., 8000 BCE). Following a Neolithic transition growth rates rise for 300 periods (9000 years), implying that $\Delta = 1/300$, so that the earliest society reaches its maximum growth rate around 1000 CE, the year when outcomes are measured in Figure 5. About 17% (60/360) of the societies are thus at a pre-Neolithic stage of development by 1000 CE.

Other parameters are set as in Section 3.5 below; see Table 5. First, γ is normalized to 0.5 and ρ to unity; these only affect levels of per-capita incomes and not gaps. The land share in the production function, α, is set to 0.4, as in Hansen and Prescott (2002). The pre-Neolithic growth rate, $\underline{g}$, is set to 0.5% per generation and the maximum growth rate, $\overline{g}$, is set to 5.5% per generation. The gap between these growth rates is chosen so that the model can match the distribution of population densities in 1000 CE.

The insight from Figure 5 is that this simple version of the model does not generate a good quantitative match with the data. When the growth rates are set to match the observed cross-country variation in population densities by 1000 CE, then the associated variation in per-capita incomes is much smaller than that observed in the data for the same year, around 0.05 log units compared to around 0.5 log units.

One conclusion could be that Angus Maddison's guesses about preindustrial per-capita income levels are not plausible. Another possibility is that the model explored here lacks some crucial component that is needed for it to generate realistic variation in per-capita incomes. The next section allows for two more changes to the standard Malthusian model, which do enable it to match the data also quantitatively.

3.5 Territorial competition and fertility constraints

This section alters the Malthusian framework explored so far in two different ways. First, land allocations are assumed to (partly) depend on population levels, in a way that allows the model to match observed trends in the size of empires. We implement this by letting land be allocated across the N societies such that in period t each society controls at least a share $(1-\lambda_t)/N$ of a unit-sized land endowment. The remaining fraction λ_t is contested and distributed in proportion to the relative (adult) population size of the societies. More formally, land holdings of society i in period t equal:

$$L_{i,t} = \frac{1-\lambda_t}{N} + \lambda_t \left(\frac{P_{i,t}}{\sum_{j=1}^{N} P_{j,t}} \right), \tag{15}$$

where λ_t increases linearly from $\underline{\lambda} \geq 0$ to $\overline{\lambda} > \underline{\lambda}$ over $1/\delta$ periods starting in period $\nu \geq 0$:

$$\lambda_t = \min \left\{ \overline{\lambda}, \max\{\underline{\lambda}, \underline{\lambda} + \delta(\overline{\lambda} - \underline{\lambda})(t-\nu)\} \right\}. \tag{16}$$

One can consider less mechanical formulations. For example, a society's relative levels of population (and/or technology) could determine its land share through a contest function, and an elite in each society could take actions which effect either or both of these inputs (see, e.g., Lagerlöf 2014 for such a setting). Also, rather than the N societies competing over the same land area of size λ_t, one could let them be located spatially in a manner that determines both which groups compete with one another, as well as the spread and timing of the Neolithic transition. While abstracting from that, the setting explored here might still be a useful starting point.[6]

The second extension is to impose an upper bound on the population growth rate. This slows down the speed of the convergence in per-capita incomes following the reallocation of territory as λ_t increases. We do this by letting fertility be capped at some maximum level, $\overline{n}$, i.e.,

$$n_{i,t} = \max \left\{ \frac{\gamma}{\rho} y_{i,t}, \overline{n} \right\}, \tag{17}$$

which is what maximizing (1) subject to $n_{i,t} \leq \overline{n}$ implies.

It is easy to see how a society which conquers land can experience a temporary rise in per-capita income levels, since land is an input in production. Eventually, land per agent falls as population expands, and per-capita

[6]There seem to be few explicit dynamic models of empire building, at least in a spatial sense, although there exists a large related literature on state building, e.g., Besley and Persson (2011) and Mayshar et al (2011, 2013).

incomes revert to their steady state levels, but the speed with which this happens depends on how fast population can grow. This is why these two extensions are complementary.

Moreover, the land conquests themselves depend on population levels, so initial population will come to matter: precisely those societies which undergo an early Neolithic transition have the highest population levels in period ν (when λ_t starts to rise), and can at that point start to build empires. As a result, they enjoy further population expansions, and greater territorial conquests, sustaining the rise in per-capita income levels a little further into the future (although not forever, since the world will eventually run out of territory).

To explore this model quantitatively, we follow the same parameterization strategy discussed in Section 3.4. The number of societies, N, is set to 500. The model is run for 360 periods, the first 300 of which – about 9000 years with 30 years per period – correspond roughly to the period of interest, 8000 BCE to 1000 CE. The periods in which the Neolithic transitions occur, τ_i, are uniformly distributed across all 360 periods. Post-Neolithic growth rates rise over a 9000-year period, or 300 periods, implying $\Delta = 1/300$. The earliest country to enter the Neolithic transition thus reaches its maximum growth rate around 1000 CE, the year when outcomes are measured.

The path for the contested land share, λ_t, is set so that, given how other parameters are calibrated, the model generates trends in the land size of the largest societies that match those in the data in Figure 3, which shows the size of empires as share of the Old World land area up until 1500 CE (i.e., excluding the New World land mass). Note that the largest empire starts to grow in size at a faster rate some time around 1000 BCE, and are still expanding by 1500, when the New World is discovered. To implement these paths, the increase in λ_t is set to occur between 1000 BCE and 1000 CE, i.e., over 2000 years, or approximately 67 periods. This implies $\delta = 1/67$, and $\nu = 234$, which with 30 years per period is roughly 7000 years after the initial period, 8000 BCE, i.e., 1000 BCE. Then $\underline{\lambda}$ is set to 0.05, to generate some modest growth in empires before 1000 BCE, and $\overline{\lambda}$ to 0.95, meaning that almost all territory is contested by 1000 CE.

The population growth rate is capped at 9% per generation ($\overline{n} = 1.09$), which can be compared to the fastest population growth rate between 1 CE and 1000 CE in the data. This occurred in Japan, which grew by a factor of 15, or about 8.5% per 30-year period. The corresponding maximum growth rate for the period 1000-1500 CE is about 10% per 30-year period and refers to the Philippines.

All societies are initially identical in terms of land holdings, and positioned in the first period on the balanced growth path associated with the pre-Neolithic growth rate, $\underline{g}$. Growth rates then start to rise in society i from period τ_i and on, as described by (14). The resulting changes in population structure are generated by $P_{i,t+1} = P_{i,t}n_{i,t}$, with $y_{i,t}$ given by (3), and $n_{i,t}$ by (17). The allocation of land, $L_{i,t}$, in each period follows from (15) with the contested share, λ_t, following the process in (16).

The main results from this version of the model are summed up in three graphs. Figure 6 shows the path of λ_t, and of the territories of the largest, and three largest, empires, in the data and as generated by the model. Whereas λ_t stops growing by 1000 CE, the transitory dynamics are still in place long after. The combined size of the three largest empires is still on an upward trajectory by the time the New World is discovered.

