Born Free

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Abstract: This paper studies coercive labor institutions in a Malthusian framework, where class is hereditary: children born by free workers are free, while children of slaves are the property of their parents' masters. When productivity increases in an urban and slave-free sector, and more free workers migrate there, slave owners respond by feeding slaves better to increase their reproduction, and thus replace migrating free workers with the slaves' offspring. As as result, slaves are made better off in the short run, while their long-run representation in the rural workforce—and possibly even the overall population—increases.

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1 Introduction

> "[I]n all societies where the institution [of slavery] acquired more than marginal significance and persisted for more than a couple of generations, birth became the single most important source of slaves. Of the great majority of slaveholding societies the stronger claim may be made that birth during *most* periods was the source of *most* slaves."
> Patterson (1982, p. 132, italics in the original).

Human societies vary greatly in how much they rely on coerced labor. No current legal system recognizes property rights in humans, certainly not as explicitly as some did in the past, and the importance of slavery has varied greatly also among preindustrial societies.

This has inspired an extensive body of theoretical research, discussed further below, where a common implicit assumption is a homogeneous workforce, i.e., that all workers are equally easy to coerce, or enslave. The models also tend to be static.

Here we study slavery in a dynamic two-sector model with urban-rural migration. Malthusian forces govern reproductive success, and workers are either slaves, or free, from birth. This offers several interesting insights about the effects of urban development on both the fraction slaves in the population, and their well-being.

According to the theory, when the slave-free urban sector grows and attracts more free workers, slave owners respond by improving the material well-being of slaves, to increase their reproductive success, and thus replace migrating free workers with the slaves' offspring. Slave owners are not compelled by any market forces to pay slaves more, since slaves have no rights, and cannot be employed in the urban sector. However, owners want to raise slaves' reproduction to substitute slave labor for that of migrating free workers.

The upshot is that urbanization improves the short-run well-being of slaves, and thereby also increases their long-run representation in the rural workforce. More surprisingly, we show that slaves' steady-state representation in the *overall* population, spanning both sectors, may increase. We pin down parametric conditions under which this is the case.

The model builds on a few, arguably plausible, assumptions:

(a) Slavery is hereditary (i.e., offspring of slaves are the property of the parent's owner).

(b) Reproductive success is an increasing function of parental resources.

(c) Only free workers can work in the urban sector.

(d) The cost of child rearing is higher in the urban sector than the rural.

(e) Free labor and slaves are substitutes in the rural sector.

These all seem empirically valid, at least for most slave societies. Slavery being hereditary can be motivated by the quote from Orlando Patterson above. There have been other sources of slave labor, in particular war captives, but the long-run survival of any slave system seems to depend on internal regeneration. For example, according to Scheidel (2011, p. 308), in the Roman Empire natural regeneration was a more important source of slave supply than war and all other sources combined. Also, most slave societies, including the Roman Empire, had codes regulating the status of slaves' offspring (Patterson 1982, Ch. 5).

Slaves' reproduction being an increasing function of resources allocated to them is consistent with an often documented interest in slaves' reproduction on part of their owners. For example, in the US South slave owners promoted early slave marriage, and spent resources on the medical treatment of slaves.[1]

The assumption that the urban sector uses only free labor could be motivated by coercion being less effective when production is more care- than effort-intensive, as argued by Fenoaltea (1984), or by different laws and institutions in cities.[2]

Child rearing being costlier in the urban sector captures the historically higher mortality rates in cities, and generates an often-documented urban wage premium, similar to e.g. Cruz and Taylor (2012).

Free workers and slaves being relatively close, if not perfect, substitutes seems reasonable, at least for small-scale farming, probably common throughout most of human history since the Neolithic transition. In the antebellum US South, slaves and free workers were substitutes on small farms, although not on big plantations (Field 1988). Moreover, when production modes are flexible some slave-free substitution should be feasible also in large-scale production. Temin (2004) suggests that slaves and free workers in the Roman Empire competed on the same labor markets.

While these assumptions all seem plausible, the long-run prediction they lead up to is arguably somewhat unexpected: increased urban productivity may lead to a more enslaved population in steady state.

The rest of this paper is organized as follows. Section 2 provides a brief overview of some existing theoretical work. Then Section 3 sets up the model and solves for the optimal choices of all agents. Section 4 studies the outcomes in steady state: first Section 4.1 treats out-migration of rural-born free workers as exogenous; then Section 4.2 endogenizes the migration decision and studies the free-slave composition of the population across both sectors. Section 5 discusses some extensions and possible applications. Section 6 ends with a concluding discussion.

[1] See, e.g., Fogel and Engerman (1974, pp. 78-86, 117-126), White (1999, Ch. 3), and Kolchin (2003, pp. 114-115, 123, 139). While Fogel and Engerman (1974) famously argue that "slave-breeding" was exceedingly rare in the antebellum South, they readily concede that slave owners rewarded reproduction.

[2] Yet another interpretation could be that slaves who do work in urban professions are better treated and therefore less exploited than in the rural sector, and in that sense closer to free on a slave-free continuum.

2 Previous literature

Much has been written on coercive labor arrangements (e.g., Domar 1970, Bergstrom 1971, Chwe 1990, Genicot 2002, Conning 2004, Lagerlöf 2009, Acemoglu and Wolitzky 2011, Fenske 2013). This contribution can be compared to two of these.

Lagerlöf (2009) models an environment where an elite can, at a cost, claim property rights over (land and/or) people, i.e., enslave them. Following Domar (1970) and Conning (2004), the incentives to enslave people are stronger when labor is scarce relative to land. In a very literal interpretation, a transition from a free-labor environment to slavery there amounts to a previously free population being completely enslaved from one generation to the next. Here the elite always own *some* slaves (as well as all land), but the *size* of the slave population is endogenous. Thus, there are no institutional "transitions" in the sense of Lagerlöf (2009), but given an institutional environment that permits slavery, the model allows us to study the determinants of the composition of the population.

Acemoglu and Wolitzky (2011) take a different approach, using a principle-agent model where the agent (a worker) controls effort, and coercion amounts to the principal (a landowner) spending resources on lowering the agent's outside options. An exogenous improvement in the agent's outside options can then induce the principal to use less coercion.[3] The key mechanism is that effort and coercion are complementary: when the agent's participation constraint becomes harder to satisfy, the principal responds by extracting less effort, thus using less coercion.

An implicit assumption in Acemoglu and Wolitzky (2011) is that all workers are equally coercible, at a cost. Here we assume that some workers (slaves) can be coerced without cost, but must be fed to produce offspring (i.e., future slaves), while others (free workers) cannot be coerced at all, but are hired on a competitive labor market, and reproduce themselves. This generates quite different mechanics. As in the Acemoglu-Wolitzky model, better outside options for free workers here induce the elite to treat slaves better in the short run, but the long-run effect is a more coerced agricultural labor force.[4]

While this paper seems to be the first to explicitly model the slave population as a capital stock, Canaday and Tamura (2009) model investment in slaves' (plantation-specific) human capital.

The link from fertility differentials between population groups to the composition of the population has been modelled in many other contexts (e.g., Galor and Moav 2001, 2002;

[3] This can account for some specific historical events, such as the decline of serfdom in the wake of the Black Death, related to the so-called Brenner Debates; cf. Brenner (1976).

[4] Acemoglu and Wolitzky (2011) also find that higher prices of the (agricultural) output that the principal produces increases coercion, which can be compared to the effects of rising rural productivity in our model; see Section 5.2.

Kremer and Chen 2002; Lucas 2002; de la Croix and Doepke 2003, 2009). Ours is perhaps closest to that of Lucas (2002, Ch. 5). However, none of these studies the dynamics of classes with different institutional status, such as slaves and free workers.

3 The model

The basic framework is a standard Malthusian model, like that of, e.g., Ashraf and Galor (2011), but with different classes and a forward-looking elite, similar in spirit to Lucas (2002, Ch. 5).

