Understanding per-capita income growth in preindustrial Europe

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Abstract: Fouquet and Broadberry (Journal of Economic Perspectives 2015) have recently compiled detailed time-series data over per-capita incomes for several European countries from as early as 1300, up to 1800. The time series are all volatile and highly persistent; per-capita incomes move in decades-long cycles of expansions and contractions. Contrary to what one might believe, this is not inconsistent with Malthusian stagnation. The current paper examines a Malthusian model with realistic life-cycle structure and stochastic growth in agricultural productivity. This model can generate per-capita income dynamics similar to the Fouquet-Broadberry data.

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1 Introduction

A common understanding of Malthusian models is that they predict per-capita incomes to be independent of land productivity. In a Malthusian world, if one economy has higher agricultural productivity than another, they should still have the same standards of living, as long as they are otherwise identical. Changes over time in land productivity should not translate into changes in living standards.

This contrasts with what we see in the data for several supposedly Malthusian societies. Fouquet and Broadberry (2015) have used various sources to compile annual per-capita GDP data for a handful of European countries from as early as 1300, up to 1800. These all display sizeable fluctuations in per-capita GDP over time, and on average the levels seem to trend upwards (see Figure 1).

However, those predictions refer to steady state outcomes in a non-stochastic environment, where levels or growth rates of land productivity are constant. If Malthusian economies are subject to productivity shocks, then per-capita incomes always differ from the steady-state level associated with a non-stochastic version of the model. In fact, as this paper tries to demonstrate, in a stochastic Malthusian environment, per-capita incomes can differ a great deal between any two economies, and they can be highly persistent over time within economies, on an order of magnitude comparable to Fouquet and Broadberry's data. Moreover, if *rates* of productivity growth trend upwards over time, so will *levels* of per-capita incomes.

To make this point, we examine a Malthusian model with realistic life-cycle structure: each period represents one year, permitting comparison to annual data. There is growth in land productivity, and that growth rate is stochastic and its mean is increasing over time.

The model otherwise relies on the standard Malthusian building blocks. There is only one sector, producing a good that we can interpret as food. When a land productivity shock raises per-capita incomes, fertility increases and mortality falls, leading to a population expansion. Because land is in fixed supply, per-capita incomes must subsequently decline. The model is thus by construction unable to generate *sustained* growth in per-capita incomes.

We then simulate 1000 model economies for 501 years, representing 1300-1800, for plausible parameter values. For each simulated economy we measure the mean, standard deviation, and serial correlation in per-capita incomes over the 501 years, and compare these to the Fouquet-Broadberry data.

Qualitatively, the model-generated paths are strikingly similar to those in the data, with several expansions and contractions lasting more than a century. Quantitatively, gaps in per-capita incomes, and levels of persistence and standard deviation over half a millennium, are all similar to what we see in the data.

The results hinge on a few assumptions that all seem realistic. First, part (but not all) of the productivity shocks must be persistent. This makes sense in this context. Consider, e.g., increases in land productivity following the introduction of New World crops, like the potato: once introduced it did not go away, so the rise in productivity was persistent.

Another important assumption, if the model is to produce the (seemingly non-Malthusian) upward trend in per-capita incomes, is that the expected productivity growth rate increases over time. This assumption is also realistic, because population growth rates increased over this period. Indeed, we set productivity growth rates to match the simulated population levels to data.[1]

Finally, the elasticities of fertility and mortality with respect to wages must not be too large. Intuitively, low elasticities amplify and prolong the effects of productivity shocks, by reducing the speed at which population levels adjust. But again, empirical studies do indicate elasticities low enough for the model to match the data.[2]

This paper relates to several papers tracing the origin of today's world income distribution to the transition of a few economies in Western Europe out of what is typically labeled Malthusian stagnation (e.g., Galor and Weil 1999, 2000; Jones 2001; Hansen and Prescott 2002; Lucas 2002; Galor 2005, 2008). Critics have pointed to the non-stagnant preindustrial environment in the Fouquet-Broadberry data as evidence against such theories. As discussed further in Section 2 below, it is in the context of that debate that this exercise becomes so important.

It is also true that variations on the standard Malthusian model can indeed alter its prediction about stagnant living standards. For example, in settings with multiple goods, and not only food, certain shocks can raise living standards permanently (Sharp et al. 2012, Voigtlander and Voth 2013, Dutta et al. 2018). Endogenous fluctuations in Malthusian models are studied by Dalgaard and Strulik (2015), who allow for investments in body mass and thus subsistence requirements.[3] These interesting extensions can be motivated independently of the lack of stagnation in the Fouquet-Broadberry data. The current exercise merely sheds light on how well a more standard Malthusian model can match those data.

This paper also relates to research on long life cycles in overlapping-generations frameworks, often in continuous time (e.g., Lee 1974, Boucekkine et al. 2002, de la Croix and Licandro 2013). These models are typically non-stochastic, but share some of the mechanisms through which shocks can propagate themselves in our simulations.

[1] Rising growth rates in productivity during the Malthusian era is also a feature of many unified growth models. For example, in Galor and Weil (2000) the rate of change in technology is an increasing function of population levels at the Malthusian stage of development.

[2] Recent empirical estimates of these elasticities can be found in Lagerlöf (2015) and Klemp and Møller (2016). For further details, see Section 5.1 below, and the Online Appendix.

[3] See the simulation of the Galor-Weil model by Lagerlöf (2006) for another example of endogenous cycles in a Malthusian context.

Several papers have calibrated growth models to produce a transition from stagnation to growth in per-capita incomes (e.g., Fernández-Villaverde 2001; Lagerlöf 2003, 2006; Bar and Leukhina 2010). This paper is probably the first to examine whether a purely Malthusian model, unable to generate sustained growth by construction, can generate transition-like time paths when adding shocks to it.

Finally, this paper is motivated by other evidence that the preindustrial world was Malthusian. Across regions defined by modern country borders, preindustrial levels of technology correlate more strongly and robustly with population densities than with living standards (Ashraf and Galor 2011). Moreover, the very same preindustrial economies displaying non-stagnant incomes in the Fouquet-Broadberry data seem to react in a Malthusian fashion to exogenous changes in per-capita incomes. Across Swedish counties in the 19th century, good harvests were associated with higher birth rates and lower death rates (Lagerlöf 2015). Similar patterns have been found for other Scandinavian countries in time-series data (Klemp and Møller 2016), and for England (Nicolini 2007, Crafts and Mills 2009, Kelly and Ó Gráda 2014, Klemp and Møller 2016).

The rest of this paper is organized as follows. Section 2 further discusses some of the debate on the importance of the Malthusian model for understanding human history. Section 3 summarizes some of the facts about per-capita incomes in preindustrial Europe that we learn from Fouquet and Broadberry (2015) and their sources. Section 4 sets up the model, first illustrating the main mechanisms in a stylized version in Section 4.1, and then presenting a setting realistic enough to be simulated in Section 4.2. Section 5 presents the simulation results and compares them to the data. Section 6 concludes.

2 The controversy

The relevance of the Malthusian model for interpreting human history has long been debated. One exchange of views flared up after the publication of Gregory Clark's book *A Farewell to Alms* (Clark 2007). Popularizing preceding theoretical work (e.g., Galor and Weil 2000, Galor and Moav 2002), it argued in favor of a Malthusian interpretation of history, pointing in particular to the absence of an upward trend in per-capita incomes prior to 1800.

A number of critical reviews followed, some gathered in a 2008 symposium published in the *European Review of Economic History.* For example, Persson (2008) cited rising preindustrial levels of urbanization in many European countries as evidence of growing per-capita incomes; his review was titled "the Malthus delusion." Voth (2008) made a similar point, citing, e.g., data on household inventories among the poor in 18th-century Britain. In his reply, Clark (2008) countered that the only direct and reliable measures of per-capita incomes, in particular from Britain, show scant evidence of any upward trend before 1800.

Some of the debate since has focussed specifically on how to reconstruct GDP data for preindustrial Britain, but even in that narrow context the conclusions are sensitive to the assumptions made. Broadberry et al. (2013) argue that per-capita incomes were growing long before 1800, while Clark (2013) argues that the patterns were rather cyclical, and that per-capita incomes by 1800 were at roughly the same level as in 1380.

The current paper enters this debate from a slightly different angle: even if we do accept that income levels in Britain (and/or other European economies) trended upwards over several centuries prior to 1800, this can be consistent with a model that is otherwise Malthusian, but where land productivity grows at accelerating rates. Indeed, using the same data for Britain as Broadberry et al. (2013), this paper proposes a more nuanced conclusion regarding the validity of the Malthusian model.

A slightly different critique of the Malthusian interpretation of history focusses not on trends per se, but rather on the absence of "stagnation" in preindustrial per-capita incomes. This view is expressed by Fouquet and Broadberry (2015, p. 227), who argue against what they call the "received wisdom," which "holds that the western European countries did not experience major phases of economic growth (or decline) prior to the Industrial Revolution." As a case in point, they quote Hansen and Prescott (2002, pp. 1214-1215), who write that "no significant permanent growth in living standards" took place before the Industrial Revolution.