Figure 7 illustrates the time paths for some other key variables, and for two (out of $N = 500$) societies: one transits earliest of them all, in 8000 BCE, and the other in the middle of the distribution, around 500 BCE; the society that transits earliest also has the largest territory in any given period.[7]

Several insights can be gained from the time paths in Figure 7. The gap between the middle and early developers rises slowly up until 1000 BCE. This is driven by the accelerating productivity growth rate in the former,

[7] Because the model is deterministic no society ever overtakes another. This would change if, e.g., the productivity growth rates, $g_{i,t}$, had a stochastic component. Then early developers with bad luck would occasionally be overtaken by later developers with good luck.

and the gradual expansion of its land holdings at the expense of other societies. (Recall that all societies start with identical land holdings.) After 1000 BCE, as the contested share, λ_t, starts to increase, per-capita incomes in the early-transition society rise rapidly, as its territory expands; the later developer experiences an associated drop in territory and per-capita incomes. Note also that fertility (i.e., population growth) rates in the early-transition society are stuck at $\overline{n}$ from 1000 BCE, because their per-capita incomes are so large. This in turn dampens the downward pressure on per-capita incomes, allowing living standards to rise for several periods with expanding territorial conquests.

Figure 8 plots per-capita incomes and population densities in 1000 CE against the year of the Neolithic transition across all 500 societies. The data points are the same as in Figures 1 and 2. Per-capita incomes are higher for societies with earlier transitions. This is reflected by their territories being larger, but population levels not as much larger. This in turn is due to the upper bound on population growth, which binds for societies which expand their territories a lot, i.e. those with very early Neolithic transitions.

As a robustness check, Figure 9 illustrates what happens when the constraint on population growth is removed altogether: the model still does capture a big part of the cross-country variation in 1000 CE, in particular the comparatively elevated levels of per-capita incomes among the earliest developers, but not by as much as in the data. It also shows the effect of letting λ_t be constant at $\lambda_0 = \underline{\lambda} = 0.05$, thus removing the expansion of empires after 1000 BCE. This essentially brings the results back to the case where all results are driven by variation in growth rates alone, as in the case illustrated in Figure 5, except there territorial competition was completely absent ($\lambda_0 = \underline{\lambda} = 0$).

4 Conclusions

This paper has explored various extensions of a simple Malthusian model to examine if it can account for an observed positive correlation between time passed since the Neolithic Revolution and per-capita income levels in preindustrial times.

The main model innovation is that the Neolithic transition is interpreted as the starting point of a *gradual rise* in the *growth rate* of land productivity. Societies with earlier transitions then have faster growth rates at any given post-Neolithic stage of development, and thus higher living standards. At the same time, the associated differences in population densities are much larger, also consistent with the data.

While that simple setting can generate the sought positive correlation, it does not do equally well quantitatively. An extension is then considered that allows for territorial competition between societies, and imposes an upper bound on the rate population growth. That extended setting can account also quantitatively for both variation in per-capita incomes and population densities, as well as global time trends in the size of empires.

There are many potential objections to this exercise. It is debatable whether one should even try to explain the correlation that we observe in the data between preindustrial per-capita incomes and time passed since the Neolithic transition. One could argue that the per-capita income measures from Angus Maddison are at best informed guesses, and should not be taken seriously. Also, as argued by Ashraf and Galor (2011), the correlation becomes insignificant when entering continental controls.

However, the most important contribution of this paper is not about the facts themselves, but about the theory with which they are confronted. The exercise is to extend an otherwise very standard Malthusian framework. Importantly, no quality-quantity choice in children is allowed for. One direct upshot is a clearer interpretation of what the Neolithic transition might represent within a Malthusian model. Moreover, these extensions matter for the predictions regarding the effects of an early Neolithic transition on per-capita incomes several millennia later. In a Malthusian model, societies that transited 8000 BCE can be richer by 1000 CE, precisely because they transited so early. One conclusion is that observing a non-zero correlation between these two variables is not itself grounds for rejecting the Malthusian model.

APPENDIX

A Dynamics with constant growth in productivity

The task undertaken here is to find expressions for population density and per-capita income in period t (i.e., $D_{i,t}$ and $y_{i,t}$) in terms of exogenous parameters, initial conditions (state variables dated 0), and t itself. Land holdings (L_i) and land productivity growth rates (g_i) are constant but may differ across societies. Using the definition of population density, $D_{i,t} = P_{i,t}/L_i$, per-capita income in (3), and $A_{i,t} = A_{i,0}(1+g_i)^t$ gives

$$D_{i,t} = \left(\frac{1}{y_{i,t}}\right)^{\frac{1}{\alpha}} A_{i,0}(1+g_i)^t. \tag{A1}$$

If an expression for $y_{i,t}$ can be found, then (A1) gives us the sought expression for $D_{i,t}$.

To find $y_{i,t}$, first use (10) and (11) to write $y_{i,t+1} = (y_i^*)^\alpha \, (y_{i,t})^{1-\alpha}$. Letting $\ln(y_{i,t}) = x_t$ and $\ln(y_i^*) = x^*$, it follows that

$$x_{t+1} - x^* = (1-\alpha)(x_t - x^*), \tag{A2}$$

which is an easily solvable difference equation with solution

$$x_t = x^* + (1-\alpha)^t (x_0 - x^*), \tag{A3}$$

or

$$\begin{aligned} y_{i,t} &= \exp(x_t) \\ &= \exp(x^*)\exp\left[(1-\alpha)^t(x_0 - x^*)\right] = y_i^* \left(\frac{y_{i,0}}{y_i^*}\right)^{(1-\alpha)^t} \\ &= (y_i^*)^{1-(1-\alpha)^t} \, (y_{i,0})^{(1-\alpha)^t} . \end{aligned} \tag{A4}$$

Using (11) and (A4) we get (12). Now (A1) and (12) give (13).

The elasticity of per-capita incomes in period t with respect to the gross growth rate (from 0 to t) is

$$\frac{dy_{i,t}}{d(1+g_i)} \frac{(1+g_i)}{y_{i,t}} = 1 - (1-\alpha)^t. \tag{A5}$$

The elasticity of per-capita incomes in period t with respect to the gross growth rate (from 0 to t) is

$$\frac{dD_{i,t}}{d(1+g_i)} \frac{(1+g_i)}{D_{i,t}} = \frac{f(t)}{\alpha}, \tag{A6}$$

where

$$f(t) = \alpha t - 1 + (1-\alpha)^t. \tag{A7}$$

It is easy to verify that $f(1) = f(0) = 0$. Intuitively, population density in period 1 does not depend on the productivity growth rate between 0 and 1, only on population density and per-capita incomes in period 0. (To see this, note that $P_{i,1} = P_{i,0} n_{i,0} = (\gamma/\rho)\, P_{i,0} y_{i,0}$, and recall that L_i is constant and $D_{i,t} = P_{i,t}/L_i$.)