There are three social classes—free workers, slaves, and a slave-owning elite—and two sectors—an urban sector, using only free labor, and a rural sector, using both slaves and free labor.

Free workers and slaves, collectively referred to as just workers, live in overlapping generations for two periods: as passive children in the first, and working adults in the second, suppling one unit of labor each. The elite are infinitely lived and of constant (and small) size, which can be interpreted as each elite agent having one offspring who inherits her property (land and slaves).[5]

In period t there are S_t (adult) slaves, and L_t and F_t (adult) free workers in the rural and urban sectors, respectively. All these evolve endogenously over time, both through free workers' migration decisions, and because class status is inherited.

3.1 Workers

Workers care about their own consumption in adulthood, c_t^X, and the number of surviving offspring, n_t^X, where $X \in \{S, L, F\}$ denotes class. Utility is given by

$$u_t^X = (1 - \widetilde{\gamma}) \ln(c_t^X) + \widetilde{\gamma} \ln(n_t^X), \tag{1}$$

where $\widetilde{\gamma} \in (0, 1)$.[6]

Each child carries a sector-specific goods cost: q in the rural sector, and $q/\varepsilon > q$ in the urban sector, where $\varepsilon \in (0, 1)$ measures the inverse of the cost gap.

Slaves can only work in the rural sector, and thus face the child-rearing cost q. For the moment, let income per slave be y_t^S; as explained below, this is chosen by the slave owner.

[5] This assumption is not important, but ensures that the elite does not expand in size, compressing their living standards to the level of workers.

[6] Allowing for altruistic links between different generations of workers would not change the analysis, since workers do not own any assets that they can bequeath to their offspring.

The slave sets n_t^S to maximize (1) subject to $c_t^S = y_t^S - qn_t^S$, giving optimal slave fertility as

$$n_t^S = \frac{\widetilde{\gamma}}{q} y_t^S = \gamma y_t^S, \tag{2}$$

where we let

$$\gamma = \frac{\widetilde{\gamma}}{q}. \tag{3}$$

Similarly, let w_t^L be the wage rate of free workers in the rural sector. Facing the child-rearing cost q, they maximize (1) subject to $c_t^L = w_t^L - qn_t^L$, giving fertility as

$$n_t^L = \frac{\widetilde{\gamma}}{q} w_t^L = \gamma w_t^L. \tag{4}$$

Finally, with w_t^F being the wage rate of free workers in the urban sector, who face the child-rearing cost q/ε, the fertility of a free urban worker becomes

$$n_t^F = \frac{\widetilde{\gamma}}{q/\varepsilon} w_t^F = \varepsilon \gamma w_t^F. \tag{5}$$

3.2 Production

3.2.1 Rural production

Rural output, Y_t^R, is produced with three inputs—land, slave labor, and free labor:

$$Y_t^R = A^R \left(M^R\right)^{1-\alpha} \left(S_t^\rho + \eta L_t^\rho\right)^{\frac{\alpha}{\rho}}, \tag{6}$$

where $A^R > 0$ is a productivity factor, M^R is (rural) land, and α is the overall labor share of rural output. Throughout the rest of the presentation, we normalize M^R to unity.

The parameter $\eta > 0$ allows the productivity of free labor to differ from that of slaves, which may capture variation in the powers elite have over free workers, relative to slaves—e.g., over their labor-leisure choices—without explicitly modelling it. For example, $\eta < 1$ would here imply that it is harder to extract effort from free workers, probably a good description of many plantation economies.

When $\rho = 1$ free workers and slaves become perfect substitutes, which would imply that all rural workers are either slaves or free in steady state (see Section 5.1). Here we assume $\rho < 1$ to rule this out. This may be a plausible assumption, if we think of the CES representation as short-hand for a more complex rural environment, where several different agricultural production modes are available, some of which are viable only with slave labor, and others only with free labor. More practically, it allows the model to generate continuous changes in steady-state outcomes when varying exogenous parameters, rather than discrete jumps from one steady-state to another.

We also assume that $\alpha < \rho$, which ensures that the two labor inputs are gross substitutes, i.e., the marginal product of free labor is decreasing in the level of slave labor input, and vice versa. It also implies that demand for each type of rural labor is increasing in the price of the other.

3.2.2 Urban production

Urban output, Y_t^U, is produced with only two inputs: free labor and some some other input, which could be land or capital. Urban output can then be written

$$Y_t^U = A^U \left(M^U\right)^{1-\delta} F_t^\delta, \tag{7}$$

where $\delta \in (0,1)$ is the labor share in urban production, $A^U > 0$ is a productivity parameter, and M^U is urban land or capital, hereafter normalized to unity. We also assume that M^U is owned by what we could call an urban elite, who consume all their income, and thus play no role in the rest of the analysis.

Much of the analysis later will be about steady-state effects when varying A^U.

Note also that the two sectors produce the same good, so we need not model any relative price between rural and urban output. (See Section 6 for a discussion.)

3.3 Elite

A representative elite agent, owning a unit land endowment and S_t slaves, earns income

$$\pi_t = \widehat{\pi}_t + w_t^S S_t, \tag{8}$$

where w_t^S is the rental price of slave labor (but not slave income, as explained below), and $\widehat{\pi}_t$ is land income. Here we set up the problem as if slave owners rent out their own slaves and hire other slaves to work on their land. Since all slave owners are identical the results are exactly the same if they employ their own slaves (see below).

As shown in Section A of the appendix, in equilibrium $\widehat{\pi}_t$ is simply the land share of total rural output in (6), with $M^R = 1$:

$$\widehat{\pi}_t = (1-\alpha) A^R (S_t^\rho + \eta L_t^\rho)^{\frac{\alpha}{\rho}}, \tag{9}$$

which depends on the equilibrium levels of S_t and L_t, and is thus taken as given by the representative elite agent.

Let s_t be the fraction of the elite's total income in (8) that is allocated to slaves. Then income per slave in period t becomes $s_t \pi_t / S_t$. Setting $y_t^S = s_t \pi_t / S_t$ in (2), we can write the dynamics of the slave population

$$S_{t+1} = n_t^S S_t = \gamma s_t \pi_t. \tag{10}$$

The elite's utility is given by

$$U_0 = \sum_{t=0}^{\infty} \beta^t \ln(c_t^E), \tag{11}$$

where c_t^E is elite consumption, and β the intertemporal (or intergenerational) discount factor. Maximizing (11) subject to (8), (10), and $c_t^E = (1 - s_t)\,\pi_t$ gives the Euler equation (see the Online Appendix for details):

$$\frac{c_{t+1}^E}{c_t^E} = \beta \gamma w_{t+1}^S. \tag{12}$$

Intuitively, the slave population is a capital stock with rate of return γw_{t+1}^S, since one unit of the consumption good allocated to a slave generates γ slave offspring, whose rental price (or marginal product) equals w_{t+1}^S in the next period.

It can now be seen that it does not matter if each elite agent uses his own slaves as labor input, or rents his own slaves out and hires others, since w_{t+1}^S is both the rental price of slaves and the marginal product of slave labor. The Euler equation in (12) looks the same with both formulations.

