Complementary notes to "Ethnic diversity, civil war and redistribution"*

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Abstract

We extend Tangerås and Lagerlöf (2009) to incorporate, in turn, (*i*) ethnic groups of varying military strength; (*ii*) total war. The qualitative results remain unchanged. In particular, the non-monotonic relationship between the number of ethnic groups and the likelihood of civil war is non-critical to the symmetry assumption and the partial war scenario considered in the main text.

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1 Introduction

We extend Tangerås and Lagerlöf (2009), henceforth TL, to (i) incorporate ethnic groups of varying military strength; (ii) consider the possibility of total war. In the first extension, there are two types of outsiders, weak and strong. The weak possess a fraction of the military strength of the strong. In the second extension, it is sufficient that one group rebels to throw the entire society into civil war. Consequently, there are only two scenarios, either all groups are involved in the conflict, or none of them are. The qualitative results remain unchanged in the two extensions. In particular, the non-monotonic relationship between the number of ethnic groups and the likelihood of civil war is non-critical to the symmetry assumption and the partial war scenario considered in the main text.

We consider a dynamic game between $N+1$ ethnic groups. In any given period, one of them is in power, the *incumbent* or *ruler*. The $N \geq 1$ other groups are *outsiders*. Each belligerent group (including the incumbent, who cannot choose whether to fight or not) incurs disutility K independently of its military strength. The timing of the stage game is the same as in the main text. In order to keep the analysis tractable, we maintain our attention on *symmetric and time-invariant* equilibria; the amount of redistribution is constant, and all outsiders with the same strength rebel with the same probability at every point in time along the equilibrium path.

2 Asymmetric strength

There are two types of outsiders. $N_1 \geq 0$ outsiders are weak in the sense that they possess $\lambda \in [0,1]$ the military strength of the $N_2 = N - N_1$ strong outsiders. By assumption, the incumbent is always strong.

Any belligerent group's likelihood of winning a conflict depends on the amount of resources invested by that group relative to the total amount invested. A weak [strong] outsider thus wins with probability $\lambda(\lambda M_1 + M_2 + 1 + \lambda)^{-1}$ $[(\lambda M_1 + M_2 + 2)^{-1}]$ whenever M_1 other weak outsiders rebel and M_2 other strong outsiders rebel. Let q_1 and q_2 be the (endogenous) probabilities that a weak, respectively, strong group rebels. The expected probability that a belligerent group of type

$i = 1, 2$ wins a conflict into which is has entered is

$$p_i(\mathbf{q}, \mathbf{N}) = \sum_{M_i=0}^{N_i-1} \sum_{M_j=0}^{N_j} \frac{(N_i-1)! q_i^{M_i} (1-q_i)^{N_i-1-M_i}}{M_i!(N_i-1-M_i)!} \frac{N_j! q_j^{M_j} (1-q_j)^{N_j-M_j}}{M_j!(N_j-M_j)!} \frac{\lambda^{2-i}}{\lambda M_1+M_2+1+\lambda^{2-i}}, \tag{1}$$

where $\mathbf{q} = (q_i, q_j)$, $\mathbf{N} = (N_i, N_j)$ and $i, j = 1, 2$, $i \neq j$. The probability that the incumbent survives to the next period is

$$p_I(\mathbf{q}, \mathbf{N}) = \sum_{M_1=0}^{N_1} \sum_{M_2=0}^{N_2} \frac{N_1! q_1^{M_1} (1-q_1)^{N_1-M_1}}{M_1!(N_1-M_1)!} \frac{N_2! q_2^{M_2} (1-q_2)^{N_2-M_2}}{M_2!(N_2-M_2)!} \frac{1}{\lambda M_1+M_2+1}. \tag{2}$$