We also need to show that $f(t) > 0$ for all $t > 1$. One way to see that this holds is to note that

$$\begin{aligned} f(t+1) - f(t) &= \alpha + (1-\alpha)^{t+1} - (1-\alpha)^t \\ &= \alpha + (1-\alpha)^t \left[(1-\alpha) - 1\right] \\ &= \alpha \left[1 - (1-\alpha)^t\right] > 0, \end{aligned} \tag{A8}$$

for all $t \geq 1$. Thus, $f(t) > 0$ for all $t > 1$ ($t \geq 2$), since $f(1) = 0$.

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Variables	Year of the Neolithic Transition	Log GDP/Capita in 1 CE	Log GDP/Capita in 1000 CE	Log GDP/Capita in 1500 CE
Log GDP/Capita in 1 CE	-0.476	1.000		
	(0.007)			
Nb. Obs.	31			
Log GDP/Capita in 1000 CE	-0.645	0.491	1.000	
	(0.000)	(0.008)		
Nb. Obs.	29	28		
Log GDP/Capita in 1500 CE	-0.533	0.318	0.093	1.000
	(0.002)	(0.099)	(0.630)	
Nb. Obs.	32	28	29	
Log GDP/Capita, average three years	-0.736	0.770	0.582	0.779
	(0.000)	(0.000)	(0.001)	(0.000)
Nb. Obs.	28	28	28	28

Table 1: Cross-correlations between Year of the Neolithic Transition (negative values for the BCE era) and per-capita incomes in different years: 1 CE, 1000 CE, 1500 CE, and the average of those three years; p-values in parentheses.

Variables	Year of the Neolithic Transition	Log Population Density in 1 CE	Log Population Density in 1000 CE	Log Population Density in 1500 CE
Log Population Density in 1 CE	-0.672	1.000		
	(0.000)			
Nb. Obs.	137			
Log Population Density in 1000 CE	-0.551	0.941	1.000	
	(0.000)	(0.000)		
Nb. Obs.	155	157		
Log Population Density in 1500 CE	-0.481	0.883	0.965	1.000
	(0.000)	(0.000)	(0.000)	
Nb. Obs.	161	157	180	
Log Population Density, average three years	-0.588	0.966	0.991	0.971
	(0.000)	(0.000)	(0.000)	(0.000)
Nb. Obs.	137	157	157	157

Table 2: Cross-correlations between Year of the Neolithic Transition (negative values for the BCE era) and population densities in different years: 1 CE, 1000 CE, 1500 CE, and the average of those three years; p-values in parentheses.

Variables	State Presence 1-500 CE	State Presence 1-1000 CE	State Presence 1-1500 CE
Log GDP/Capita in 1 CE	0.612	0.483	0.427
	(0.000)	(0.006)	(0.016)
Nb. Obs.	31	31	31
Log GDP/Capita in 1000 CE	0.557	0.506	0.421
	(0.002)	(0.005)	(0.023)
Nb. Obs.	29	29	29
Log GDP/Capita in 1500 CE	0.394	0.496	0.568
	(0.026)	(0.004)	(0.001)
Nb. Obs.	32	32	32

Table 3: Cross-correlations between State Presence over various periods and per-capita incomes in 1 CE, 1000 CE, and 1500 CE; p-values in parentheses.

	Dependent variable is log GDP/capita in 1000 CE								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	World sample, no controls			World sample, geogr. controls			Drop New World, no controls		
Millennia since NT	0.035***	0.013***	0.009	0.036***	0.015***	0.011*	0.053***	0.016*	0.013
	(3.340)	(3.068)	(1.249)	(4.151)	(4.258)	(1.886)	(3.394)	(1.780)	(1.184)
Empire dummy, 1000 CE		0.285***	0.286***		0.267***	0.262***		0.275***	0.273***
		(3.672)	(3.512)		(4.088)	(3.981)		(3.190)	(3.087)
State Presence 1-1000 CE			0.044			0.058			0.055
			(0.875)			(0.955)			(1.033)
R^2	0.42	0.77	0.77	0.64	0.83	0.84	0.47	0.75	0.75
No. Obs.	29	29	29	29	29	29	24	24	24

Table 4: Log GDP per capita in 1000 CE regressed on the number of millennia passed since the Neolithic Transition and various controls. Columns (4)-(6) include the same geographical controls as in Ashraf and Galor (2011, Table 5), as detailed in the text; Columns (7)-(9) drop New World countries (Australia, Canada, Mexico, New Zealand, and the United States); t statistics are reported in parentheses; * indicates $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$

Parameter	**Value**	**Comment**
N	500	Number of societies
γ	0.5	Normalization
ρ	1	Normalization
α	0.4	As in Hansen and Prescott (2002)
$\underline{g}$	0.005	Some productivity growth before Neolithic Transition
$\overline{g}$	0.055	Variation in pop. densities in 1000 CE consistent with data
Δ	1/300	Rise in growth rates takes 300 periods (9000 years)
$\underline{\lambda}$	0.05	5% of land contested from start until 1000 BCE
$\overline{\lambda}$	0.95	95% of land contested by 1000 CE; matches Taagepera's data
ν	234	Rise in λ_t starting in period 234 (around 1000 BCE)
δ	1/67	Rise in λ_t takes 67 period (2000 years)
$\overline{n}$	1.09	Upper bound on pop. growth 9% per generation (max observed in the data)

Table 5: Parameter values for the baseline simulation. The first seven parameters (N, γ, ρ, α, $\underline{g}$, $\overline{g}$, and Δ) describe the model simulated in Section 3.4. The remaining ones refer to the extension in Section 3.5.