3.4 Migration, factor prices and population ratios

Because urban workers have fewer offspring than rural, any steady state with a constant (and positive) ratio of rural to urban free workers involves migration of agents born by rural workers to the urban sector. Let θ_{t+1} be the fraction of the $n_t^L L_t$ agents born by free workers in the rural sector who stay in the rural sector, so that, recalling (4):

$$L_{t+1} = \theta_{t+1} n_t^L L_t = \theta_{t+1} \gamma w_t^L L_t. \tag{13}$$

Similarly, $\varepsilon \gamma w_t^F F_t$ agents are born by urban free agents, all of whom stay in that sector. The dynamics of the urban population can thus be written

$$F_{t+1} = \varepsilon \gamma w_t^F F_t + (1 - \theta_{t+1}) \gamma w_t^L L_t. \tag{14}$$

By equalizing the utility in (1) between urban and rural free workers, using the expressions for fertility in (4) and (5), and the associated expressions for consumption, $c_t^L = w_t^L - q n_t^L$ and $c_t^F = w_t^F - (q/\varepsilon) n_t^F$, it can be seen (and is shown in the Online Appendix) that free workers are indifferent between working in the rural and urban sectors when the wage rates satisfy

$$w_t^F = \varepsilon^{-\gamma q} w_t^L, \tag{15}$$

where $\gamma q = \widetilde{\gamma} \in (0, 1)$; recall (3). Note that $\varepsilon < 1$ implies $w_t^F > w_t^L$. That is, the higher cost of fertility in the urban sector generates an urban wage premium. Using (4) and (5), and multiplying (15) by $\varepsilon\gamma$, we also note that the richer urban workers still have lower fertility: $n_t^F = \varepsilon^{1-\gamma q} n_t^L < n_t^L$.

Using (13) to (15), it can also be seen that the ratio of rural to urban free workers evolves according to

$$\frac{L_{t+1}}{F_{t+1}} = \frac{\theta_{t+1}\frac{L_t}{F_t}}{\varepsilon^{1-\gamma q} + (1-\theta_{t+1})\frac{L_t}{F_t}}. \tag{16}$$

Next, the first-order conditions for the elite's hiring of free workers and slaves, evaluated in a factor market equilibrium, can be written

$$w_t^L L_t = \alpha A^R (S_t^\rho + \eta L_t^\rho)^{\frac{\alpha-\rho}{\rho}} \eta L_t^\rho, \tag{17}$$

and

$$w_t^S S_t = \alpha A^R (S_t^\rho + \eta L_t^\rho)^{\frac{\alpha-\rho}{\rho}} S_t^\rho, \tag{18}$$

respectively (see Section A of the appendix). It follows that the ratio of slaves to free rural workers becomes

$$\frac{S_t}{L_t} = \left(\frac{w_t^L}{\eta w_t^S}\right)^{\frac{1}{1-\rho}}. \tag{19}$$

Intuitively, a higher cost of free workers relative to slaves implies a higher slave-to-free ratio in rural production. The optimal composition of the two labor inputs is more sensitive to changes in their relative cost (w_t^L/w_t^S) when they are closer substitutes (ρ closer to one).

Finally, using (7) with $M^U = 1$, wages in the urban sector become

$$w_t^F = \delta A^U F_t^{\delta-1}. \tag{20}$$

4 Steady-state analysis

4.1 Exogenous migration

To understand the workings of the model, it helps to first examine the steady-state composition of the population within the rural sector. This is easiest if we treat the fraction free workers who stay in the rural sector as exogenous, here denoted $\overline{\theta}$. To study the composition of the total population (i.e., across both sectors), we need to endogenize migration, which is left to Section 4.2.

Let bars denote steady-state levels, so that $\overline{L}$ is the steady-state level of L_t, etc. The free rural population evolves according to (13), but with $\overline{\theta}$ being exogenous for now. That is, $L_{t+1} = \overline{\theta}\gamma w_t^L L_t$, with w_t^L given by (17). In steady state, where the free rural population is constant ($L_{t+1} = L_t = \overline{L}$), the free-worker wage becomes

$$\overline{w}^L = \frac{1}{\overline{\theta}\gamma}. \tag{21}$$

Intuitively, out-migration in this Malthusian setting works the same way as mortality. More out-migration (lower $\overline{\theta}$) implies that fertility, and thus wages, must be higher for the free rural population to be constant.

Next, evaluating the Euler equation in (12) in steady state ($c_{t+1}^E = c_t^E = \overline{c}^E$), the steady-state marginal product of slave labor equals

$$\overline{w}^S = \frac{1}{\beta\gamma}, \tag{22}$$

which can be interpreted as a condition for constant slave population. Intuitively, the elite increase the slave population, by feeding slaves better, until the intertemporal rate of substitution, which equals $1/\beta$ in steady state, equals the marginal return to slave investment, $\gamma\overline{w}^S$. The more the elite value the future (the higher is β), the lower is the steady-state return to slave investment.

The Malthusian steady-state conditions in (21) and (22), and the expressions for the marginal products in (17) and (18), jointly determine the steady-state free and slave populations, $\overline{L}$ and $\overline{S}$, as illustrated in Figure 1(a). Both curves have negative slope, reflecting the substitutability between the two inputs: more input of slave labor implies lower marginal product of free labor, and vice versa; thus, keeping marginal products constant, more of one input implies less of the other.[7]

Figure 1(b) shows the effect of increased out-migration, captured by a fall in $\overline{\theta}$ from $\overline{\theta}^H$ to $\overline{\theta}^L$. This shifts down the locus along which $w_t^L = 1/\left(\gamma\overline{\theta}\right)$, leading to a fall in $\overline{L}$ and a rise in $\overline{S}$. (To see why the locus shifts the way it does, note that a higher free-labor wage is associated with less free labor input, at given slave labor inputs.) Note that the fall in $\overline{\theta}$ exerts both a direct negative effect on the free rural workforce, and an indirect effect by which increased slave investment crowds out free rural labor further.

The transition to the new steady state with larger slave population involves higher slave fertility, and thus higher slave incomes; recall (2).[8] In the short run, out-migration by free workers thus improves slaves' conditions. This is an interesting result, because slaves themselves have no rights and cannot migrate. Rather, the elite's optimal response to the higher returns to slave labor involves better treatment of slaves, since allocating resources to slaves is how they control slave fertility.

These insights can be summed up as follows:

[7] One can use (17) to confirm that $\partial w_t^L/\partial L_t < 0$ and $\partial w_t^L/\partial S_t < 0$ (recall $\alpha < \rho$). Thus, the slope of a curve along which w_t^L is constant becomes: $dL_t/dS_t = -\left(\partial w_t^L/\partial S_t\right)/\left(\partial w_t^L/\partial L_t\right) < 0$. Similarly, one can use (18) to show that the slope of the curve along which w_t^S is constant has negative slope.

[8] The Online Appendix shows formally that slave per-capita incomes, in the wake of a fall in $\overline{\theta}$, are on average higher during the transition to the new steady state than in the old steady state. (Note that slave per-capita income always equals $1/\gamma$ in steady state, due to the Malthusian population dynamics.)

Result 1 *An exogenous increase in out-migration by free rural workers (a fall in $\overline{\theta}$) leads to:*
(a) Higher steady-state slave population ($\overline{S}$);
(b) Lower steady-state rural free-worker population ($\overline{L}$);
(c) Higher slave incomes (y_t^S) in the transition.

As we shall see, the same comparative statics carry over when $\overline{\theta}$ is endogenous. Then a fall in $\overline{\theta}$ can be interpreted as caused by a rise in urban productivity, A^U (see Result 2 below).

However, when more free workers migrate out of the rural sector, the free population in the urban sector should increase as well. An interesting question is thus what happens to the proportion slaves in the *overall* population, which we explore in the next section.

4.2 Endogenous migration

4.2.1 Determining $\overline{\theta}$

Next we let the θ_t adjust endogenously in each period to make free workers' utilities equalize across sectors, meaning the condition in (15) must hold. As with other endogenous variables, we let bars denote steady state levels, so $\overline{\theta}$ now denotes the (endogenous) steady-state level of θ_t.

We cannot solve for $\overline{\theta}$ explicitly, but we can define it implicitly from two relationships between $\overline{L}$ and $\overline{\theta}$. The first, depicted as $\mathcal{L}^I(\theta)$ in Figure 2(a), shows where the two rural populations, S_t and L_t, are simultaneously stationary. Graphically, this is derived from Figure 1(b), where a fall in $\overline{\theta}$ from 1 to 0, shifts down the curve along which $w_t^L = 1/\left(\gamma\overline{\theta}\right)$, leading to a fall in $\overline{L}$.