Dutta et al. (2018, p. 359) concur with Fouquet and Broadberry (2015), and write that the alleged mischaracterization of preindustrial incomes as stagnant "has led to the development of theories that can accommodate long-run stagnation followed by explosive and then sustained economic growth. [...] Perhaps the best-known example is unified growth theory." As examples, they cite, e.g., Galor and Weil (1999), Jones (2001), and Galor and Moav (2002).

It stands to argue that few of the contributors to Unified Growth Theory would use the term "stagnation" to mean literally *constant* per-capita incomes. For example, Galor (2005, p. 180) suggests that "in the Malthusian epoch [...] income per capita fluctuated significantly within regions deviating from their sluggish long-run trend over decades and sometimes over several centuries."

However, to assess how *much* per-capita incomes can fluctuate in a Malthusian model when subjecting it to shocks, a useful first step seems to be to simulate it under plausible assumptions.

3 The data

Figure 1 shows log per-capita GDP for five economies (with periods indicated in parenthesis): England/Britain (1300-1800), Italy (1310-1800), the Netherlands (1348-1800), Portugal (1530-1800), and Sweden (1560-1800). The data are here reported in logs and normalized so that the logged series equal zero when averaged over time and across countries.

The source for this data is Fouquet and Broadberry (2015), who in turn rely on various in-depth studies for the respective countries.[4] They also report data from Spain but not on annual frequency so we do not use those numbers here. Compiling some of these data relies on certain assumptions about, e.g., elasticities and sector specific productivity levels. For the rest of this paper, we shall take the data as face value, but also remember that they probably come with some measurement error.

Four things can be noted from Figure 1. First, per-capita GDP was not constant over this period, but fluctuated a lot for all countries. Any model that seeks to replicate such time series arguably needs some stochastic component.

Second, because of these fluctuations, there are noticeable differences in GDP per capita across these countries in any given year, and some of these gaps stay when averaged over time. Portugal is poorest on average, and the Netherlands richest; cf. Table 5.

Third, average log GDP per capita across the five countries shows a mild upward trend over time. This is easiest to see for the period after 1560 when data is available for all five countries, but the same holds for earlier periods when considering only countries with available data. In short, levels for most countries grow a little over time.

Fourth, the time series show a great deal of persistence. A country's level of per-capita GDP in any given year is highly correlated with where it was in the previous year, and even two or three years back. Put another way, GDP per capita expands and contracts in long cycles, rather than jumping all over the place from one year to another.

The next section discusses how these patterns could be reconciled with a Malthusian model.

[4] The original sources are as follows: for England/Britain: Broadberry et al. (2011, 2015); for Italy (more precisely its central and northern parts): Malanima (2011); for the Netherlands (more precisely the province of Holland): van Zanden and van Leuwen (2012); for Sweden: Schön and Krantz (2012); for Portugal: Palma and Reis (2016). Fouquet and Broadberry (2015) also cite a paper by Reis, Martins, and Costa as source for Portugal, but the actual data is from Palma and Reis (2016); I thank Nuno Palma for pointing this out to me.

4 Theoretical framework

4.1 A simplified setting

To compare any annual data to model-generated time paths it helps to let each model period correspond to one year. Section 4.2 below considers such a model. However, it is useful to first illustrate some of the mechanisms driving the results using an overlapping-generations framework where agents live for only two periods. In the first phase of life, they are inactive children; in the second, adult, phase, they earn income, consume, and rear children. To fix notation, let agents who are adult in period t earn wage w_t, consume c_t, and rear n_t children.

There are two twists to the framework presented here, compared to most textbook Malthusian models: (1) an income-fertility elasticity less than one, and (2) sustained (but for the moment constant and non-stochastic) growth in land productivity.

To capture the first of these model innovations, let the cost of rearing n_t children be $qn_t^{1/\delta}$, where $q > 0$, and where $\delta \in (0, 1]$ measures the degree of returns to scale in child production. Most standard Malthusian models assume $\delta = 1$. We may interpret $\delta < 1$ (i.e., decreasing returns to scale in the production of children) as stemming from an implicit production function for child survival, where parental time and food are inputs, and where more children (less time per child) means that higher per-child input of the consumption good is needed to ensure each child's survival.

The budget constraint can now be written

$$c_t = w_t - qn_t^{\frac{1}{\delta}}. \tag{1}$$

Utility is logarithmic and defined over the number of children, and (adult) consumption, with weight $\widetilde{\gamma} \in (0, 1)$ on the former:

$$U_t = (1 - \widetilde{\gamma}) \ln (c_t) + \widetilde{\gamma} \ln (n_t) . \tag{2}$$

Maximizing (2) subject to (1), some algebra gives the agent's optimal fertility as follows:[5]

$$n_t = \gamma w_t^{\delta}, \tag{3}$$

where $\gamma = (\delta\widetilde{\gamma}/[q [1 - \widetilde{\gamma}(1 - \delta)]])^{\delta}$. That is, the elasticity of fertility with respect to wages equals δ.

Total output in period t equals

$$Y_t = (MA_t)^{\alpha} L_t^{1-\alpha}, \tag{4}$$

[5] The Online Appendix shows how to derive (9), which is the corresponding relationship in the extended model set up in Section 4.2.

where α is the land share of output, M is total land size, A_t is a land-augmenting productivity factor, and L_t is the size of the labor force, which is the same as the adult population.

Labor is paid its marginal product.[6] Using (4), and normalizing land size to unity ($M = 1$), the wage rate can thus be written

$$w_t = (1-\alpha)\left(\frac{A_t}{L_t}\right)^{\alpha}. \tag{5}$$

The second novelty compared to most other Malthusian models is the assumption of sustained growth in land productivity, here set to some exogenous and constant rate $g > -1$:

$$A_{t+1} = (1+g)A_t. \tag{6}$$

Each adult agent has n_t children—all of whom are assumed (for now) to survive until adulthood—and since all agents die after the adult phase of life, the labor force evolves according to $L_{t+1} = n_t L_t$. Forwarding (5) to period $t+1$, and applying (3), (6), and $L_{t+1} = n_t L_t$, some algebra gives a first-order difference equation for the wage rate:

$$w_{t+1} = \left(\frac{1+g}{\gamma}\right)^{\alpha} w_t^{1-\alpha\delta}, \tag{7}$$

which has a unique and stable steady-state equilibrium, defined by

$$\overline{w} = \left(\frac{1+g}{\gamma}\right)^{\frac{1}{\delta}}. \tag{8}$$

Two qualitatively important insights can be gained from (8). First, the steady-state wage rate, $\overline{w}$, is higher in economies with faster productivity growth, higher g. This is quite intuitive. When the wage rate is constant at $\overline{w}$, the ratio A_t/L_t is also constant; recall (5). Thus, population grows at the same (gross) rate as land productivity, $n_t = 1+g$, which from (3) requires higher w_t in steady state. In other words, for population to keep up with faster productivity growth, living standards must be higher.

The second insight from (8) is a corollary of the first. The positive effect on $\overline{w}$ from increasing g is stronger when δ is small. Intuitively, the less elastic is fertility to changes in wages, the more wages must increase in response to an increase in productivity growth to keep productivity-population ratio constant. If δ is small (and it seems to be at least less than one in the data; see Section 5.1 below), then small differences in g can generate large gaps $\overline{w}$.

This may explain how two otherwise identical Malthusian economies can have different living standards, with relatively modest differences in productivity growth rates. Moreover,

[6] Output not paid to labor is here implicitly assumed to be allocated to a landowning elite of small fixed population size, playing no role in the rest of the analysis.

patterns like those in Figure 1 could possibly occur if growth rates fluctuate over time. To explore this possibility requires that these mechanisms are nested in a less stylized framework, a task undertaken in the next section.

4.2 A more realistic setting

4.2.1 Fertility

Consider now an overlapping-generations model where agents live for at most T periods. They are reproductive from period $\underline{B}$ to $\overline{B}$, and earn wages from period $\underline{B}$ to R, where $1 < \underline{B} < \overline{B} < R \leq T$.

In each model period t, an agent in period $j \in \{\underline{B}, ..., R\}$ of life earns a wage $w_{j,t}$, which determines the number of children conceived in that period (born in the next), analogously to (3):

$$n_{j,t} = \gamma_j w_{j,t}^{\delta}, \tag{9}$$

where $\delta > 0$ is the elasticity of fertility with respect to wages, and $\gamma_j > 0$ is here an age-specific parameter, such that $\gamma_j = 0$ for $j \notin \{\underline{B}, ..., \overline{B}\}$.

The Online Appendix presents a simple model, which generates the behavior postulated in (9), as well as consumption of all agents (including those who do not earn incomes). For the purpose of the current modeling exercise, we do not need to know where the behavior described in (9) comes from.