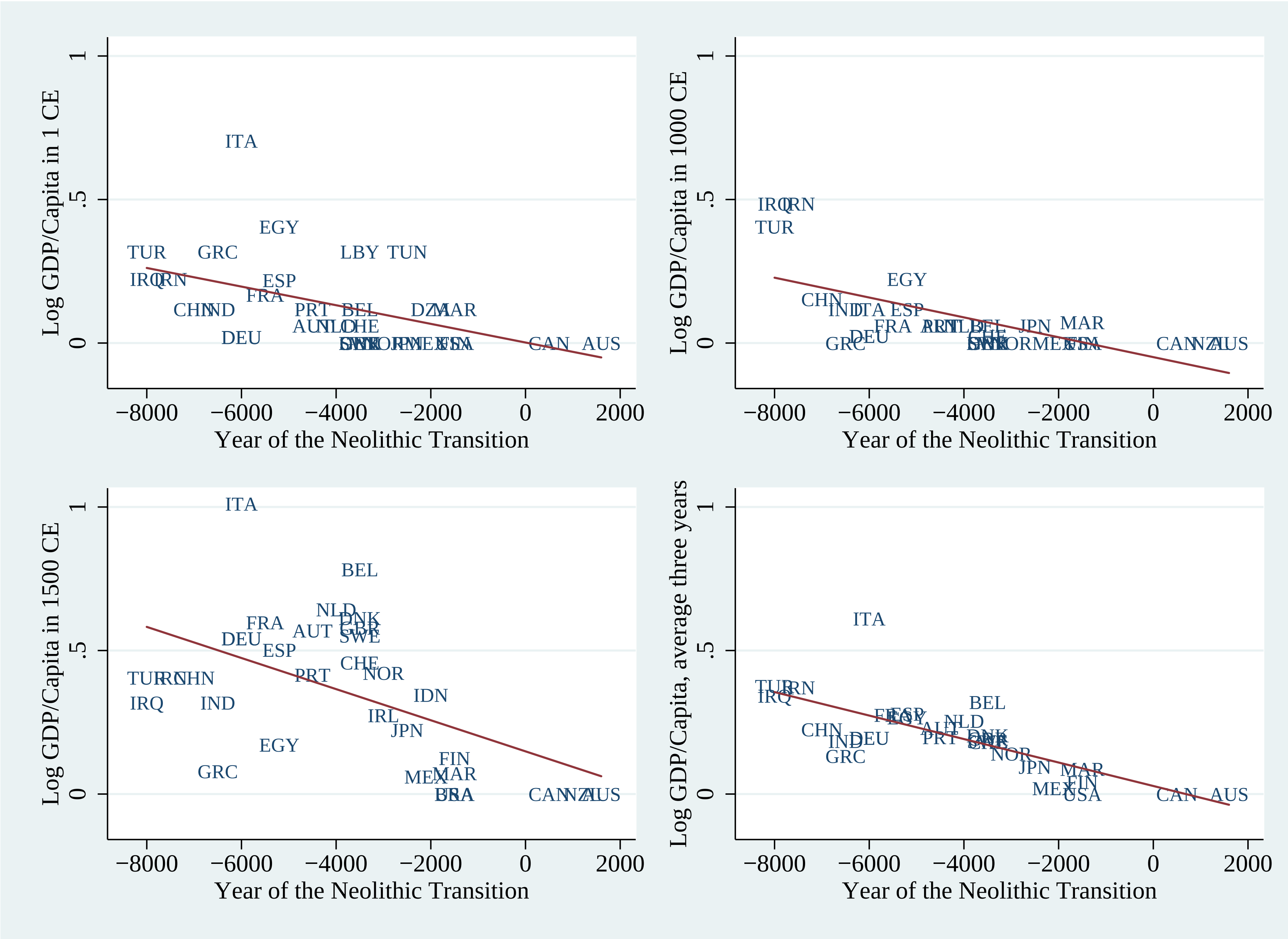

Figure 1: Year of the Neolithic Transition and per-capita incomes in 1 CE, 1000 CE, 1500 CE, and the average across those three years. Per-capita incomes have been normalized to one (zero in logs) for the poorest country in 1 CE.

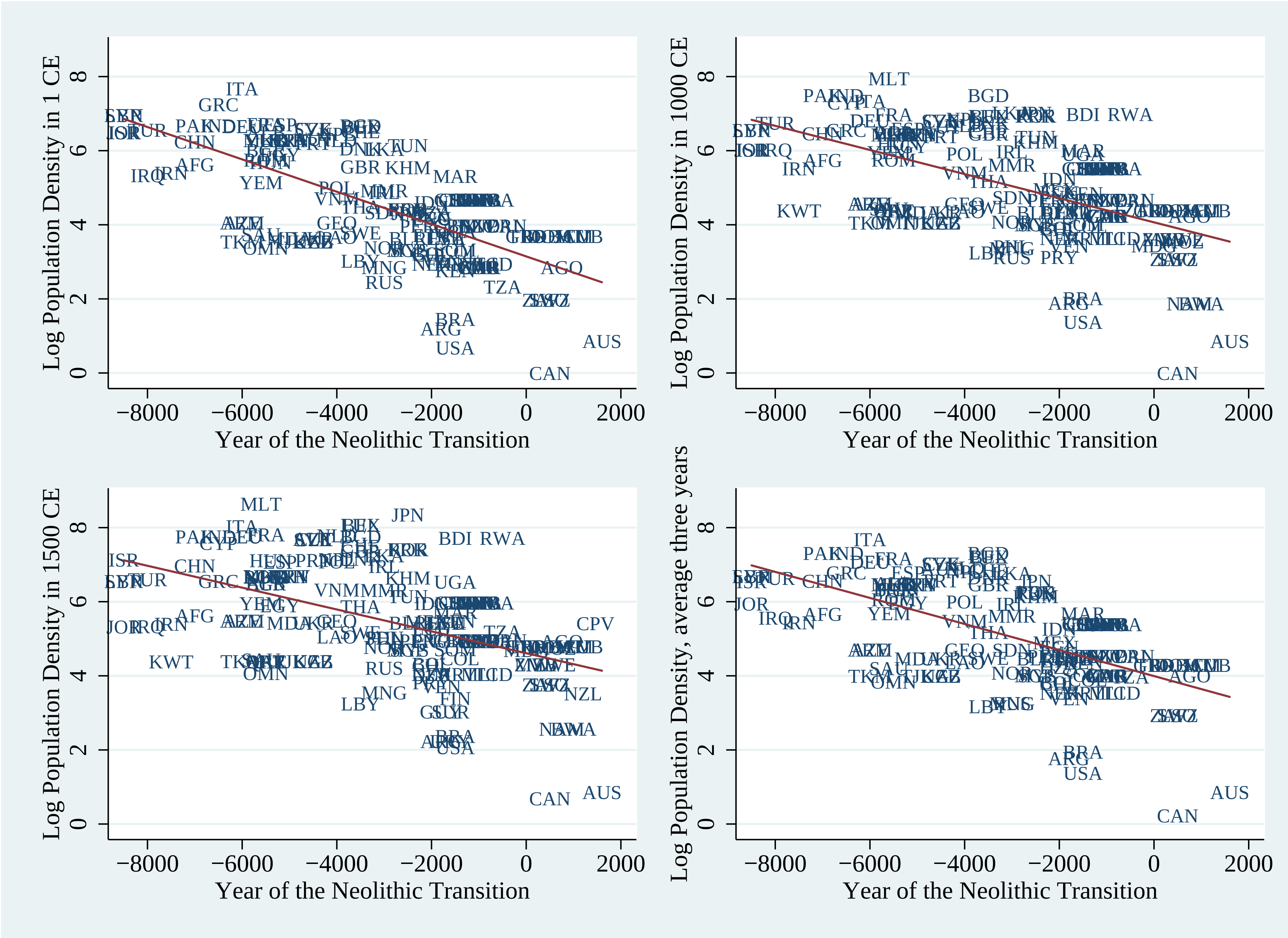

Figure 2: Year of the Neolithic Transition and population densities in 1, 1000, and 1500 CE. Density has been normalized to one (zero in logs) for the most sparsely populated country in 1 CE.