To derive $\mathcal{L}^I(\theta)$ explicitly we first solve (17) and (18) for the demand for free labor in terms of w_t^L and w_t^S; see (A7) in the appendix. Then imposing steady state on L_t, w_t^L, and w_t^S, using (21) and (22), shows us that $\overline{L}$ and $\overline{\theta}$ must satisfy $\overline{L} = \mathcal{L}^I(\overline{\theta})$, where

$$\mathcal{L}^I(\theta) = \left(\alpha A^R\right)^{\frac{1}{1-\alpha}} (\eta\gamma\theta)^{\frac{1}{1-\rho}} \left[(\beta\gamma)^{\frac{\rho}{1-\rho}} + \eta^{\frac{1}{1-\rho}} (\gamma\theta)^{\frac{\rho}{1-\rho}}\right]^{\frac{\alpha-\rho}{\rho(1-\alpha)}}. \tag{23}$$

It is straightforward to show that $\partial\mathcal{L}^I(\theta)/\partial\theta > 0$. [This can be seen directly from Figure 1(b) but is shown more formally in the Online Appendix.] Intuitively, less free-worker out-migration (higher $\overline{\theta}$) has a positive effect on the free rural steady-state population, both directly and by crowding out slave labor.

Next, we write the steady-state wage rate in the urban sector as

$$\overline{w}^F = \frac{\overline{w}^L}{\varepsilon^{\gamma q}} = \frac{1}{\varepsilon^{\gamma q}\overline{\theta}\gamma}, \tag{24}$$

where the first equality implies that free workers are indifferent between working in the two sectors [cf. (15)], and the second that the free rural working population is constant, $\overline{w}^L = 1/\left(\overline{\theta}\gamma\right)$ [cf. (21)].

Now the second relationship that must hold between $\overline{L}$ and $\overline{\theta}$ shows where the free rural and urban populations, F_t and L_t, are simultaneously stationary. This holds when $\overline{L} = \mathcal{L}^{II}(\overline{\theta}, A^U)$, where

$$\mathcal{L}^{II}(\theta, A^U) = \left[\frac{\theta - \varepsilon^{1-\gamma q}}{1-\theta}\right]\left(\delta A^U \varepsilon^{\gamma q}\theta\gamma\right)^{\frac{1}{1-\delta}}, \tag{25}$$

which is also depicted in Figure 2(a). The factor within square brackets gives the steady-state ratio of rural to urban free workers, and is derived by imposing steady state on (16); the second factor denotes the steady state size of the urban population, derived from (24) and (20).

It is easy to see that $\mathcal{L}^{II}(\varepsilon^{1-\gamma q}, A^U) = 0$ and that $\mathcal{L}^{II}(\theta, A^U)$ is increasing in θ (again, see the Online Appendix). This reflects a direct positive effect on the free rural population from less out-migration, and an amplifying effect by which free workers reproduce faster when fewer live in the urban sector.

Now $\mathcal{L}^{I}(\overline{\theta}) = \mathcal{L}^{II}(\overline{\theta}, A^U)$ defines a steady state non-migration rate, $\overline{\theta} \in (\varepsilon^{1-\gamma q}, 1)$, at which *all three* working populations are simultaneously stationary at $\overline{L}$, $\overline{S}$, and $\overline{F}$, respectively. This steady-state equilibrium is unique under the very plausible and sufficient condition that $\delta \geq (2\rho - 1)/\rho$ (see the Online Appendix), which we shall assume holds from now on. In particular, this always holds if $\delta > \rho$, which in turn also implies that labor is a more important input in urban production than rural ($\delta > \alpha$).[9]

As is illustrated in Figure 2(b), and shown formally in the Online Appendix, an increase in A^U (from $A^{U,L}$ to $A^{U,H}$ in the figure) shifts up $\mathcal{L}^{II}(\theta, A^U)$, resulting in a fall in $\overline{\theta}$. That is, higher urban productivity means that rural-born workers migrate at a higher rate to the urban sector. This can be summarized as follows:

Result 2 *An increase in urban productivity, A^U, leads to higher steady-state out-migration from the rural sector (a fall in $\overline{\theta}$).*

This means that the effects from exogenous changes in $\overline{\theta}$, as summarized in Result 1, can be interpreted as driven by changes in A^U. A higher A^U implies a lower $\overline{\theta}$, and thus larger slave population and smaller free rural population in steady state.

[9] Recall that $\rho > \alpha$. In terms of Figure 2(a), the condition $\delta \geq (2\rho - 1)/\rho$ ensures that $\mathcal{L}^{II}(\theta, A^U)$ is sufficiently steep relative to $\mathcal{L}^{I}(\theta)$ so that they intersect only once. This probably holds regardless; it seems impossible to find parameter values generating multiple $\overline{\theta}$.

4.2.2 Composition of the overall population

Next we derive an expression for the ratio of slaves to total free working population. First we use (16) to find the steady-state rural-urban ratio among free workers, $\overline{L}/\overline{F}$. Then (19), (21), and (22) give the steady-state ratio of slaves to free rural workers, $\overline{S}/\overline{L}$. Section C of the appendix shows that the ratio of slaves to total free working population now becomes:

$$\frac{\overline{S}}{\overline{L}+\overline{F}} = \Gamma\left(\overline{\theta}\right)^{-\frac{1}{1-\rho}}\left(\overline{\theta}-\varepsilon^{1-\gamma q}\right), \tag{26}$$

where $\Gamma = (\beta/\eta)^{\frac{1}{1-\rho}}\left(1/[1-\varepsilon^{1-\gamma q}]\right)$ is constant and does not depend on $\overline{\theta}$ (or A^U).

The slave-free ratio, as expressed in (26), depends on A^U only through the fraction non-migrating free workers, $\overline{\theta}$, where we recall from Result 2 that $\overline{\theta}$ is a decreasing function of A^U. If $\varepsilon^{1-\gamma q} \geq \rho$, then the slave-free ratio in (26) is monotonically increasing in $\overline{\theta}$, and thus monotonically decreasing in A^U. Higher urban productivity then increases the total free population $(\overline{L}+\overline{F})$ more than the slave population $(\overline{S})$. This is what most Malthusian models would predict: increasing productivity in the urban sector raises the relative size of the population group allowed to work there.

The more interesting case occurs when $\varepsilon^{1-\gamma q} < \rho$. Some algebra then shows that the slave-free ratio is maximized when $\overline{\theta} = \varepsilon^{1-\gamma q}/\rho < 1$, associated with some (positive and finite) level of A^U, call it $\widehat{A}^U$. Thus, a marginal increase in urban productivity, from low enough levels $(A^U < \widehat{A}^U)$, raises the steady-state proportion slaves in the population. In other words, urbanization—or "modernization", if you like—tilts the total population's composition toward a higher representation of slaves.

We summarize this as follows (shown more formally in Section C.1 of the appendix):

Result 3 *An increase in urban productivity, A^U, has the following effect on the composition of the population:*
(a) If $\varepsilon^{1-\gamma q} \geq \rho$, then the slave-to-free ratio, $\overline{S}/(\overline{L}+\overline{F})$, is monotonically decreasing in A^U.
(b) If $\varepsilon^{1-\gamma q} < \rho$, then there exists some positive level of A^U, denoted $\widehat{A}^U$, at which the slave-to-free ratio is maximized.

The intuition behind Result 3 relates to a few opposing effects, the relative strengths of which depend on ε and ρ.

First, when the urban sector becomes more productive all free workers earn more, due to free migration. In a Malthusian steady state, this translates to a larger free population.

However, as we saw earlier, this also makes the elite invest more in substitute slave labor, implying an increase in the slave population. Moreover, when more workers choose the urban sector, this lowers the average free worker's fertility, leading to a smaller steady-state free population. These latter two effects can dominate if (1) the child-cost gap between the sectors is large (ε small), and (2) slave owners can easily replace free workers with slaves, implying high substitutability between free and slave labor in the rural sector (high ρ).