The probability that the incumbent is being replaced by an outsider of type $i = 1, 2$ is

$$\overline{p}_i(\mathbf{q}, \mathbf{N}) = \sum_{M_1=0}^{N_1} \sum_{M_2=0}^{N_2} \frac{N_1! q_1^{M_1} (1-q_1)^{N_1-M_1}}{M_1!(N_1-M_1)!} \frac{N_2! q_2^{M_2} (1-q_2)^{N_2-M_2}}{M_2!(N_2-M_2)!} \frac{\lambda^{2-i} M_i}{\lambda M_1+M_2+1}.$$

Civil war breaks out with probability $y(\mathbf{q}, \mathbf{N}) = 1 - (1 - q_1)^{N_1}(1 - q_2)^{N_2}$ along the time-invariant equilibrium path. As is easily verified, $N_i q_i p_i = \overline{p}_i$, so that

$$N_1 q_1 p_1(\mathbf{q}, \mathbf{N}) + N_2 q_2 p_2(\mathbf{q}, \mathbf{N}) + p_I(\mathbf{q}, \mathbf{N}) = 1. \tag{3}$$

The expected value v_i^O of being an outsider of type $i = 1, 2$ is

$$v_i^O = (1 - q_i)\delta v_i^O + q_i[p_i \delta v^I + (1 - p_i)\delta v_i^O - K] \tag{4}$$

along a symmetric and time-invariant equilibrium path without redistribution. The group stays peaceful with probability $1 - q_i$ and rebels with probability q_i. In the first event, the group remains an outsider even the next period, which has value v_i^O discounted by $\delta \in (0, 1)$. In case of conflict, the belligerent incurs disutility K with certainty, expects to win and gain power with probability p_i, the discounted value of which is δv^I, and to lose and remain an outsider with probability $(1 - p_i)$, the discounted value of which is δv_i^O.

The value v^I of being an incumbent along the same equilibrium path is

$$v^I = \theta + p_I \delta v^I + \overline{p}_1 \delta v_1^O + \overline{p}_2 \delta v_2^O - yK. \tag{5}$$

The ruling group keeps θ for itself in the current period. It survives until the next period with probability p_I and is ousted with probability $(1 - p_I)$. The incumbent is replaced by a weak (strong) outsider with probability $\overline{p}_1$ ($\overline{p}_2$). In the first case the ousted group replaces the new incumbent as a weak (strong) outsider. This assumption ensures that the number of weak and strong outsiders is constant across periods.

The net benefit of rebellion is

$$p_i\delta[v^I - v_i^O] - K$$

in group $i = 1, 2$. Rewrite (4) as

$$v_i^O = \frac{p_i q_i \delta v^I - q_i K}{(1-\delta(1-p_i q_i))},$$

substitute v_i^O into the expression above and simplify to get the net benefit of rebellion

$$\frac{1-\delta}{1-\delta+\delta p_i q_i}\left(p_i \delta v^I - K\right).$$

In interior equilibrium, the net benefit of rebellion is zero. Therefore, the conditional probability of winning the conflict is

$$p_i = \frac{K}{\delta v^I},\ i = 1, 2 \tag{6}$$

in interior equilibrium. The right-hand side is independent of whether the group is strong or weak. In interior equilibrium, therefore, the probability of winning the conflict conditional of entering it, is identical for both groups, i.e.,

$$p_1(\mathbf{q}, \mathbf{N}) = p_2(\mathbf{q}, \mathbf{N}) = p(\mathbf{q}, \mathbf{N}). \tag{7}$$

Using (6), we can solve for the equilibrium value of being an outsider, $v_1^O = v_2^O = 0$, and an insider, $v^I = (\theta - yK)(1 - \delta p_I)^{-1}$, in interior equilibrium. Plug v^I into (6), use the identities (3) and (7) to solve for the equilibrium probability of winning the conflict:

$$p(\mathbf{q}, \mathbf{N}) = \frac{(1-\delta)K}{\delta[\theta - (y(\mathbf{q}, \mathbf{N}) + N_1 q_1 + N_2 q_2)K]} \tag{8}$$

We now have two (non-linear) equations (7) and (8) in two unknowns q_1 and q_2.