4.2.2 Production and land productivity

In any period t, total output of a single good, Y_t, is produced using a Cobb-Douglas production function, with land and effective labor as inputs, similar to the model in Section 4.1:

$$Y_t = (X_t A_t)^{\alpha} L_t^{1-\alpha}, \tag{10}$$

where L_t is effective labor (explained further below), α is the land share of output, and the amount of land is (again) normalized to unity. Land productivity in period t equals $X_t A_t$, where X_t and A_t are subject to temporary and permanent shocks, respectively. These shocks are distributed as follows:

$$\ln(X_t) \sim N(0, \sigma_X), \tag{11}$$

and

$$\ln(A_{t+1}) - \ln(A_t) \sim N(\mu_t, \sigma_A), \tag{12}$$

where μ_t is the expected productivity growth rate, which is assumed to be time-dependent (but non-stochastic), allowing mean growth in land productivity to change over time. The parameters σ_X and σ_A denote the standard deviations in the temporary and permanent shocks, respectively. Temporary shocks could represent fluctuations in weather. Permanent shocks could capture innovations to agricultural technology, or the effects of newly introduced crops; see Nunn and Qian (2011) for evidence of a positive effect on population levels from the introduction of potato.

In any model period t, let $P_{j,t}$ be the population in the jth period of life, and let each age group $j \in \{\underline{B}, ..., R\}$ supply one unit of labor. Effective labor is determined by a CES aggregation function, allowing flexible substitutability between labor inputs of the different working age groups:

$$L_t = \left[\sum_{j=\underline{B}}^{R} \beta_j P_{j,t}^{\rho} \right]^{\frac{1}{\rho}}, \tag{13}$$

where $\rho < 1$, and $\sum_{j=\underline{B}}^{R} \beta_j = 1$. The most standard assumption might be that $\rho = 1$, and that β_j is constant across age groups, implying perfect substitutability between labor supply of different cohorts. The alternative assumption ($\rho < 1$, and β_j being different across cohorts) can be interpreted as different age cohorts performing different work tasks, perhaps because older workers have more experience, and younger workers more physical strength. This allows the age distribution in any given period to have an effect on output, and thus incomes, and reproduction.

4.2.3 Wages

Recall that an agent in period j of life earns wage $w_{j,t}$, which equals the marginal product of that age group's labor input. Using (10) and (13), some algebra shows that

$$w_{j,t} = \frac{\partial Y_t}{\partial L_t} \frac{\partial L_t}{\partial P_{j,t}} = (1-\alpha) \frac{Y_t}{L_t} \beta_j \left(\frac{L_t}{P_{j,t}} \right)^{1-\rho}. \tag{14}$$

Although it does not matter for the analysis, we here implicitly assume that the output not paid to landowners (which can be seen to equal αY_t) is allocated to landowners, who we can think of as old and non-active agents, using their income for consumption.[7]

4.2.4 Population dynamics

The new-born population in period $t+1$, $P_{1,t+1}$, is made up of children conceived in period t. A reproductive agent in the jth period of life conceives $n_{j,t}$ children, and since there are

[7] See the Online Appendix. Alternatively, land income could be allocated to a class of elite agents of small and fixed size, rearing only one offspring per agent, and using the reminder of their income for consumption.

$P_{j,t}$ in each such cohort, it follows that

$$P_{1,t+1} = \sum_{j=\underline{B}}^{\overline{B}} n_{j,t} P_{j,t}. \tag{15}$$

For all other cohorts, population levels evolve according to

$$P_{j+1,t+1} = s_{j,t} P_{j,t}, \tag{16}$$

where $s_{j,t} \in [0, 1]$ denotes the rate at which agents survive from the jth period of life to the next (and from model period t to the next). We describe $s_{j,t}$ further below.

4.2.5 Total population and per-capita incomes

Let total population be denoted

$$P_t = \sum_{j=1}^{T} P_{j,t}. \tag{17}$$

For any given levels of output, Y_t, and total population, P_t, the economy-wide per-capita income level becomes

$$y_t = \frac{Y_t}{P_t}. \tag{18}$$

In what follows, we shall let y_t in the model correspond to GDP per capita in the data.

4.2.6 The survival rate

The model allows for three mortality factors: starvation, disease, and age. We want to incorporate each of these, since they may all to some extent have impacted per-capita income dynamics in the European data. For example, disease shocks could be one factor contributing the volatility in per-capita incomes. To that end, the survival rate is defined as

$$s_{j,t} = s_t^y s_t^d s_j^{\text{age}}, \tag{19}$$

where s_t^y, s_t^d, and s_j^{age} all lie on $[0, 1]$, and represent survival from starvation, disease, and age, respectively. Survival from starvation (s_t^y) is specified as

$$s_t^y = \min \left\{ 1, \left(\frac{y_t}{\overline{y}} \right)^{\kappa} \right\} \in (0, 1], \tag{20}$$

where y_t is per-capita output in (18), $\overline{y}$ is the corresponding (non-stochastic, non-growing) steady-state level of y_t, and κ is a parameter measuring the elasticity of the survival rate with respect to falls in per-capita incomes. In this formulation, the mortality effects are present only when living standards are sufficiently low. This is broadly consistent with data from

19th-century Sweden, where good harvests do not lower mortality much, but bad harvests raise mortality (Lagerlöf 2015). It also seems intuitive that mortality from malnutrition is constrained to zero when food intake is above some threshold level. Here that threshold is set at the level associated with a Malthusian steady state absent shocks and starvation.[8]

The disease component in (19) is defined as

$$s_t^d = \exp\left(-\phi m_t^2\right) \in (0, 1], \tag{21}$$

where $m_t \sim N(0,1)$ is a mortality shock, such that larger deviations of m_t from its zero mean imply lower survival rates, and ϕ is a parameter capturing the size of the effect of these shocks. It can be shown that $E(s_t^d) = (1+2\phi)^{-.5} \approx 1 - \phi$, so ϕ is approximately the expected annual death rate from disease.

Finally, the age component in (19), s_j^{age}, varies with age, but is constant over time. It is set to match data from Sweden, as explained below.

5 Quantitative analysis

5.1 Benchmark parameter values

To generate simulated time paths to compare to data we first need to make assumptions about parameter values. Most of these are summed up and explained in Table 1.

First, the life-cycle parameters $\underline{B}$, $\overline{B}$, R and T can be set very intuitively. Agents live for at most $T = 90$ years, reproduce between life periods $\underline{B} = 15$ and $\overline{B} = 49$, and work from $\underline{B} = 15$ to $R = 70$, all conditional on survival.[9]

The land share of output (α) is set to 0.4, as in Hansen and Prescott (2002).

The reproduction parameters (γ_j), the age-dependent weights in the production function (β_j), and the elasticity of substitution between labor inputs of different age groups (ρ), are set to jointly match age-specific fertility and wage data from Sweden. Fertility data are averages from 1751-1800, and the wage data refer to agricultural workers in 1940 (the earliest year for which such age-wage data is available, but Sweden was at the time particularly dependent on its agricultural sector due to the second world war); see the Online Appendix for further details. Values of γ_j and β_j are shown in Table 3 for three age groups. The model-data match is illustrated in Figure 2.

[8] To be precise, we let $\overline{y}$ be given by the level derived in the Online Appendix, which applies to the case with unit elasticity of fertility ($\delta = 1$), enabling us to easily find an analytical expression for $\overline{y}$.

[9] Setting $\overline{B} = 49$ means that, in the last period of life in which an agent can *conceive* a child, the agent's *age* is 48; the agent is thus in the 49th year of life. The child is born in the 50th period of the parent's life, when the parent is of age 49. Similarly, agents can start to conceive (a small but positive number of) children when they are of age 14 (in the 15th year of life).

Consider next the elasticity parameters, δ and κ. Klemp and Møller (2016, Appendix B) provide a summary of estimated elasticities of fertility and mortality with respect to wages. These are so-called long-run elasticities (sums of elasticities with respect to wages at various time lags), and based on aggregated time-series data over Crude Birth and Death Rates (i.e., total births and deaths over total population), mostly from England and starting as early as 1540. Estimates of fertility elasticities range from .12 to .32, and mortality elasticities from $-.47$ to $-.08$; most studies find numbers at the lower end of those intervals.

Since many of these are based on aggregate data one may worry about causality; part of the variation could be driven by economic activity declining in periods when agents reduce child rearing for other reasons than direct Malthusian checks, e.g., disease or war. Partly addressing such concerns, Lagerlöf (2015) exploits cross-county harvest fluctuations in Sweden 1816-1856. Those elasticity estimates are somewhat smaller: .1 and $-.09$ for fertility and mortality, respectively.

The Online Appendix describes in more detail how to compare the model-generated survival and fertility rates to these estimates. In short, we compute the CBR and CDR from our simulated data. We then regress the logarithm of these variables on logged wages at various lags (see the Online Appendix). We set $\delta = .15$ and $\kappa = .01$, putting the elasticities estimated from the simulated data at around .13 and $-.2$, for birth and death rates, respectively. These fall within the ranges of existing empirical estimates.

We set $\phi = .01$, implying that the expected annual death rate from disease is about 1%.

Finally, s_j^{age} is set to make $E(s_t^d)s_j^{\text{age}} \approx (1-\phi)\, s_j^{\text{age}}$ roughly fit mortality data by age from Sweden 1751-1800, with s_{90}^{age} set to zero, so that all agents who survived for $T = 90$ periods die with certainty after that; see Table 2.