The relationship between the slave-to-free ratio in (26) and urban productivity is illustrated in Figure 3 for a few different values of ε. Other parameters are set as realistically as possible, given the stylized nature of the model; see Section D in the appendix. As seen, the lower is ε, the stronger is the tendency for the relationship to be inversely U-shaped. In this numerical example, with a cost gap of 5 ($\varepsilon = .2$) the ratio peaks a bit below 0.45, implying roughly two free workers for every slave.

The broader qualitative insight is perhaps most interesting: how demographically important slavery becomes in a society depends on the extent to which urbanization alters free workers' reproductive behavior, and the degree of substitutability between slave and free labor.

5 Extensions and applications

5.1 Perfect substitutability

An interesting special case arises when slaves and free workers are perfect substitutes ($\rho = 1$). Then the rural sector will use only slave labor, or only free labor, in steady state.[10]

Moreover, absent an urban sector (or other outside options for free workers, implying $\overline{\theta} = 1$), and without productivity differences between free and slave labor ($\eta = 1$), it can be seen that only free labor is used in steady state. This follows because the slave-owning elite discount the future ($\beta < 1$), and therefore always pay slaves less than their marginal product—which is what free workers earn—so the population of free workers always crowds out the slave population.

5.2 Rural productivity

The analysis above considered changes in A^U, holding A^R (and other parameters) constant. A more plausible scenario might be that both increase in tandem, However, it can be seen that a partial rise in A^U has the same effect as an increase in both productivity variables simultaneously, if the ratio $\left(A^U\right)^{1/(1-\delta)} / \left(A^R\right)^{1/(1-\alpha)}$ increases.

A partial rise in A^R works the same way as a rise in A^U, but in reverse. It leads to less out-migration by free workers, a lower steady-state ratio of slaves to free workers in the rural sector, and thus worse treatment of slaves in the transition. The short-run effect somewhat

[10] To see this, substitute the steady-state costs of free and slave labor in (21) and (22) into (19), and note that the steady-state slave-free ratio in the rural sector approaches zero or infinity as ρ approaches unity, depending on whether $\eta\overline{\theta}/\beta$ falls above or below one.

resembles the result in the static principle-agent model of Acemoglu and Wolitzky (2011), where higher prices of the rural output leads to more coercion, by raising the value of effort. However, our model also predicts a lower fraction slaves in the long run, as free workers crowd them out.

Interestingly, an urban sector is needed for rural productivity to matter in our model. If $A^U = 0$, then changes in A^R have no effect on the steady-state composition of the population. Intuitively, all effects on the slave-free composition work through the migration rate. Absent an urban sector, there is no out-migration by rural-born free workers, implying $\overline{\theta} = 1$, and the steady-state ratio of slaves to free workers becomes just $\overline{S}/\overline{L} = (\beta/\eta)^{1/(1-\rho)}$.[11]

5.3 The discount factor

The steady-state composition of the population also depends on the elite's discount factor, β. This can be interesting for at least two reasons. First, Galor and Özak (2014) show that geographical factors seem to impact variation in time preferences. Insofar as such variation also affects the elite, it would in this model impact the population's steady-state composition, suggesting a link from geography to how slave-dependent a society becomes.

The second reason the elite's discount factor can be interesting relates to elite behavior when they anticipate emancipation. Suppose that all slaves are set free with some exogenous probability p in each period. Then the discount factor in the current model would be replaced by $\beta(1-p)$, making the probability of abolition affect the elite's behavior.

More precisely, an increase in p would work the same way as a fall in β, which implies the following in steady state: a smaller slave population, $\overline{S}$; a larger free rural population, $\overline{L}$; a larger free urban population, $\overline{F}$; and a lower slave-to-free ratio, $\overline{S}/(\overline{L}+\overline{F})$.

This is very intuitive. The higher is the risk of abolition, the less the elite invest in slaves, so rural production relies more on free labor. Interestingly, this also increases the urban population, since the free rural population, with its higher rates of fertility, partly populates the urban sector through migration.

5.4 Barriers to mobility of free workers

In models without slave labor, when an urban sector attracts free workers, the rural elite might respond by trying to reduce worker mobility. If they could, the elite would want to enslave all workers. If that is not feasible, they may undertake actions to impose a utility loss of migration. For example, the elite could spend on things that make rural-born workers attached to their homeland.

[11] To see this, impose steady state on (19), (21) and (22), with $\overline{\theta} = 1$.

An extension that incorporates labor-augmenting human capital could work in similar ways, if human capital is more important in urban production than in rural. The elite may then find it optimal to strategically underinvest in human capital of free workers to make them less mobile, or even hinder human capital accumulation. (This mechanism relates to, e.g., Canaday and Tamura 2009.)

This contrasts with the results in our model, where the elite rather replace the migrating free workforce by treating the unfree workers better, thus increasing their reproduction.

Interestingly, the availability of slave labor can also reduce the elite's incentives to hinder migration by mobile free workers. (The Online Appendix gives an example of such a model.) More precisely, the lower is the rental price of slave labor, the less the elite prefer to spend on reducing free workers' mobility. Intuitively, the elite is less concerned with the out-migration of free workers if these can easily be replaced by a substitute production factor unable to migrate. In that sense, the presence of an institutionally enslaved population can make the non-enslaved population more free, or at least less restricted in its mobility.

5.5 Slave and free fertility in the United States

Result 3 refers to steady-state effects, which can take many generations to materialize. However, in the transition we should observe associated differences in fertility rates. That is, in the wake of an increase in urban productivity we should be able to infer from differences in fertility rates between free workers and slaves which steady state the economy is converging to.

Table 1 shows slave, free-black, and white fertility rates in the United States, estimated from the 1860 census, the last before abolition.[12] Free blacks may best represent free workers in the model, insofar as their labor was a closer substitute to slave labor than whites, but regardless of which group we use for comparison, slave fertility was higher than that of both other groups. Absent white immigration, and the subsequent abolition of slavery, this would imply an economy in transition toward a steady state with a higher slave-free population ratio.

Our model is consistent with this, if we interpret the US in 1860 as an economy reacting to an increase in A^U from low enough levels, and if $\varepsilon^{1-\gamma q} < \rho$ (cf. Result 3). Such a rise in A^U would push the economy toward a higher slave-free ratio, implying higher slave than free fertility in the transition.

Of course, the model could explain the reverse pattern too (e.g., if $\varepsilon^{1-\gamma q} > \rho$), but *if* fertility of free workers were indeed higher than that of slaves, then *any* Malthusian

[12] This is based on census data from the Inter-university Consortium for Political and Social Research (ICPSR 2005). Fertility is calculated as the number of children of ages 0-9 years, divided by the number of women of ages 20-39 years.

model could account for that, since incomes of whites, and (most probably) free blacks, were higher than those of slaves.[13] Moreover, among blacks freedom itself often induced higher fertility—for example, previously childless slave women often bore children when freed (Allen 2015)—making the pattern in Table 1 even more puzzling. Here the facts are explained by free workers being pulled into the urban sector, where they have lower fertility.

In fact, the observed US fertility differences in Table 1 are not only *consistent* with a growing urban sector (and other changes allowing better outside options for free workers), but our theory suggests that such changes may have partly *caused* these fertility differences. Put another way, the fact that good outside options were available to free workers can explain why the slave population in the US showed so much faster natural growth than the free population.

5.6 Other applications

Regardless of the applicability of our theory to the antebellum US, the insight that urbanization, and/or modernization, need not imply less slavery may have bearing elsewhere. According to Murdock and White's (1969) Standard Cross-Cultural Sample, containing information about 186 supposedly representative preindustrial human societies, many of the most urban and densely populated societies, like the Romans, the Hebrews, and the Aztecs, practiced slavery.