Assume that $\delta\theta \in (2(1-\delta)K, 2K)$. This is sufficient to render the equilibrium interior for all $N \geq 1$ in the symmetric case when all groups are equally strong. By continuity, this is is sufficient to render even asymmetric equilibria interior for λ sufficiently close to one. The probability of civil war is strictly decreasing in the number of ethnic groups in the symmetric case $\lambda = 1$. By continuity, therefore, there exists a $\underline{\lambda} < 1$ such that the probability of civil war is decreasing in N for all $\lambda \in (\underline{\lambda}, 1]$.

2.1 Power sharing coalitions

Let a *Pacific Coalition Equilibrium* (PCE) refer to a coalition structure in which civil war does not break out in equilibrium and where deviations (joint or unilateral) from the equilibrium path are punished by reversion to the non-redistributive equilibrium. Assume that an $\mathbf{A} = (A_i, A_j)$ coalition has formed, $i, j = 1, 2$, $i \neq j$. The net discounted value of a non-coalition member of type $i = 1, 2$ of remaining at the proposed equilibrium path is $x_i(1-\delta)^{-1}$. Deviating yields

$$x_i + \frac{\lambda^{2-i}}{\lambda A_1 + A_2 + \lambda^{2-i} + 1}\delta\left(v^I - v_i^O\right) + \delta v_i^O - K$$

under the assumption that everyone else behaves according to the proposed equilibrium strategy. The non-coalition outsider remains pacific only if $x_i \geq \underline{x}_i(\mathbf{A}, \mathbf{N})$ where

$$\underline{x}_i(\mathbf{A}, \mathbf{N}) = (1-\delta)\max\{\frac{\lambda^{2-i}(v^I - v_i^O)}{\lambda A_1 + A_2 + \lambda^{2-i} + 1} + v_i^O - \frac{K}{\delta}; 0\},\ i = 1, 2. \tag{9}$$

The corresponding threshold for the coalition member is

$$\underline{x}_i^A(\mathbf{A}, \mathbf{N}) = (1-\delta)\max\{\frac{\lambda^{2-i}(v^I - v_i^O)}{\lambda A_1 + A_2 + 1} + v_i^O - \frac{K}{\delta}; 0\},\ i = 1, 2. \tag{10}$$

Adding up yields a lower bound

$$\underline{X}(\mathbf{A}, \mathbf{N}) = \underline{X}_1(\mathbf{A}, \mathbf{N}) + \underline{X}_2(\mathbf{A}, \mathbf{N}) \tag{11}$$

on total transfers necessary to uphold peace, where

$$\underline{X}_i(\mathbf{A}, \mathbf{N}) = A_i \underline{x}_i^A(\mathbf{A}, \mathbf{N}) + (N_i - A_i)\underline{x}_i(\mathbf{A}, \mathbf{N}).$$

The ruler cares only about the total amount of redistribution as long as the transfers are sufficient to preserve peace. Hence, the upper bound on transfers is given by

$$\overline{X}(\mathbf{N}) = \theta - (1-\delta)v^I.$$

PCE can be sustained by an $\mathbf{A}$ coalition only if $\overline{X}(\mathbf{N}) \geq \underline{X}(\mathbf{A}, \mathbf{N})$. A *minimal-cost* coalition is a coalition that minimizes $\underline{X}(\mathbf{A}, \mathbf{N})$ and thus maximises the possibility of having a pacific equilibrium. Introducing asymmetric strength among ethnic groups has no effect on the minimal-cost coalition:

Lemma 2.1 *The grand coalition is a minimal-cost coalition.*

Proof. We demonstrate below that $A_i = N_i$ minimizes $\underline{X}(\mathbf{A}, \mathbf{N})$ for every A_j. Consequently, $\mathbf{N}$ is a minimal-cost coalition. First, note that $\underline{X}_j(\mathbf{A}, \mathbf{N})$ is weakly decreasing in A_i since $\underline{x}_j^A(\mathbf{A}, \mathbf{N})$ and $\underline{x}_j(\mathbf{A}, \mathbf{N})$ are both weakly decreasing in A_i. Thus, if we can show that even $\underline{X}_i(N_i, A_j, \mathbf{N}) \leq \underline{X}_i(\mathbf{A}, \mathbf{N})$ for all $A_i \in \{0, .., N_i\}$, we are done. There are two cases to consider.