The time-dependent expected growth rate in land productivity, μ_t, is set to increase from 0% per year in 1300 to 1.25% in 1800, with more of the increase coming toward the end of that period. This makes the model match the double exponential trends in population levels for the five countries, as shown in Figure 3, and also implies a rising trend in levels of GDP per capita in the simulations.

The parameters measuring the dispersion in the shocks in (11) and (12), σ_X and σ_A, are set so that the simulations generate moments of $\ln y_t$ measured over 501 years as close as possible to (or with at least some overlap with) the corresponding numbers in the data. This is explained in connection to the results in Figure 6 below.

5.2 Initial conditions

We need to set start values so that the initial distributions are close to the steady-state distributions to which they would eventually converge absent the trend in expected productivity growth. Otherwise, the model might artificially generate high persistence, due to a standard convergence process over the first several periods. To that end, we set initial values at the steady-state levels associated with a deterministic version of the model, with unit fertility elasticity; analytical expressions for these initial values are derived in the Online Appendix.

We then simulate the model (without productivity growth, $\mu_t = 0$, but with all other parameters set as in the benchmark case above) over 500 periods before starting to measure outcomes. The 501st simulated period represents the year 1300.

5.3 Simulation results

Among 1000 simulated economies under our benchmark parametrization, Figure 4 shows the time paths of $\ln y_t$ for the first four; recall that y_t in (18) corresponds to GDP per capita in the data. The paths are normalized to equal zero when averaged over time and across simulated economies, similar to the data in Figure 1. As seen, in any given year we observe large gaps in $\ln y_t$ across these four economies, and each of them displays long cycles of expansions and contractions over time. The patterns are strikingly similar to those based on actual data in Figure 1. All the economies are parametrically identical, with all shocks drawn from the same distributions, so the differences between them are driven only by different realizations of the shocks.

Recall also that the model which generates the paths is stagnant by construction, in the sense that per-capita incomes cannot exhibit sustained growth; each of the growth spurts in Figure 4 is eventually followed by a decline. Not knowing this, one could easily interpret some of the growth phases as break-outs from stagnation.

The mean of $\ln y_t$ across all 1000 runs shows a mild upward trend in Figure 4. This is generated by the upward drift in productivity growth rates, as captured by the rise in μ_t. To understand why, recall the simplified Malthusian model in Section 4.1, where a higher productivity growth rate is associated with higher per-capita incomes (and wages) on the balanced growth path; cf. (8). When this growth rate increases gradually over time the result is a gradual rise in per-capita incomes.

While Figure 4 shows just a few random paths, Figure 5 shows the 5th and 95th percentiles of $\ln y_t$ in any given year among the 1000 simulated economies. That is, in any given year, 90 percent of the simulated economies fall between these two percentiles. The corresponding paths for the five countries in Figure 1 are also shown. The paths sometimes fall outside of the interval, but mostly within. As shown in Table 4, there is variation across the countries, but roughly 5% of all the country-years in the data fall below the 5th percentile, and about 3% above the 95th.

The simulations allow us to examine how likely we are to observe a country as "extreme" as those we have data for. For example, in Table 4 we see that Portugal's GDP per capita falls below the 5th percentile in about 29% of the years, which seems high. Among the 1000 simulated economies, only about 2.1% had an experience as poor as that of Portugal (i.e., falling below the 5th percentile in 29% of the years, or more). However, the probability that we should see at least one such country if we draw five at random is $1-(1-.021)^5 \approx 0.1$. That is, we would expect to draw a country as poor as Portugal (by this particular measure) with 10% probability.

Figure 6 gives a different picture of how well the model can match the data, by displaying histograms over some time-series moments of $\ln y_t$: mean, standard deviation, and serial correlation coefficients at one- and two-year lags, each calculated over the 501-year period representing 1300-1800. The figure shows the corresponding numbers in the data as well, also displayed in Table 5, together with the 5th and 95th percentiles of the histograms from the simulations.

Consider first the top-left panel of Figure 6, which shows the distribution of means. Reflecting the different realizations of the shocks, some of the simulated economies have higher means in $\ln y_t$ than others. In principle, we can generate any amount of dispersion with large enough standard deviations of the productivity shocks, σ_X and σ_A. But if we set these too large it becomes difficult for the model to match the corresponding distribution for standard deviations of $\ln y_t$ across the 501 years, as illustrated in the top-right panel of Figure 6.

Table 5 illustrates this with some numbers. Under the benchmark setting, the model can generate just enough variation in means to make (almost) all five economies fall within the 5th and 95th percentiles of the simulated economies (Portugal being on the border), but Italy falls slightly below the 5th percentile for standard deviations. We could make the model account for Italy's low levels of standard deviation, but then it would be harder to match the low mean for Portugal.

A similar point can be made about the one- and two-year serial correlation coefficients, as shown in the bottom two panels of Figure 6, with numbers in Table 5. In data for Sweden and Italy these two coefficients fall below the bottom 5th percentile of the simulated economies. However, the differences are not huge, when considering that correlation coefficients can range from -1 to 1; note the scales used in Figure 6. There would be less serial correlation with more dispersion in the temporary shocks (higher σ_X), and/or with less dispersion in the permanent shocks (lower σ_A), but that would worsen the fit with standard deviations and means, respectively.

Another way to compare the model to data is to examine how these moments co-vary across the 1000 simulated economies, and the five real ones. This illustrated in Figure 7. The right-hand panel shows that more serial correlation is associated with higher measures of standard deviation, both across simulations and in the data. In the model, some of the shocks to land productivity are persistent, and thus generate persistence in $\ln y_t$, making higher measured serial correlation be associated with higher measures of standard deviation. As shown in the left-hand panel, there is no similar relationship between standard deviations and means, either in the data or across simulated economies. Intuitively, shocks can be both good and bad, so more volatile paths can be associated with lower or higher incomes on average.

Of course, σ_X and σ_A were set to match these patterns, but it is far from obvious that the model should be able to do as well as it does. Recall that the differences in outcomes are the result only of variation in the realized shocks across simulations. Indeed, the fact that the match is not perfect illustrates this point. If we were to let these countries differ also in some exogenous parameter—e.g., in terms of the γ_j's, which determine long-run per-capita incomes—then it would be a trivial exercise to match model and data.

Moreover, given that the data probably also come with some measurement error, the most reasonable interpretation seems to be that they do not reject the Malthusian model.

5.4 Robustness checks

We have learned that the Malthusian model can conform relatively well with the data. What assumptions drive this result? Panel B of Table 5 indicates how the 5th and 95th percentiles for the various moments change when altering some parameter values. More details are provided in the Online Appendix.

An informative exercise is to close down the permanent shocks by setting $\sigma_A = 0$, and increase the dispersion of the temporary shocks to $\sigma_X = .35$. That way we roughly match the average across the simulated standard deviations to data, given the constraint that all shocks are temporary. Interestingly, the simulations still generate some serial correlation, due to the model's internal mechanisms, but not nearly enough to match the data. In other words, we need some persistence in the shocks that go into the model for it to match the data.

Data coverage starts in different years for each country. Sweden, the country with the least coverage, lacks data before 1560. If we consider only the period 1560-1800, for which all five countries have data, the simulations generate more dispersion in mean outcomes, since they are calculated on smaller samples (over shorter time periods).

Raising the elasticities of fertility and mortality, determined by δ and κ, respectively, tends to shrink the dispersion in mean per-capita incomes, and lower standard deviations. The reason is that larger elasticities imply larger effects of productivity shocks, making population levels adjust faster. This shortens the effects on per-capita incomes.

Increasing the parameter ϕ implies lower expected survival rates from disease, as well as higher variance in the survival rate. As seen in Table 5, when we double ϕ from .01 to .02, mean outcomes become more dispersed, and the distribution of standard deviations shifts up. Intuitively, feeding larger shocks into the model makes outcomes vary more.

The last row of Table 5 considers a reduction in the land share of output, α, from .4 to .2. This has two opposing effects. First, because the productivity shocks are land augmenting, it shrinks the effective variation in the shocks. Second, it makes wages less sensitive to population changes, thus increasing the persistence of the shocks.[10] In our numerical exercise, the first effect dominates, in the sense that mean outcomes become less dispersed and standard deviations smaller when we reduce α.

5.5 Extensions and other tests of the model

5.5.1 Trends after 1800

We can extend the time horizon of our simulations and compare the results to an era where we would not expect to see any Malthusian stagnation. In the Online Appendix we merge the data from Fouquet and Broadberry (2015) with post-1800 data from Bolt and van Zanden (2014). There are some discrepancies in the data sources for overlapping years, but ignoring these we get an (unbalanced) panel with GDP per capita for five countries 1300-2010.

We then simulate the model for another 210 years, representing the period 1300-2010. We assume benchmark parameter values and let the expected productivity growth rate (μ_t) stay constant after 1800 at the level it had reached then (i.e., 1.25% per year). We find big differences in how well the model can match the data before and after 1800. After 1800, 28% of the country-years fall above the 95th percentile of the simulated data, compared to less than 1% in the pre-1800 era. The model's stagnant features are thus much more consistent with the data before 1800 than after.