The decline of slavery that came with the fall of the Roman Empire seems to have also come together with a movement of people out of cities, and free workers replacing slaves in rural production (Phillips 1985, p. 36).[14]

6 Conclusions

This paper sets up a Malthusian model with hereditary slavery. Children born by free workers are free, and children of slaves are the property of their parents' masters. The slave population then becomes a form of capital to the slave-owning elite, and the composition of the population depends on the reproductive rates of free workers and slaves, where the latter in turn depends on an elite's choices about how well to treat slaves.

The main theoretical result refers to the effects of increasing urban productivity. This unambiguously leads to a higher slave-to-free ratio in steady state in the rural sector, as slave owners substitute migrating free workers with slaves, and thus to better treatment of slaves in the transition. More interestingly, it can raise the steady-state slave-to-free ratio

[13] At least living standards were higher for freed blacks after abolition (Ransom and Sutch 2001, pp. 2-7).

[14] These non-slave rural workers later lost many of their freedoms by being tied to the land, forming a new class of workers, called *coloni*, a precursor to European serfs.

in the population as a whole, if slaves and free workers are close enough substitutes in the rural sector, and the fertility gap between urban and rural workers is large enough.

This may help us understand, e.g., the high fertility rates of slaves, relative to whites and free blacks, in the antebellum US. The deeper insight is that modernization does not necessarily lead to more freedom, but can set in motion demographic forces leading to an overall more enslaved population.

The model can be altered in many ways. One interesting extension would be to allow for enslavement of free workers and/or manumission of slaves. To model that one would need to be explicit about who gains property rights over the newly enslaved, and what induces the elite to manumit slaves.

It would also be interesting to let the two sectors produce different goods. Depending on application, the rural good could be either food, or an input in urban production. A related extension could be to introduce physical capital as an input in the urban sector, accumulated by either a separate urban elite, or by a single elite whose asset holdings include both slaves and physical capital. Such general equilibrium extensions could alter the workings of the model a great deal.

Hopefully, the framework presented here may serve as a useful theoretical stepping stone toward a model that incorporates some of these extensions.

APPENDIX

A Elite income

Set $M^R = 1$ in (6) and let

$$\widehat{\pi}_t = \max_{(\widetilde{L},\widetilde{S})\in\mathbf{R}_+^2} \left[A^R(\widetilde{S}^\rho + \eta\widetilde{L}^\rho)^{\frac{\alpha}{\rho}} - w_t^L\widetilde{L} - w_t^S\widetilde{S} \right]. \tag{A1}$$

The first-order conditions, evaluated at $\widetilde{S} = S_t$ and $\widetilde{L} = L_t$, can be written as in (17) and (18). Adding (17) and (18) up gives

$$w_t^L L_t + w_t^S S_t = \alpha A^R (S_t^\rho + \eta L_t^\rho)^{\frac{\alpha-\rho}{\rho}} (S_t^\rho + \eta L_t^\rho) = \alpha A^R (S_t^\rho + \eta L_t^\rho)^{\frac{\alpha}{\rho}}. \tag{A2}$$

Evaluating the maximized expression in (A1) at $\widetilde{S} = S_t$ and $\widetilde{L} = L_t$, and using (A2), gives (9).

B Demand for free labor and slaves

Letting $Z_t = S_t^\rho + \eta L_t^\rho$, the first-order conditions in (17) and (18) can be reorganized as follows:

$$\eta L_t^\rho = \eta^{\frac{1}{1-\rho}} \left(\frac{\alpha A^R}{w_t^L} \right)^{\frac{\rho}{1-\rho}} Z_t^{\frac{\alpha-\rho}{1-\rho}}, \tag{A3}$$

and

$$S_t^\rho = \left(\frac{\alpha A^R}{w_t^S} \right)^{\frac{\rho}{1-\rho}} Z_t^{\frac{\alpha-\rho}{1-\rho}}. \tag{A4}$$

Adding (A3) and (A4) we get

$$Z_t = S_t^\rho + \eta L_t^\rho = \left(\alpha A^R\right)^{\frac{\rho}{1-\rho}} Z_t^{\frac{\alpha-\rho}{1-\rho}} \left[\left(\frac{1}{w_t^S} \right)^{\frac{\rho}{1-\rho}} + \eta^{\frac{1}{1-\rho}} \left(\frac{1}{w_t^L} \right)^{\frac{\rho}{1-\rho}} \right], \tag{A5}$$

which can be solved for Z_t:

$$Z_t = \left(\alpha A^R\right)^{\frac{\rho}{1-\alpha}} \left[\left(\frac{1}{w_t^S} \right)^{\frac{\rho}{1-\rho}} + \eta^{\frac{1}{1-\rho}} \left(\frac{1}{w_t^L} \right)^{\frac{\rho}{1-\rho}} \right]^{\frac{1-\rho}{1-\alpha}}. \tag{A6}$$

Now (A3) and (A6) give the elite's demand for free workers in terms of the two factor prices:

$$L_t = \left(\alpha A^R\right)^{\frac{1}{1-\alpha}} \left(\frac{\eta}{w_t^L} \right)^{\frac{1}{1-\rho}} \left[\left(\frac{1}{w_t^S} \right)^{\frac{\rho}{1-\rho}} + \eta^{\frac{1}{1-\rho}} \left(\frac{1}{w_t^L} \right)^{\frac{\rho}{1-\rho}} \right]^{\frac{\alpha-\rho}{\rho(1-\alpha)}}. \tag{A7}$$

Similarly, (A4) and (A6) give the elite's demand for slaves:

$$S_t = \left(\alpha A^R\right)^{\frac{1}{1-\alpha}} \left(\frac{1}{w_t^S}\right)^{\frac{1}{1-\rho}} \left[\left(\frac{1}{w_t^S}\right)^{\frac{\rho}{1-\rho}} + \eta^{\frac{1}{1-\rho}} \left(\frac{1}{w_t^L}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{\alpha-\rho}{\rho(1-\alpha)}}. \tag{A8}$$

A negative relationship between w_t^S and L_t in (A7) requires that the exponent on the expression in square brackets is negative, i.e., $\alpha < \rho$, and the same holds for the relationship between w_t^L and S_t in (A8).

C Levels and composition of the population with endogenous migration

Evaluating (16) in steady state, some algebra gives:

$$\frac{\overline{L}}{\overline{F}} = \frac{\overline{\theta} - \varepsilon^{1-\gamma q}}{1-\overline{\theta}}. \tag{A9}$$

Then evaluating (19) in steady state, using (21) and (22), we see that

$$\frac{\overline{S}}{\overline{L}} = \left(\frac{\beta}{\eta\overline{\theta}}\right)^{\frac{1}{1-\rho}}. \tag{A10}$$

Next note that

$$\frac{\overline{S}}{\overline{L}+\overline{F}} = \frac{\left(\frac{\overline{S}}{\overline{L}}\right)\left(\frac{\overline{L}}{\overline{F}}\right)}{\left(\frac{\overline{L}}{\overline{F}}\right)+1}. \tag{A11}$$

Using (A9) to (A11), and applying some algebra, gives (26), with $\Gamma = (\beta/\eta)^{\frac{1}{1-\rho}} \left(1/[1-\varepsilon^{1-\gamma q}]\right)$.

Using (20) in steady state, and the expression for $\overline{w}^F$ in (24), gives

$$\overline{F} = \left(\delta A^U \varepsilon^{\gamma q}\overline{\theta}\gamma\right)^{\frac{1}{1-\delta}}. \tag{A12}$$

Then (A9), (A10), and (A12) give

$$\overline{L} = \left(\frac{\overline{\theta} - \varepsilon^{1-\gamma q}}{1-\overline{\theta}}\right)\left(\delta A^U \varepsilon^{\gamma q}\overline{\theta}\gamma\right)^{\frac{1}{1-\delta}}, \tag{A13}$$

and

$$\overline{S} = \left(\frac{\beta}{\eta\overline{\theta}}\right)^{\frac{1}{1-\rho}}\left(\frac{\overline{\theta} - \varepsilon^{1-\gamma q}}{1-\overline{\theta}}\right)\left(\delta A^U \varepsilon^{\gamma q}\overline{\theta}\gamma\right)^{\frac{1}{1-\delta}}. \tag{A14}$$

We now see that (A13) is $\mathcal{L}^{II}(\overline{\theta}, A^U)$ in (25).