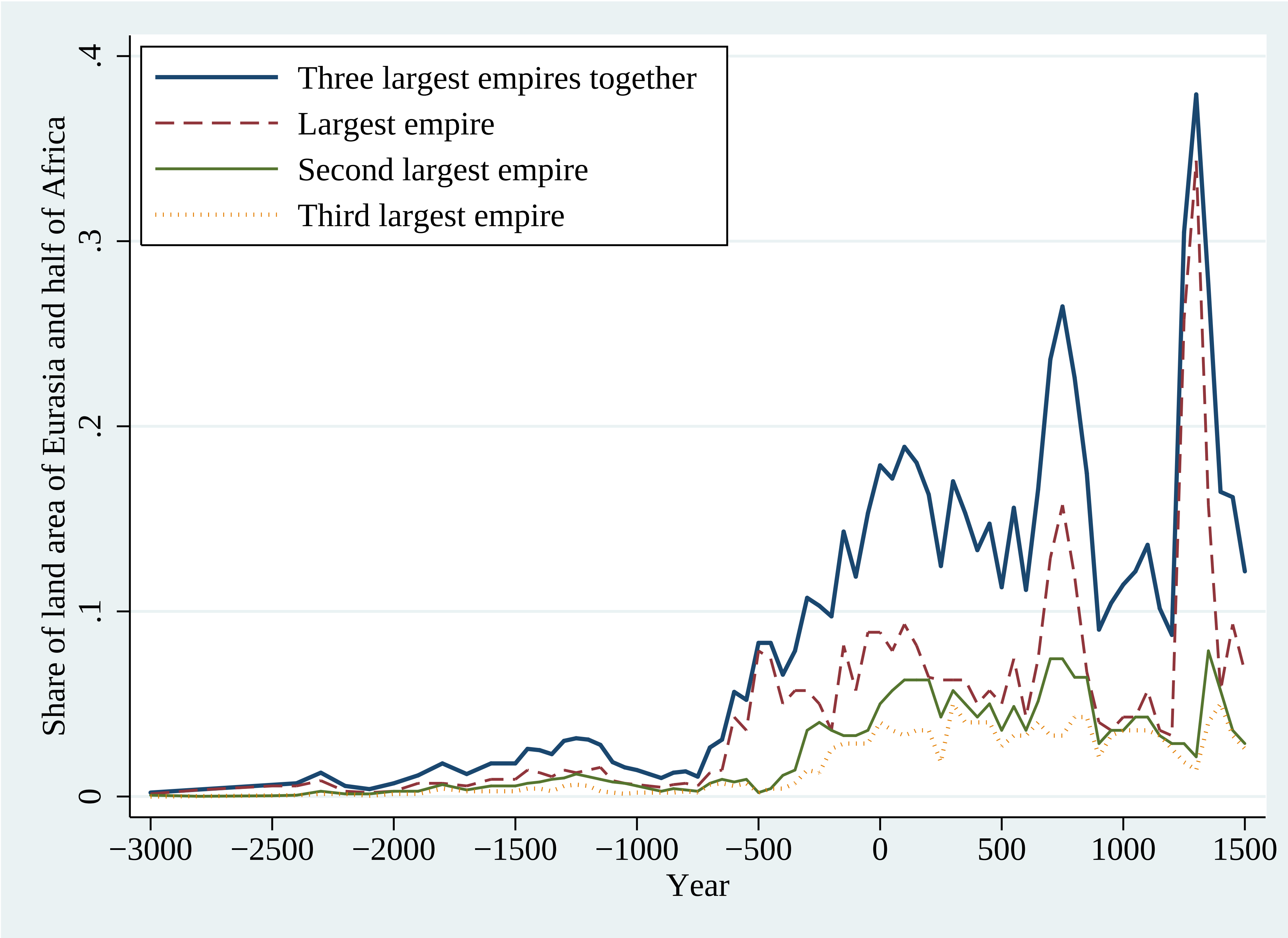

Figure 3: Trends in the size of empires.

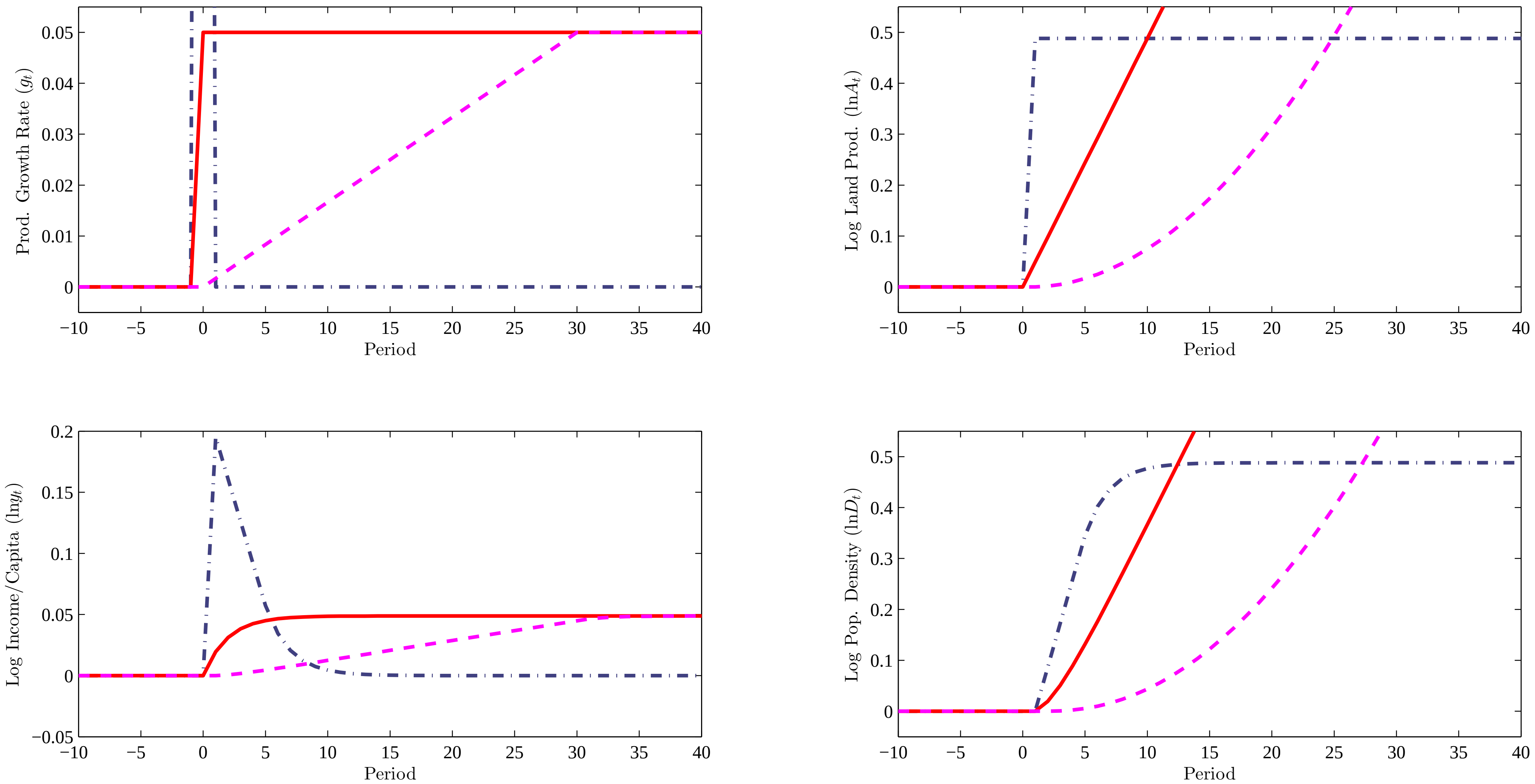

Figure 4: The time paths for some key variables for an economy which experiences a Neolithic transition in period 0, for different interpretations of what the Neolithic transition means.