5.5.2 Birth and death rates

From our simulated data we can also calculate Crude Birth and Death Rates, i.e., annual birth and deaths over total population. These can be compared to the corresponding numbers for available years from England and Sweden, compiled by Wrigley et al. (1997) and Statistics Sweden (1969), respectively.

In the Online Appendix we find that mean levels of both CBR and CDR are similar in the data and the simulations. The CBR fluctuations are somewhat larger in the data than in the simulations, while CDR fluctuations are smaller. However, the differences are not huge. For example, among the 1000 simulated economies, as many as 8.4% had CBR levels above the 95th percentile as often England did (in 17% of the years, or more), and 10.4% had CBR

[10] This can be understood intuitively from the simplified model in Section 4.1, where we expressed the dynamics in terms of wages; see (7). The exponent on the lagged wage rate is decreasing in α, so a smaller α implies more persistence.

levels below the 5th percentile as often England. If we consider possible measurement errors the deviations between model and data arguable seem quite reasonable, especially since we did not try to directly match these data.

5.5.3 Marriage

In a Malthusian world, positive income shocks can also affect reproductive rates by inducing more marriage. Here we have relied on a one-sex model, making it hard to even think about marriage. In the Online Appendix we explore a stylized way to model marriage within a one-sex structure. Agents transition from a state called "non-married" into another state, "married," at rates that are random, and dependent on age and wage rates. The only difference between the two states is that agents have higher fertility when married.

The Online Appendix performs a simple quantitative exercise of this model. We assume that the marriage and fertility rates have the same wage elasticities, identical to the benchmark simulation ($\delta = .15$). While very preliminary, some results are both intuitive and interesting. Fluctuations in fertility become larger, which is very intuitive: productivity shocks now affect fertility both directly in the same period, and by increasing current marriage rates, thus rasing future fertility rates. For the same reason, fertility tends to be positively correlated with incomes at deeper lags.

One shortcoming of this simple extension is that the Crude Marriage Rate (after correcting for each marriage in the data representing two agents entering marriage) tends to be lower in the simulations than in both English and Swedish data. While the model abstracts from dissolution of marriages, in the data agents often marry more than once over a life cycle, since marriages are dissolved when one of the spouses dies. Such dissolutions of marriages are hard to model without a two-sex setting.

5.5.4 Stability of the linearized dynamical system

The dynamics of our model can be written in terms of deviations in the log size of each of the T age groups from its steady state level. The Online Appendix studies a linearization of this system, when closing down deterministic productivity growth, and shocks to productivity and mortality (i.e., setting $\mu_t = \sigma_A = \sigma_X = \phi = \kappa = 0$). We first derive the Jacobian matrix, which updates the log population deviations in one period to those in the next. It is then shown that all eigenvalues of this matrix are located within the unit circle, so each age group converges in the long run to its steady state level. At the same time, the imaginary parts of the eigenvalues are large, and most eigenvalues are relatively close to one in absolute terms. This explains why the dynamics display a high degree of persistence.

While the absolute value of the largest real eigenvalue is always smaller than one for

$\delta > 0$, it gets closer to one as δ gets closer zero. Intuitively, when δ is small fertility is less sensitive to wage shocks, so it takes longer for the Malthusian forces to push the economy back to steady state.

5.5.5 Population trends

The Online Appendix also examines population trends using the data from McEvedy and Jones (1978) that we saw in Figure 3. For each country, we first select the 25 simulated economies out of 1000 whose GDP per capita paths most closely resemble those in the data, and refer to the mean among those 25 simulated economies as an "artificial" version of that country. The artificial paths for log GDP per capita closely follow the corresponding paths in the country data, since they were constructed that way. For population levels the results are somewhat mixed. The artificial population paths resemble the data relatively well for the Netherlands and Sweden, overshoot the data for Italy, and undershoot them for England and Portugal. In other words, if we take the model seriously, we should expect to see more variation in the population paths between countries than is reported in the data: faster population growth in richer countries, such as Italy, and slower population growth in poorer countries, such as Portugal. Intuitively, population differences in a Malthusian model tend to be proportional to productivity differences, and productivity levels here follow a random walk, thus diverging over time across countries.

However, relatively small shifts in the GDP per capita paths, in the order of 20% up or down, are sufficient to capture most of the observed variation in population trends. We argue in the Online Appendix that the observed mismatch is reasonable if we consider measurement errors in both GDP per capita and population levels. We also show that the match improves greatly when allowing per-capita income levels in steady state to differ across countries, e.g., by letting the fertility parameters, γ_j, differ. However, such a setting would not attribute observed differences in mean per-capita incomes to variation in shocks only, as in the benchmark setting.

6 Concluding remarks

Over the last several years researchers have compiled high-quality comprehensive per-capita income data for a couple of European countries from 1300-1800, recently summarized by Fouquet and Broadberry (2015). The data display big fluctuations in per-capita incomes over time, with a high degree of persistence, as per-capita incomes can move in long cycles of expansions and contractions. The average per-capita income across these countries even shows an upward trend. Can these observations really be consistent with the predictions of a standard Malthusian model, which says that per-capita incomes should be stagnant?

This paper proposes a simple exercise to come up with a tentative answer to this question. We set up a Malthusian model that is stagnant by construction, in the sense that it cannot exhibit sustained growth in per-capita incomes, and simulate it 1000 times to compare the time paths to Fouquet and Broadberry's data. The life-cycle structure is such that each model period corresponds to one year, enabling us to compare the results to annual data. Land productivity is subject to shocks, both temporary and persistent, and the average productivity growth rate increases slightly over time.

We try to be as informed as possible when setting parameter values: parameters guiding age-profiles for mortality, wages, and fertility are set based on data from preindustrial Sweden; elasticities of mortality and fertility with respect to wages are set to match empirical studies on English and Scandinavian preindustrial time-series and panel data; the rise over time in the mean productivity growth rate is set to make the model roughly match the accelerating pace of population growth over the period.

It turns out that the model can match the data decently, as measured by means, standard deviations and serial correlations in per-capita incomes over time, given the right amount of variation in the land productivity shocks. The simulated time paths can show centuries-long cycles of expansions and contractions in per-capita incomes. Not knowing that they are generated by a Malthusian model, one could mistakenly believe that some of the expansions constitute transitions out of Malthusian stagnation. Moreover, mean per-capita income across all 1000 simulated economies shows an upward trend, mirroring the gradual rise in the productivity growth rate, in turn set to match the accelerating population growth rate over this period.

A few assumptions are important for these results. There must be growth in land productivity, and that growth rate must be stochastic and increase over time in expectation. Fertility and mortality cannot be too elastic with respect to fluctuations in wages. Like discussed, the numbers used in the simulations are consistent with empirical estimates.

This is not proof that the Malthusian model set up here is the only one to explain preindustrial growth in Europe. Other models may match the data equally well, or better. This may include models that allow for (some modest amount of) quality-quantity trade-off in children, and/or the presence of a non-agricultural sector. The aim of this paper is to start off with a model without these features—one that is closer to how, e.g., Galor and Weil (2000) and Hansen and Prescott (2002) model the pre-transition, or Malthusian, stage of development—and assess whether it can at all be consistent with the data compiled by Fouquet and Broadberry (2015). Indeed, the model-data match is not perfect, so one could argue in favor of a *somewhat* more sophisticated model. However, it seems wrong to claim that these new and important data obviously refute the Malthusian model.

Regardless of one's preferred conclusion, this paper provides a parametric and numerical framework that can be extended and used for other applications. For example, it could serve as a starting point for constructing a unified growth framework with realistic life cycles, where existing simulations of such models tend to assume that each generation lives for only two periods of 20-30 years each. Exercises such as that are left for future research.

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Tables

Table 1: Benchmark parameter values.

Parameter	**Value**	**Interpretation/comment**
T	90	Last period of life
$\underline{B}$	15	Period of life when agents become economically and reproductively active
$\overline{B}$	49	Last period of life being reproductively active
R	70	Last period of life being economically active
α	.4	Land share of output; same as Hansen and Prescott (2002)
δ	.15	Gives fertility-wage elasticity of .13; similar to Klemp and Møller (2016), Lagerlöf (2015)
κ	.01	Gives mortality-wage elasticity of $-.2$; similar to Klemp and Møller (2016), Lagerlöf (2015)
ϕ	.01	Expected death rate from disease is about 1% per year
γ_j	See Table 3	To match age-fertility profile to Swedish data in Figure 2
β_j	See Table 3	To match age-income profile to Swedish data in Figure 2
ρ	.9	Close to perfect substitutability between age cohorts when determining effective labor
μ_t	$.0125(t/500)^2$; $t = 0$ in 1300	Mean annual productivity growth rate rises at accelerating rate from 0 to 1.25 percent from 1300-1800; set to match population data in Figure 3
σ_A	.07	Standard deviation in permanent shock; set to match moments in the data
σ_X	.07	Standard deviation in temporary shock; set to match moments in the data

Table 2: Age-specific survival probabilities, in Swedish data and in the model. The model values are the expected survival rate from age and disease, but disregarding starvation.