C.1 Showing Result 3

Rewriting (26) as $\overline{S}/(\overline{L}+\overline{F}) = \Gamma H(\overline{\theta})$, where

$$H(\theta) = \theta^{-\frac{1}{1-\rho}} \left(\theta - \varepsilon^{1-\gamma q}\right), \tag{A15}$$

some algebra shows that

$$H'(\theta) = \frac{\theta^{-\frac{1}{1-\rho}}}{(1-\rho)\theta} \left(\varepsilon^{1-\gamma q} - \rho\theta\right). \tag{A16}$$

It follows immediately from (A16) that $H'(\theta) \geq 0$ if $\varepsilon^{1-\gamma q} \geq \rho$, since $\theta \leq 1$. Moreover, if $\varepsilon^{1-\gamma q} < \rho$, then $H(\theta)$ [and thus $\overline{S}/(\overline{L}+\overline{F})$] is maximized at $\theta = \varepsilon^{1-\gamma q}/\rho$, since $H'(\theta) > (=,<)0$ for $\theta < (=,>)\varepsilon^{1-\gamma q}/\rho$. Note also that $\varepsilon^{1-\gamma q}/\rho \in (\varepsilon^{1-\gamma q}, 1)$ for $\varepsilon^{1-\gamma q} < \rho$.

Next recall from Result 2 that $\overline{\theta}$ is decreasing in A^U. We can now let $\overline{\theta}$ vary on $[0,1]$ by letting A^U vary on $[0,\infty)$, so there must exist some $\widehat{A}^U > 0$ such that $\overline{\theta} = \varepsilon^{1-\gamma q}/\rho$ for $A^U = \widehat{A}^U$. Since the ratio in (26) is proportional to $H(\theta)$, it too is maximized at $A^U = \widehat{A}^U$.

D Parameter values for the simulation in Figure 3

First, we set $\alpha = .6$, as in Hansen and Prescott (2002, Table 3).

We need to set $\rho > \alpha$, to make slave and free labor gross substitutes, and below one to make them imperfect substitutes; we choose $\rho = .75$.

Given ρ, we set $\delta = (2\rho - 1)/\rho$, i.e., at the lower bound needed to ensure that $\overline{\theta}$ is unique. This gives $\delta = 2/3$, putting the labor share in the urban sector close to conventional wisdom, and greater than that of the rural sector.

We set $\beta = .5$. If each period is 25 years, this implies an annual steady-state return to slave investment of $(1/\beta)^{1/25} - 1 \simeq 2.8\%$.

Recall that η measures the productivity of free, relative to slave, labor. We set $\eta = .75$, which implies that a rural economy relying only on slave labor, and in which all slaves are suddenly manumitted, would experience a drop in output by about 16% ($\eta^{\alpha} \simeq 0.84$). In the US South, plantation output fell by about 50% after abolition, depending on what end-year is used for comparison; some of this may be attributed to the Civil War.[15] Plantation slavery in the US South, and the Americas at large, is perhaps also uniquely unsuited for free labor, so 16% may not be unreasonable.

We set $q = 1$, as normalization, and let $\gamma = \widetilde{\gamma} = .5$, meaning workers spend half their income on own consumption and half on offspring.

Also as normalization, we set $A^R = 1$.

[15] Ransom and Sutch (2001, Table F.3) report a fall in crop output of 54% between 1859 and 1867; the reduction is smaller between 1859 and later years, but some of that can be due to mechanization.

To understand the implications of the various values for ε in Figure 3, note that, given $\rho = .75$ and $\gamma q = .5$, the relationship becomes non-monotonic for $\varepsilon < \rho^{1/(1-\gamma q)} = .5625$. If $\varepsilon = .25$, then a rural worker would have four $(1/\varepsilon)$ times as many surviving children as an urban worker with the same income, and the equilibrium urban-to-rural wage gap equals $\varepsilon^{-\gamma q} = 2$; cf. (15). Taking this wage differential into account, the rural agent actually has twice as many children as the urban agent $(1/\varepsilon^{1-\gamma q} = 2)$.

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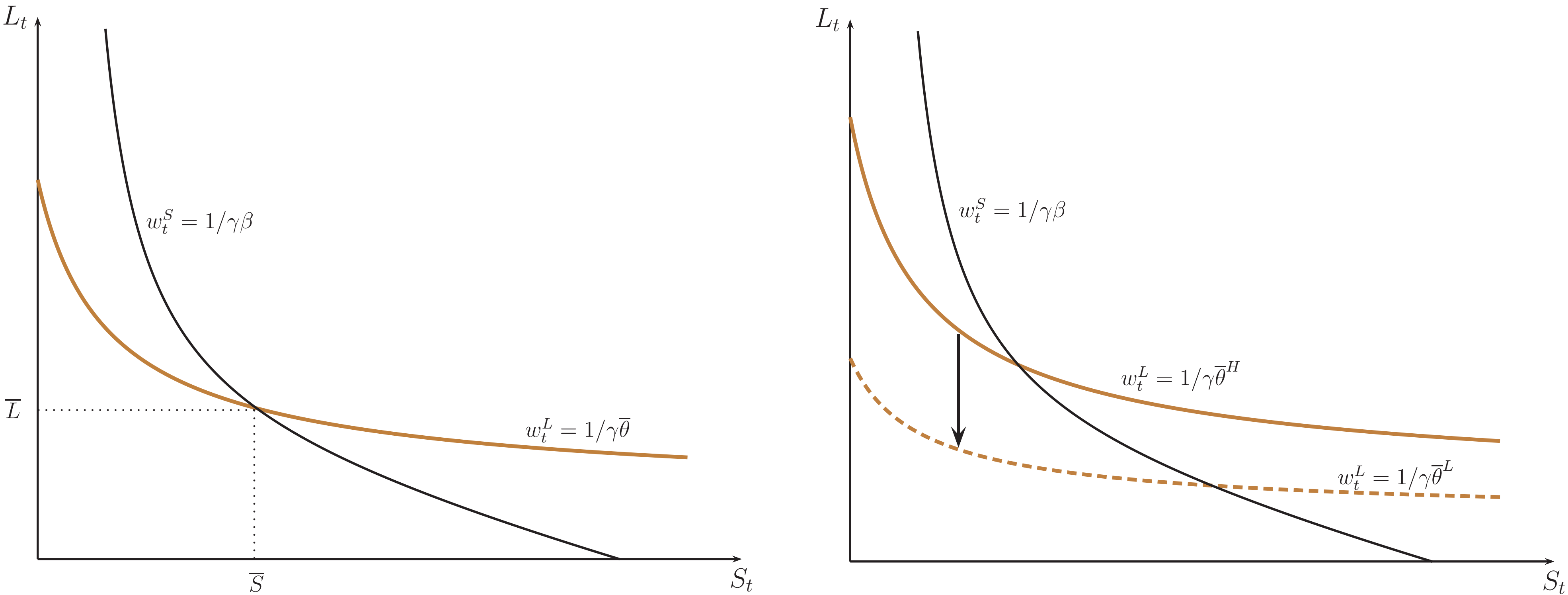
	Estimated US fertility, 1860.
Slaves	2.18
Free blacks	1.62
Whites	1.88