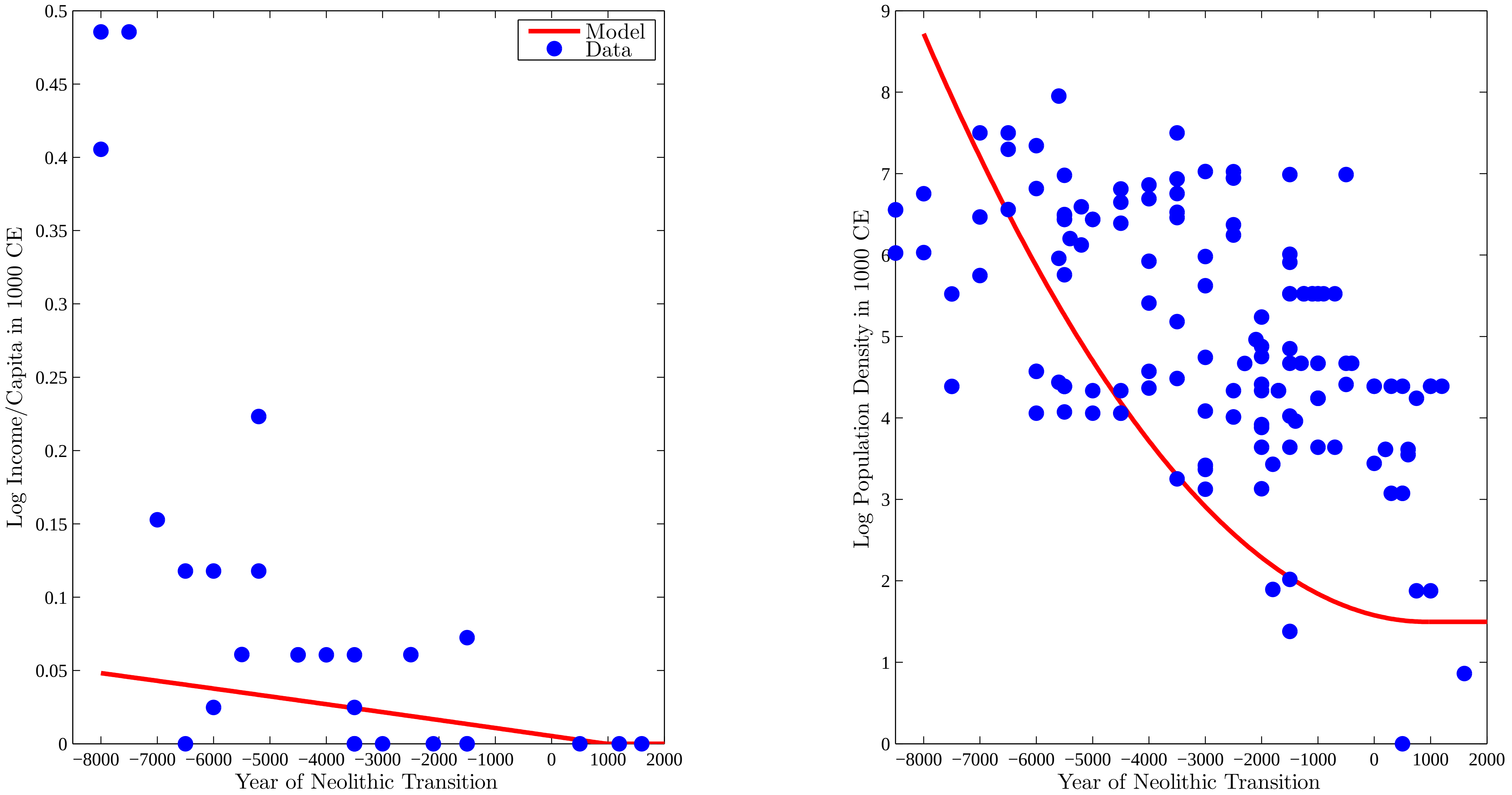

Figure 5: Illustration of the cross-sectional patterns for population densities and per-capita incomes with gradually rising productivity growth rates.