Ages $(j-1)$	**Survival rate in the data**	$E[s_t^d]s_j^{\text{age}}$
0	.797	.797
1-2	.947	.947
3-4	.972	.972
5-9	.987	.990
10-14	.993	.990
15-19	.993	.990
20-24	.992	.990
25-29	.990	.990
30-34	.988	.988
35-39	.988	.988
40-44	.984	.984
45-49	.982	.982
50-54	.978	.978
55-59	.973	.973
60-64	.959	.959
65-69	.941	.941
70-74	.908	.908
75-79	.870	.870
80-88	.775	.775
89	NA	0

Table 3: Values for γ_j and β_j for three select age groups.

j	β_j	γ_j
18	0.0181	0.0104
32	0.0208	0.1128
40	0.0192	0.0468

Table 4: Fraction of the years in which log GDP per capita fell below the 5th percentile, and above the 95th percentile, across 1000 simulated economies. The results are shown by country and for all country-years together.

Country	**Fraction years below 5th percentile**	**Fraction years above 95th percentile**
England	.024	0
Netherlands	0	.097
Sweden	.004	0
Italy	0	.026
Portugal	.29	0
All	.046	.029

Table 5: Comparing moments in the data to the simulation results for the benchmark calibration and when changing some of the parameter values.

Panel A: Moments in log GDP/capita time-series data

Country	Mean	Standard deviation	1-year lag correlation	2-year lag correlation
England	$-.16$	.25	.96	.95
Netherlands	.31	.3	.96	.94
Italy	.18	.097	.85	.74
Sweden	.015	.13	.86	.74
Portugal	$-.35$	.17	.94	.92
Mean across five countries	.00	.19	.91	.86

Panel B: Moments in simulations [5th percentile, 95th percentile]

Benchmark	$[-.34, .41]$	$[.12, .43]$	$[.92, .99]$	$[.9, .99]$
$\sigma_A = 0$, $\sigma_X = .35$	$[-.063, .068]$	$[.18, .24]$	$[.37, .66]$	$[.37, .66]$
Start year 1560	$[-.41, .51]$	$[.098, .35]$	$[.87, .99]$	$[.83, .99]$
$\delta = .3$	$[-.28, .31]$	$[.12, .39]$	$[.92, .99]$	$[.89, .99]$
$\kappa = .02$	$[-.27, .36]$	$[.12, .41]$	$[.92, .99]$	$[.89, .99]$
$\phi = .02$	$[-.57, .62]$	$[.15, .55]$	$[.94, 0.996]$	$[.92, .99]$
$\alpha = .02$	$[-.26, .29]$	$[.065, .23]$	$[.93, .99]$	$[.91, .99]$

Figures

Figure 1: Data over log GDP per capita. The series are normalized to equal zero when averaged first over time for each country, and then across all countries.

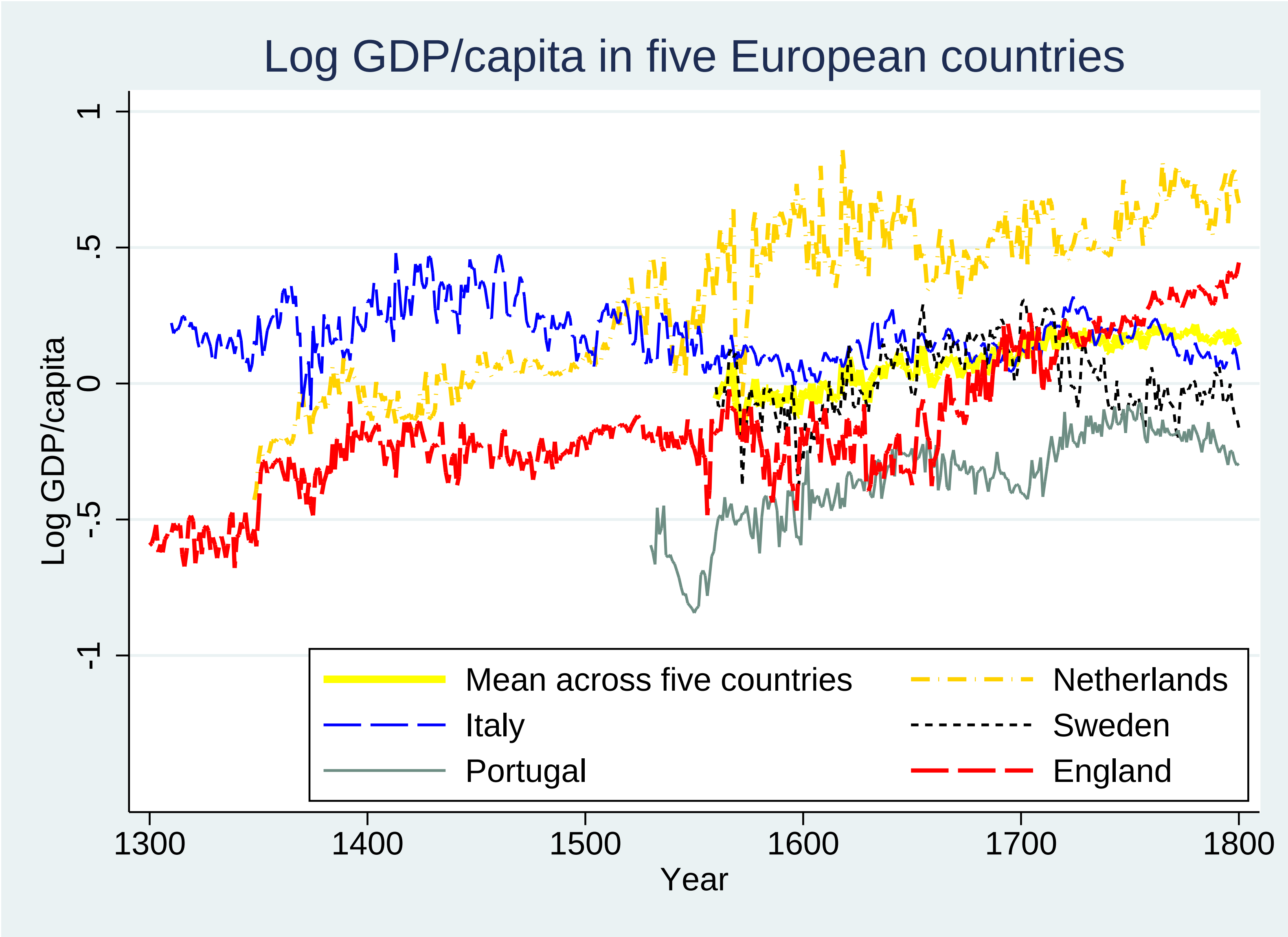

Figure 2: Fertility and wages by age group, in the data and in the model, guiding how γ_j, β_j and ρ are set. The data are from Sweden. Fertility rates are averages 1751-1800, and wages refer to rural Swedish workers in 1940, the earliest year available. The model-generated profiles are averages over 1000 runs for the last 50 years of the simulation, corresponding to the years 1751-1800.