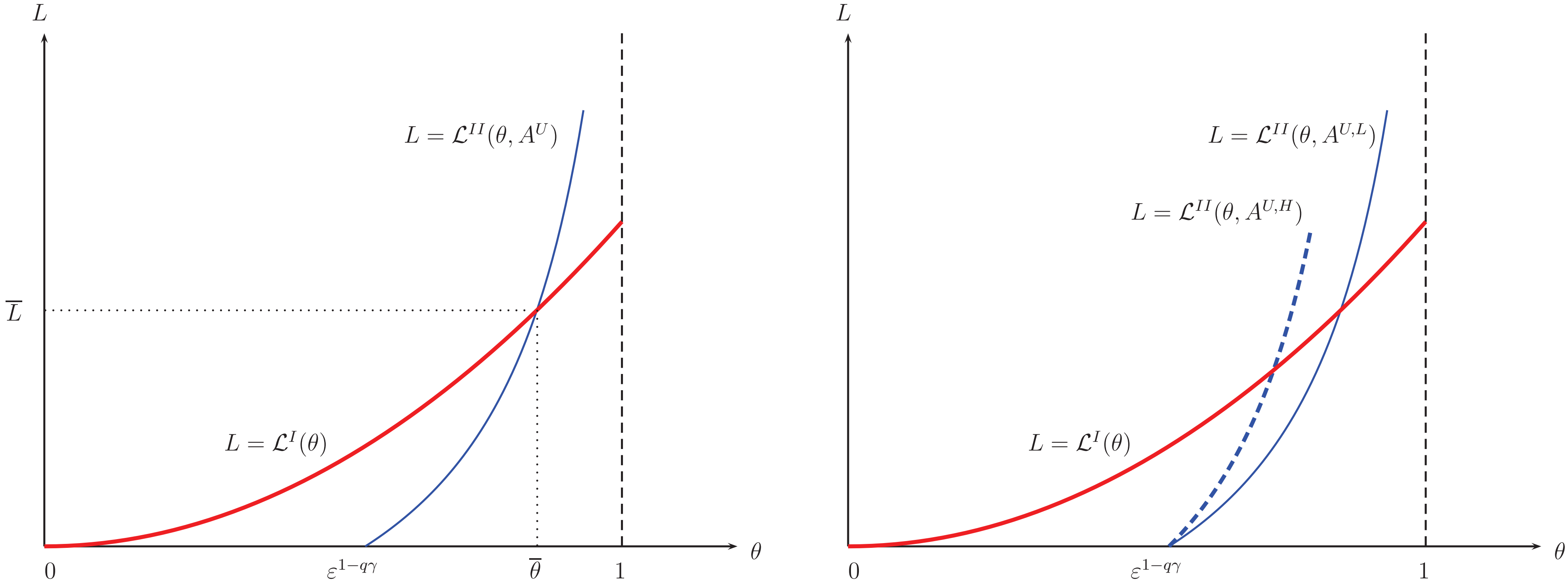
Table 1: Number of children ages 0-9, per woman of ages 20-39.



(a) Determining $\overline{L}$ and $\overline{S}$.

(b) Shifting $\overline{\theta}$.

Figure 1: Panel (a) shows how the steady-state levels of L_t and S_t are determined, given the fraction rural-born free workers who do not migrate, $\overline{\theta}$. The free rural population is constant when $w_t^L = 1/\gamma\overline{\theta}$, and the slave population is constant when $w_t^S = 1/\gamma\beta$. Panel (b) shows the effects of an exogenous fall in $\overline{\theta}$ from $\overline{\theta}^H$ to $\overline{\theta}^L$.



(a) Determining $\overline{\theta}$.

(b) Shifting A^U.

Figure 2: Panel (a) shows how the steady-state non-migration rate, $\overline{\theta}$, is determined, jointly with the steady-state level of the free rural population, $\overline{L}$. $\mathcal{L}^{I}(\theta)$ shows where the two rural populations (S_t and L_t) are simultaneously stationary; $\mathcal{L}^{II}(\theta, A^U)$ shows where the two free populations (F_t and L_t) are simultaneously stationary, given urban productivity, A^U. The intersection shows where all three populations are simultaneously stationary. Panel (b) shows the effects of a rise in A^U from $A^{U,L}$ to $A^{U,H}$.

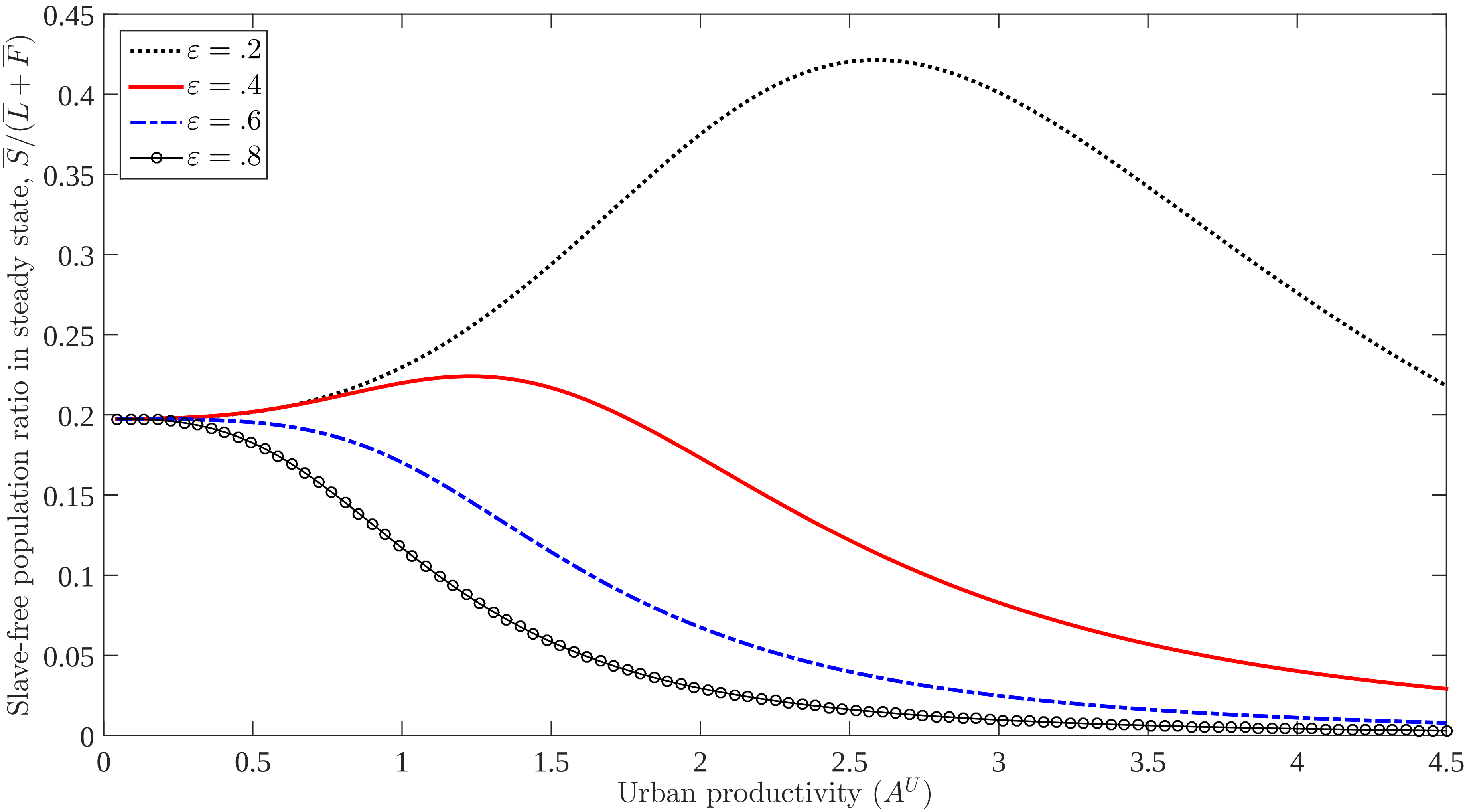


Figure 3: Steady-state ratio of slaves to free working population for different levels of urban productivity (A^U) and different values of the fertility-cost gap between the rural and urban sector (ε). In this numerical example, the relationship becomes non-monotonic for $\varepsilon < \rho^{1/(1-\gamma q)} = 0.5625$.