Case (i): $\underline{x}_i^A(N_i, A_j, \mathbf{N}) = 0$. Observe, $\underline{X}_i(N_i, A_j, \mathbf{N}) = N_i \underline{x}_i^A(N_i, A_j, \mathbf{N}) = 0 \leq \underline{X}_i(\mathbf{A}, \mathbf{N})$ for all $A_i \in \{0, .., N_i\}$, where the weak equality follows from non-negativity of $\underline{x}_i^A(\mathbf{A}, \mathbf{N})$ and $\underline{x}_i(\mathbf{A}, \mathbf{N})$. Thus $A_i = N_i$ minimizes $\underline{X}_i(\mathbf{A}, \mathbf{N})$ for every A_j in this first case.

Case (ii): $\underline{x}_i^A(N_i, A_j, \mathbf{N}) > 0$. We proceed by differentiating $\underline{X}_i(\mathbf{A}, \mathbf{N})$ wrt A_i. There are two sub-cases to consider. In the first sub-case $\underline{x}_i(\mathbf{A}, \mathbf{N}) = 0$. Now,

$$\frac{\partial \underline{X}_i(\mathbf{A},\mathbf{N})}{\partial A_i} = \underline{x}_i^A(\mathbf{A}, \mathbf{N}) + A_i \frac{\partial \underline{x}_i^A}{\partial A_i} = \frac{(1-\delta)\lambda^{2-i}(v^I - v_i^O)}{\lambda A_1 + A_2 + 1} - (1-\delta)\left(\frac{K}{\delta} - v_i^O\right) - \frac{A_i(1-\delta)\lambda^{2(2-i)}(v^I - v_i^O)}{(\lambda A_1 + A_2 + 1)^2}.$$

Observe that

$$\underline{x}_i(\mathbf{A}, \mathbf{N}) = 0 \Leftrightarrow \frac{K}{\delta} - v_i^O \geq \frac{\lambda^{2-i}(v^I - v_i^O)}{\lambda A_1 + A_2 + \lambda^{2-i} + 1}.$$

Plugging this result into $\partial \underline{X}_i / \partial A_i$ above produces an inequality:

$$\frac{\partial \underline{X}_i(\mathbf{A},\mathbf{N})}{\partial A_i} \leq -\frac{(A_i \lambda^{2-i} + (A_i - 1)(\lambda A_1 + A_2 + 1))(1-\delta)\lambda^{2(2-i)}(v^I - v_i^O)}{\left(\lambda A_1 + A_2 + \lambda^{2-i} + 1\right)(\lambda A_1 + A_2 + 1)^2},$$

which is negative for all $A_i \geq 1$. In the other sub-case, $\underline{x}_i(\mathbf{A}, \mathbf{N}) > 0$. After some manipulations

$$\frac{\partial \underline{X}_i(\mathbf{A},\mathbf{N})}{\partial A_i} = -\left(\frac{A_1 \lambda^{2-i} + (A_i - 1)(\lambda A_1 + A_2 + 1)}{\left(\lambda A_1 + A_2 + \lambda^{2-i} + 1\right)(\lambda A_1 + A_2 + 1)^2} + \frac{N_i - A_i}{\left(\lambda A_1 + A_2 + \lambda^{2-i} + 1\right)^2}\right)(1-\delta)\lambda^{2(2-i)}(v^I - v_i^O),$$