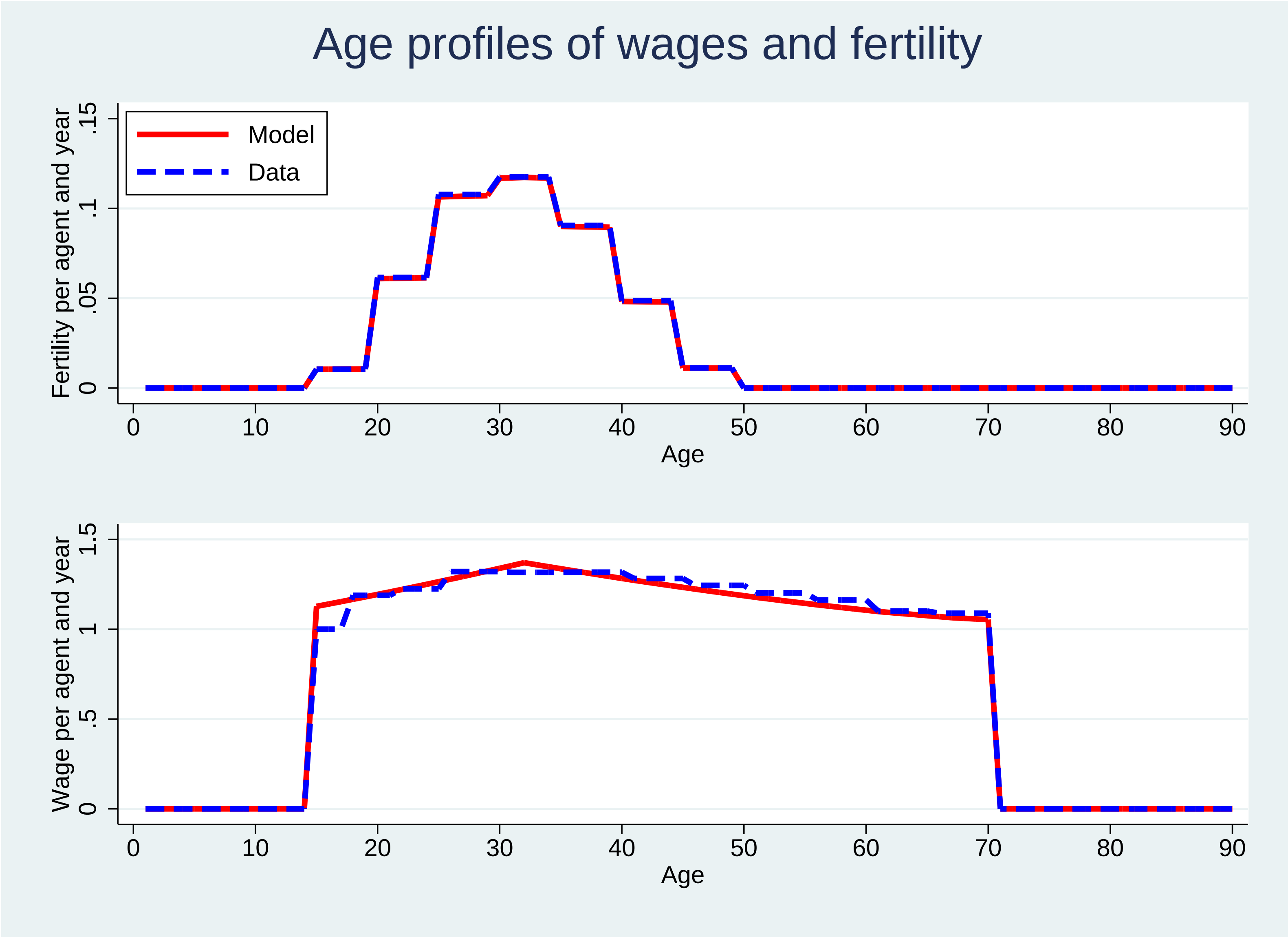

Figure 3: Population level data for the five countries from McEvedy and Jones (1978), and the mean and median population levels across 1000 simulated economies.

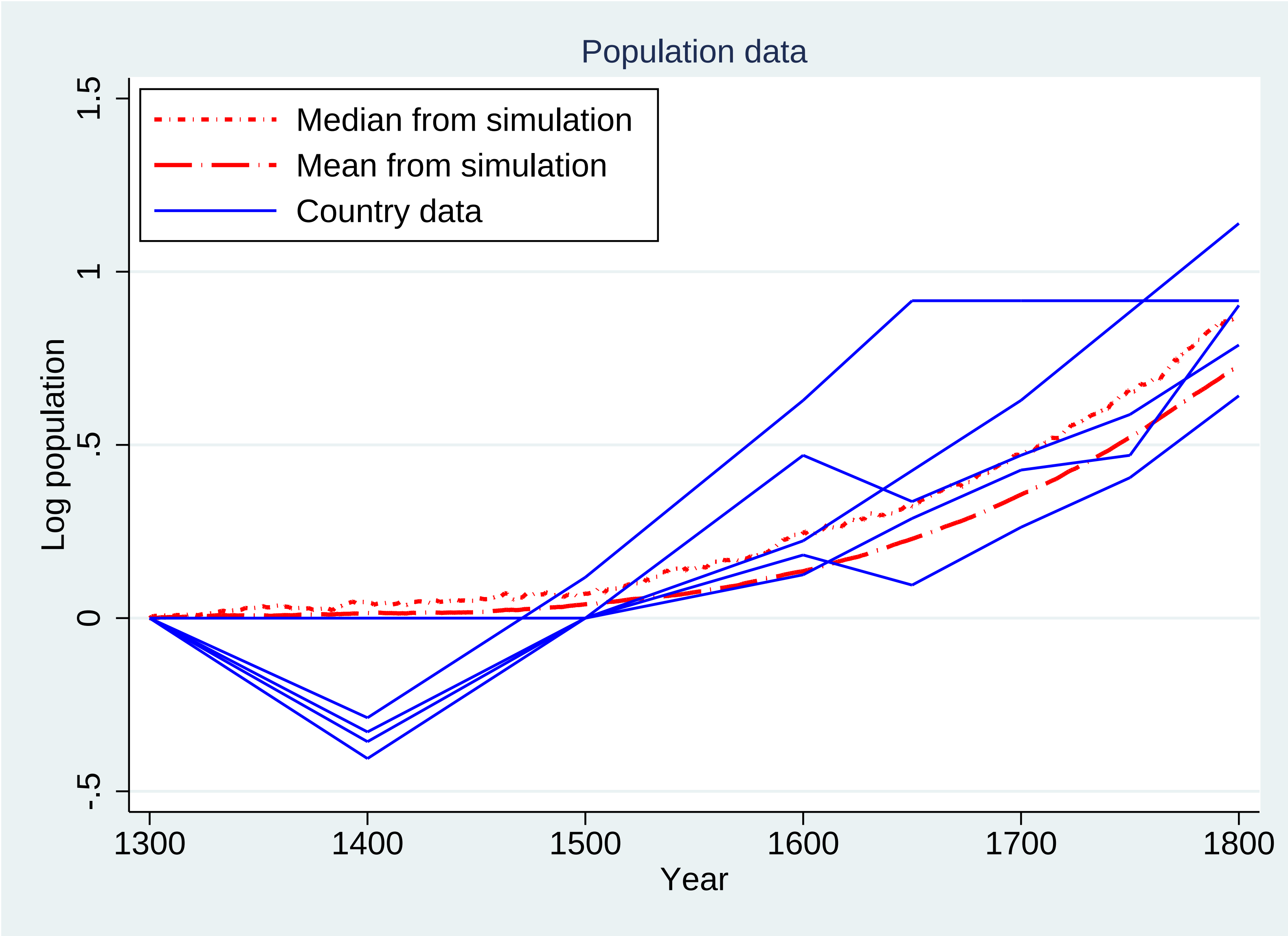

Figure 4: Examples of paths of log GDP per capita for four different simulated economies, and the average across 1000 runs (including the ones shown).

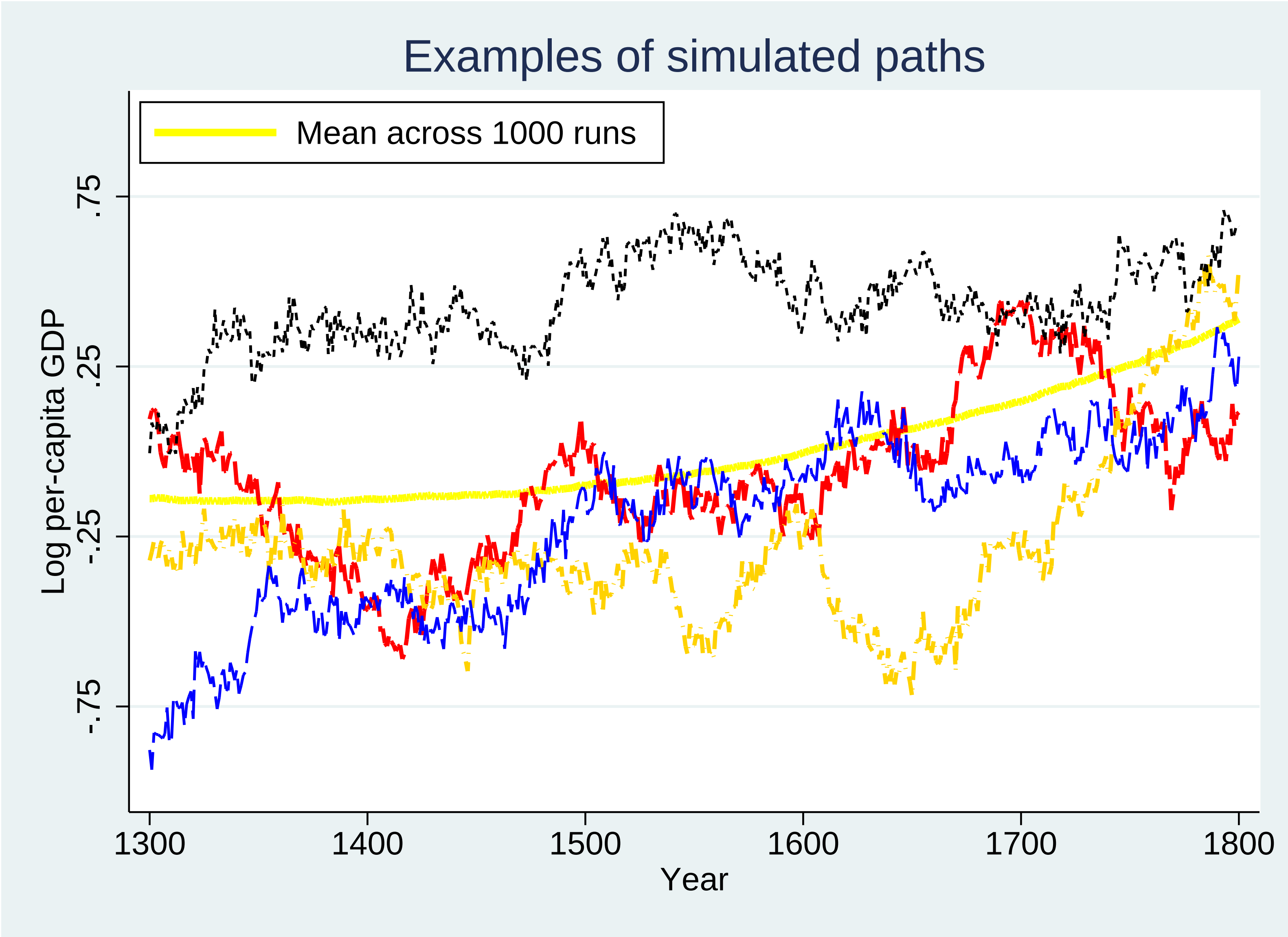

Figure 5: This figure compares the time paths in log GDP per capita for the five countries in Figure 1 to the median across 1000 runs by year, and the associated 5th and 95th percentiles for each year.