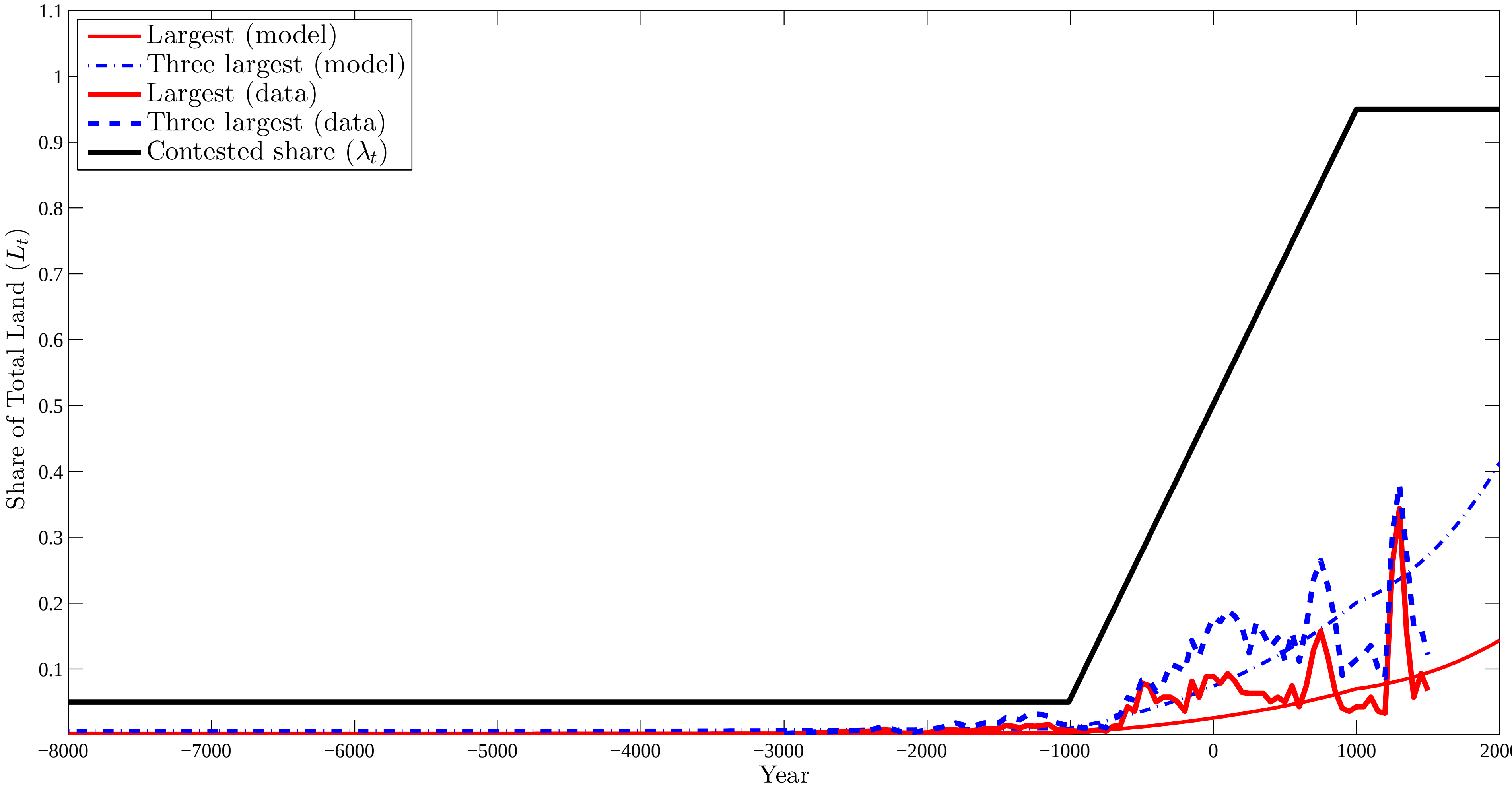

Figure 6: How the extended setting with territorial competition and fertility constraints fits the Taagepera data.

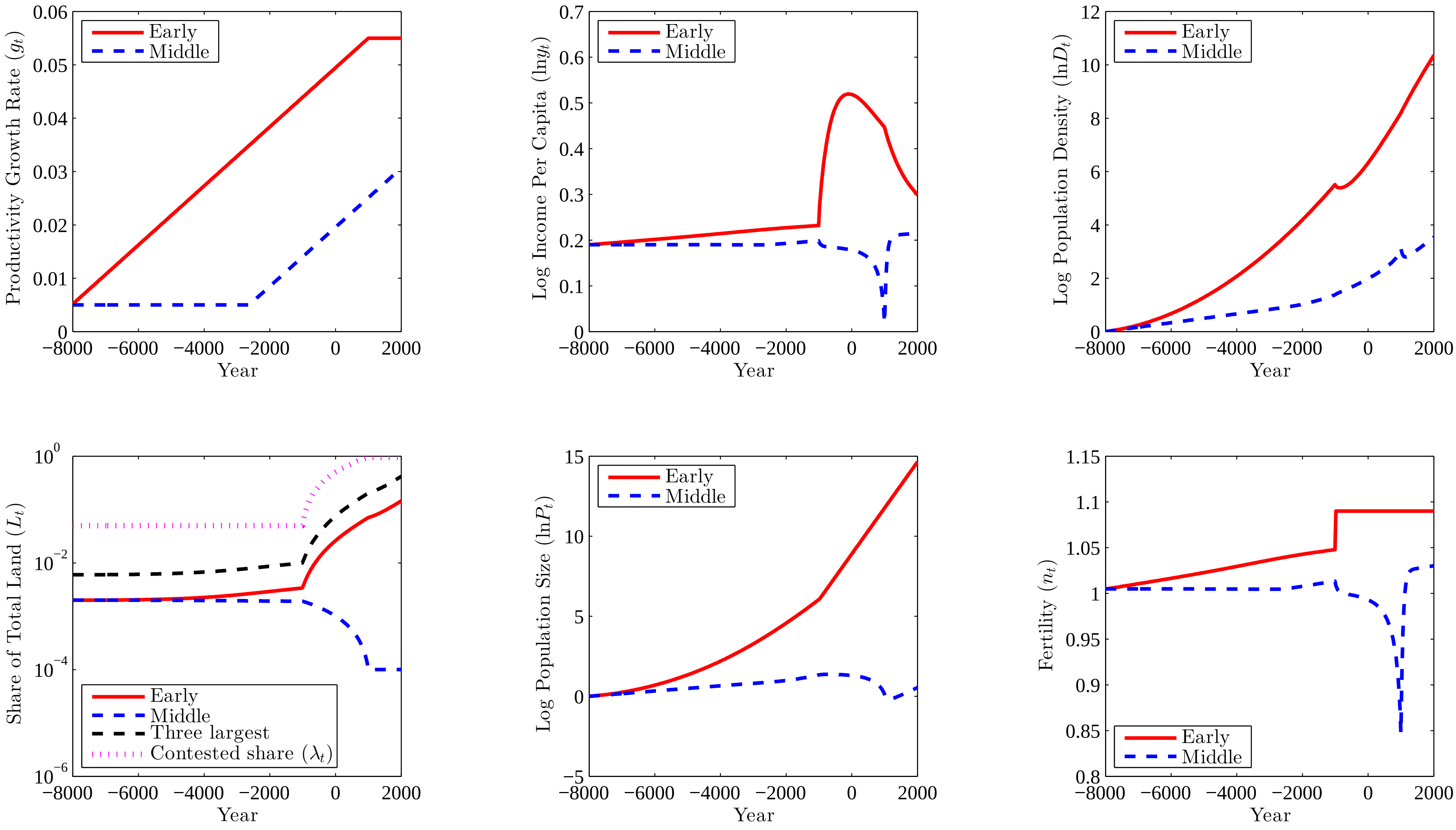

Figure 7: Time paths in the extended setting with territorial competition and fertility constraints.