which is still negative for $A_i \geq 1$. The results for these two sub-cases, hold the implication that either $A_i = 0$ or $A_i = N_i$ minimizes $\underline{X}_i(\mathbf{A}, \mathbf{N})$. The larger coalition is better since

$$\underline{X}_i(0, A_j, \mathbf{N}) - \underline{X}_i(N_i, A_j, \mathbf{N}) = N_i \left(\underline{x}_i(0, A_j, \mathbf{N}) - \underline{x}_i^A(N_i, A_j, \mathbf{N})\right) \geq 0$$

owing to $\underline{x}_i(0, A_j, \mathbf{N}) \geq \underline{x}_i^A(N_i, A_j, \mathbf{N})$ is trivially satified for $\underline{x}_i^A(N_i, A_j, \mathbf{N}) = 0$, and

$$\underline{x}_i(0, A_j, \mathbf{N}) - \underline{x}_i^A(N_i, A_j, \mathbf{N}) = \frac{(N_i - 1)(1-\delta)\lambda^{2(2-i)}(v^I - v_i^O)}{\left(\lambda^{i-1} A_{-i} + \lambda^{2-i} + 1\right)\left(\lambda^{2-i} N_i + \lambda^{i-1} A_{-i} + 1\right)} \geq 0$$

whenever $\underline{x}_i^A(N_i, A_j, \mathbf{N}) > 0$. ■

Obviously, PCE can be sustained by the grand coalition only if $\overline{X}(\mathbf{N}) \geq \underline{X}(\mathbf{N}, \mathbf{N})$. Otherwise, there exists no redistribution scheme which can prevent unilateral deviations. As the grand coalition is a minimal-cost coalition, it immediately follows that *no* defence coalition $\mathbf{A}$ can uphold a pacific equilibrium if the grand coalition cannot. Hence, $\overline{X}(\mathbf{N}) \geq \underline{X}(\mathbf{N}, \mathbf{N})$ is a necessary condition for the existence of a PCE.

We finally show that non-ruling groups have no incentive to pool their resources in a joint effort to overthrow the government. Assume that $\mathbf{J} = (J_i, J_{-i})$ non-ruling groups have pooled their resources in a joint attack on the grand coalition. Subsequently, the members of a winning sub-coalition are assumed to fight a winner-take all battle among themselves for the prize, in this case the benefit of becoming the ruler in the subsequent period. The probability of becoming the subsequent leader is $(\lambda J_1 + J_2)^{-1}$ for a strong member of the sub-coalition and $\lambda(\lambda J_1 + J_2)^{-1}$ for a weak member. The expected value of belonging to a rebellious sub-coalition $\mathbf{J}$ is given by

$$\begin{aligned} & \underline{x}_i^A(\mathbf{N},\mathbf{N}) - K + \tfrac{\lambda J_1+J_2}{\lambda N_1+N_2+1}\{\tfrac{\lambda^{2-i}}{\lambda J_1+J_2}\delta v^I + (1-\tfrac{\lambda^{2-i}}{\lambda J_1+J_2})\delta v_i^O\} + (1-\tfrac{\lambda J_1+J_2}{\lambda N_1+N_2+1})\delta v_i^O \\ = \; & \underline{x}_i^A(\mathbf{N},\mathbf{N}) - K + \tfrac{\lambda^{2-i}\delta\left(v^I - v_i^O\right)}{\lambda N_1+N_2+1} + \delta v_i^O, \end{aligned}$$

for a type $i = 1, 2$ group, which is independent of the coalition $\mathbf{J}$. Thus, no outsider group can benefit more from joining a sub-coalition than from a unilateral deviation. It follows that $\overline{X}(\mathbf{N}) \geq \underline{X}(\mathbf{N},\mathbf{N})$ is also a sufficient condition for the existence of a PCE. The result from TL, that PCEs are sustainable if and only if the grand coalition is immune to unilateral deviations, generalizes to the asymmetric setting.