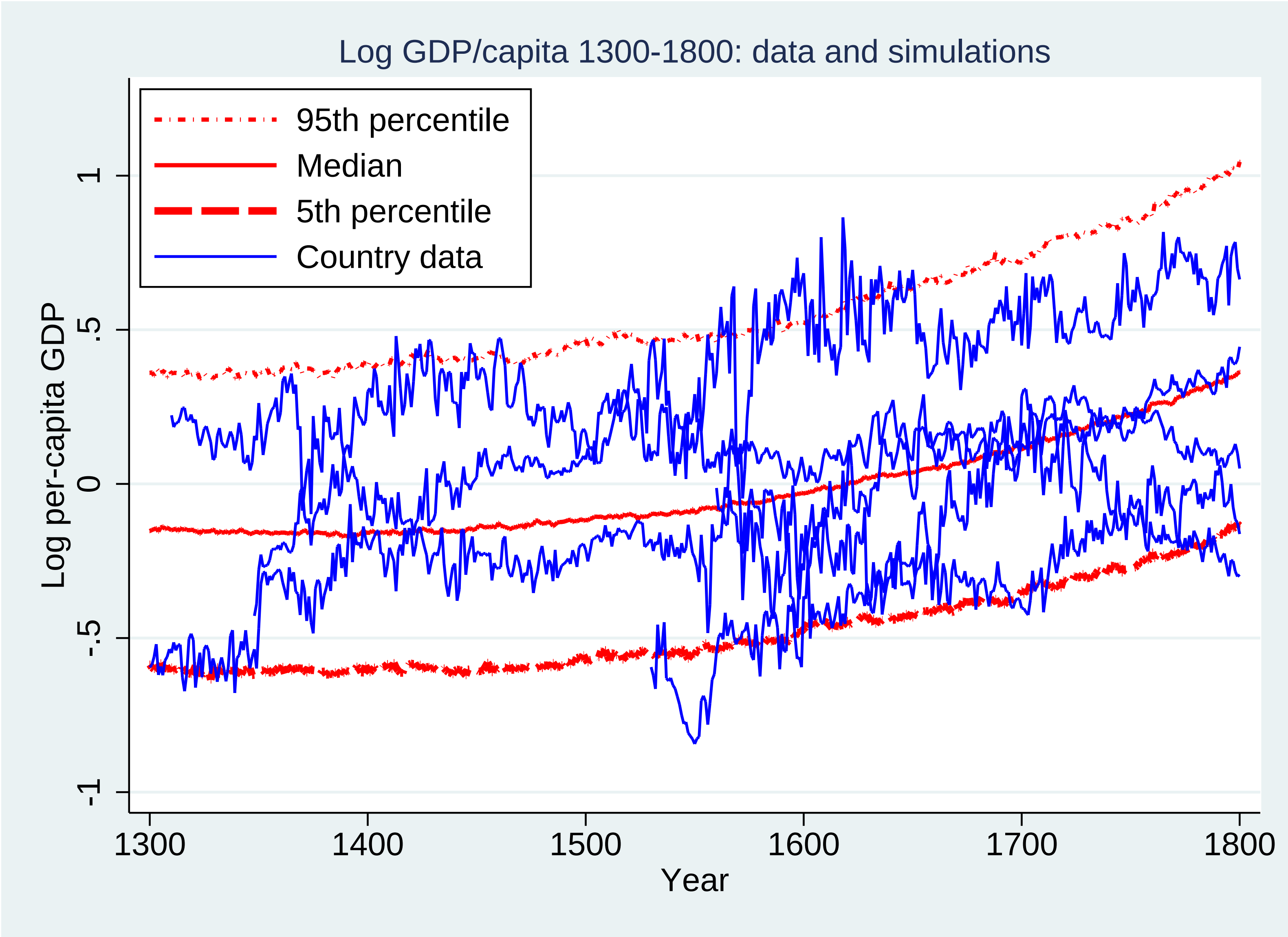

Figure 6: For each of the 1000 simulated economies, the means, standard deviations, and serial correlation coefficients (at one and two year lags) were calculated. This figure shows how these moments were distributed, as well as the corresponding values for the five countries.

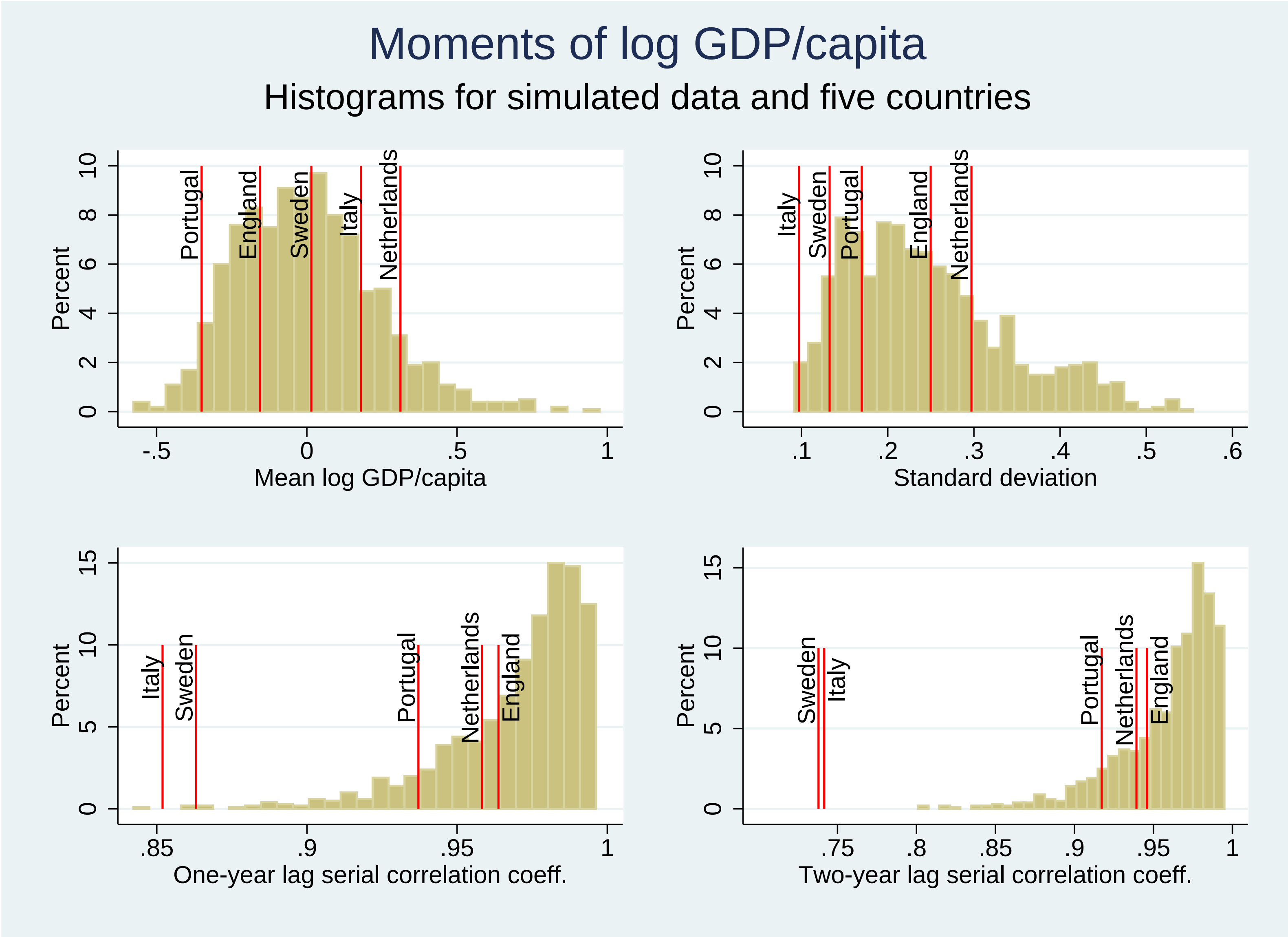

Figure 7: The same data as in Figure 6 presented in two plots.