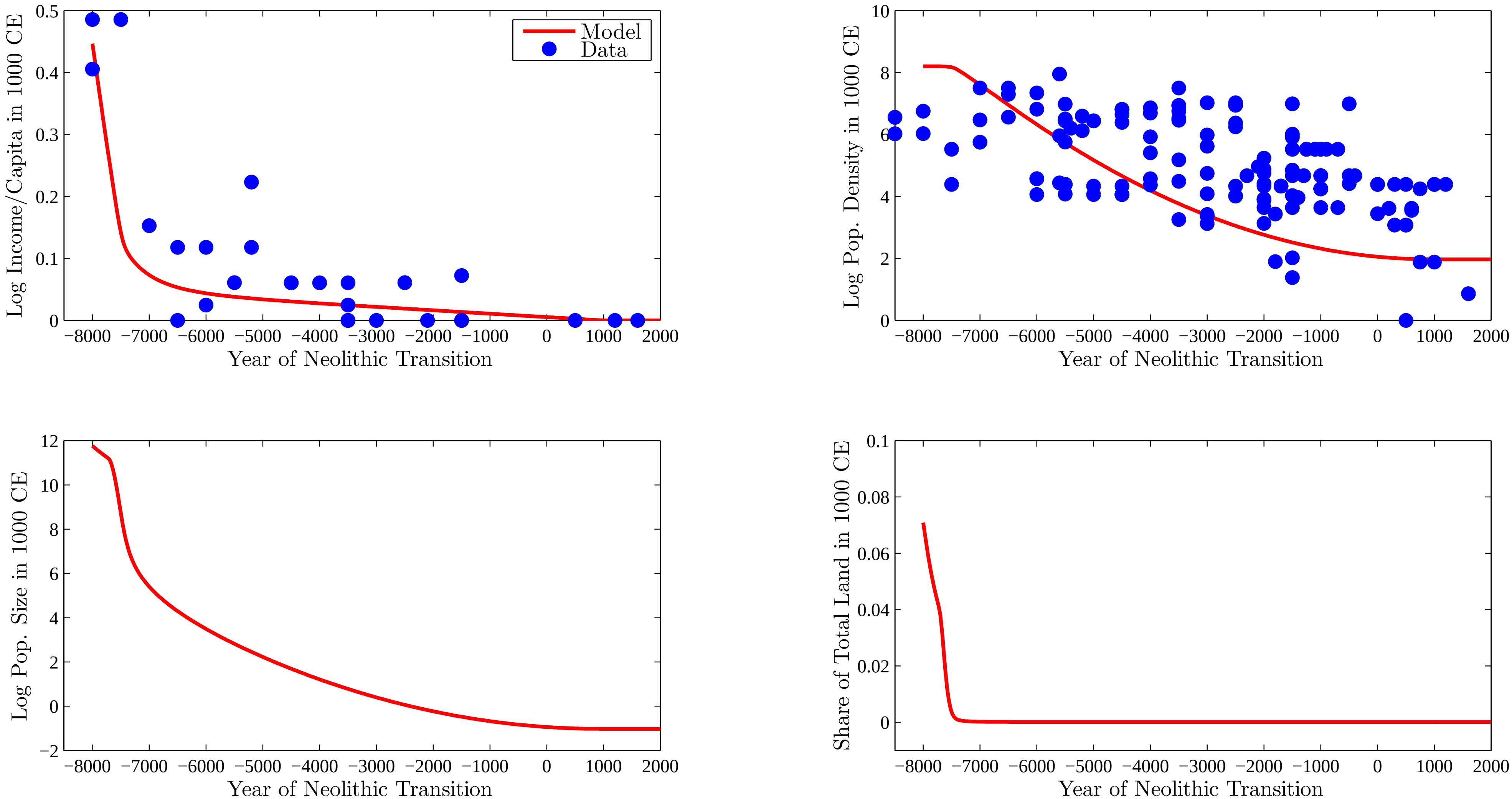

Figure 8: Cross-sectional patterns in 1000 CE in the extended setting with territorial competition and fertility constraints.

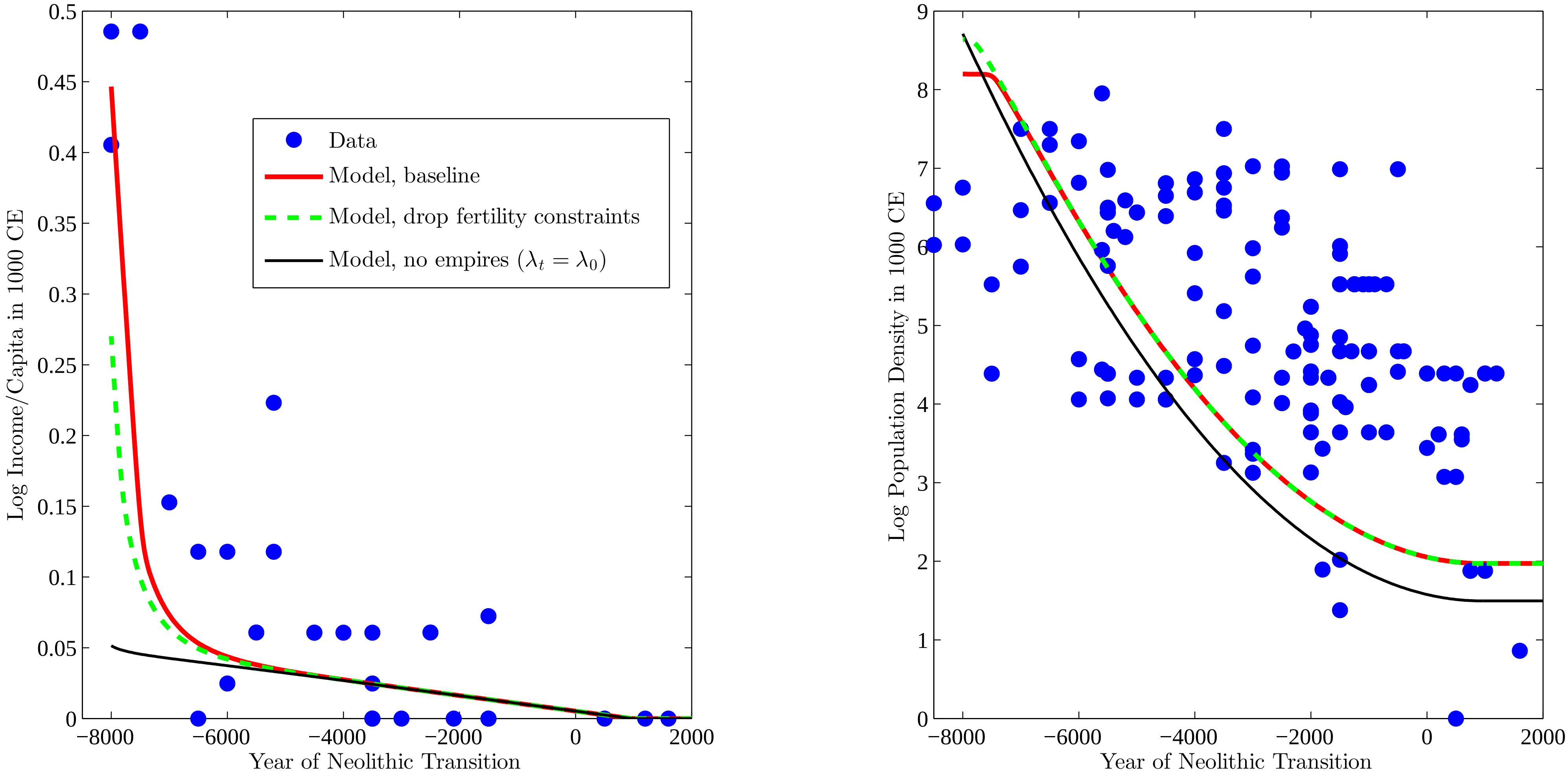

Figure 9: Cross-sectional patterns in 1000 CE under alternative assumptions.