3 Total war

The analysis in the main text is conducted under the assumption that any outsider unilaterally can decide whether to engage in conflict or stay outside. Any group staying outside the conflict is unaffected by it. This section assumes instead that war affects everybody. All groups will be engaged in conflict if at least one of them initiates it. One can see this modelling assumption as the escalation of conflict being unavoidable.

The value of being an outsider in the total war scenario is

$$V^O(\gamma, q) = (\gamma + (1-\gamma)(1-(1-q)^{N-1}))\left(\tfrac{1}{N+1}\delta v^I + \tfrac{N}{N+1}\delta v^O - K\right) + (1-\gamma)(1-q)^{N-1}\delta v^O$$

and an incumbent

$$v^I = \theta + y\left(\tfrac{1}{N+1}\delta v^I + \tfrac{N}{N+1}\delta v^O - K\right) + (1-y)\delta v^I,$$

where γ is the outsider's likelihood for rebelling. Note that

$$V^O(\gamma, 1) = \tfrac{1}{N+1}\delta v^I + \tfrac{N}{N+1}\delta v^O - K$$

is independent of γ. If an outsider expects the other groups to rebel with probability one, it is indifferent between rebelling and staying pacific; war will break out anyhow. Consequently, perpetual civil war is a Subgame-Perfect Equilibrium of the game of total war. However, other equilibria may coexist with the perpetual war equilibrium. For $q \in (0,1)$, the net benefit of rebellion is

$$V_{\gamma}^{O}(\gamma,q) = (1-q)^{N-1}\left(\tfrac{1}{N+1}\delta(v^{I}-v^{O}) - K\right).$$

In interior equilibrium $V_{\gamma}^{O}(\gamma,q) = 0$, which implies

$$\delta(v^{I}-v^{O}) = (N+1)K. \tag{12}$$

By substituting this relation back into $V^{O}(\gamma,q)$ and v^{I} above, one can solve for the two value functions $v^{O} = V^{O}(q,q) = 0$ and $(1-\delta)v^{I} = \theta - y\,(N+1)\,K$ in interior equilibrium. Substitute these expressions back into (12) and solve explicitly for the equilibrium probability of civil war

$$y^{w} = \tfrac{1}{N+1}\tfrac{\theta}{K} - \tfrac{(1-\delta)}{\delta}, \tag{13}$$

which is an equilibrium for all $\delta\theta/K \in [(N+1)(1-\delta),(N+1)]$. The interior equilibrium has the same qualitative features as in TL. The decisions to rebel are strategic substitutes. More ethnic groups mean a lower probability that each group rebels. The probability of civil war is increasing in θ/K and in δ and is decreasing in $N \geq 1$.

3.1 Redistribution

As shown in TL, a Pacific Transfer Equilibrium (PTE) can be sustained only if $\overline{X}(N) \geq N\underline{x}(N)$, where

$$\underline{x}(N) = \tfrac{1}{2}\,(1-\delta)\,(v^{I}+v^{O}) - \tfrac{(1-\delta)}{\delta}K$$

is the lower bound on transfers to each group and

$$\overline{X}(N) = \theta - (1-\delta)v^{I}$$

is the upper bound on total transfers. Assume that $\delta\theta/K \in [(N+1)(1-\delta),(N+1)]$ and that the economy reverts to the interior equilibrium following a deviation from PTE. By using $v^{O} = 0$, $(1-\delta)v^{I} = \theta - y\,(N+1)\,K$ and (12), we get

$$\overline{X}(N) = \theta - \tfrac{(1-\delta)}{\delta}(N+1)K,\ N\underline{x}(N) = \tfrac{1-\delta}{2\delta}N(N-1)K.$$

As in the main text, PTE can be sustained only for N sufficiently low, since $\overline{X}(N)$ is decreasing and $N\underline{x}(N)$ is increasing in N.

References

[1] Tangerås, Thomas and Nils-Petter Lagerlöf (2009): "Ethnic diversity, civil war and redistribution", *Scandinavian Journal of Economics* **111**: 1-27.