Population, Technology and Fragmentation: Supplementary Notes

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Abstract: These notes discuss some facts and theories, which did not fit into the paper "Population, Technology and Fragmentation" (Lagerlöf 2013).

1 Wars in East Asia and Western Europe

This section presents some descriptive statistics over wars fought in Western Europe and East Asia, as cited in Lagerlöf (2013). The source is Brecke (1999, 2012) and the data file can be found here:

http://www.cgeh.nl/data

The period considered here runs from 1400 (the earliest year for which data is currently available) and 1700 (which in the paper is the end year for the rise in the competition parameter). Western Europe is defined as countries west of 15 degrees east longitude plus Sweden and Italy; East Asia as China, Japan, and Korea.

The table below lists some key statistics:

	Western Europe	East Asia
Total number of wars started 1400-1700	445	403
Average length per war, in years	4.62	3.00
Average number killed per war	40,118	29,992
Average number of contestants	3.18	2.16

To measure the length of the war we use the reported number of years, which includes some observations of zero.

Data on fatalities here refer to total (as opposed to military) fatalities. Data are missing for many wars and there are also many outliers.

Of all the wars started between 1400 and 1700, the deadliest, as measured by total fatalities, is the Thirty Years' War fought in Europe 1618-1648, with a reported 8,000,000 fatalities, or about 267,000 per year. The deadliest war in China (and East Asia) over the same period refers to the fighting when the Qing dynasty ousted the Ming 1644-1662, with 817,000 fatalities in total, or about 45,000 per year.

In other words, the deadliest Chinese war had only 10% as many fatalities in total as Europe's deadliest war, or 17% when comparing annual rates.

2 Trends in population and per-capita incomes in China and Europe from 1000 to today

Figure 1 shows the log population levels for China and Europe from year 1 until today, based on the file "Statistics on World Population, GDP and Per Capita GDP, 1-2008 AD" from Maddison's posthumous website, available here:

http://www.ggdc.net/maddison/oriindex.htm

The numbers are in thousands (before logging); note that the graphs refer to different axes. There is a spurt in population growth after 1000 in both regions. Europe initially grows a little faster after that take-off, up until around 1700, and its overall path is also less volatile. This suggests that Europe's faster population growth may be due to either less devastating mortality shocks, or its elites being better at handling or mitigating such shocks, in particular famines and epidemics.[1] The latter explanation would fit with the mechanism that is at work in the model: that the elites in the fragmented Europe extracted fewer resources from their subjects, to build up a population in response to external threats.

Figure 2 shows the gap between Europe's population and that of China, as well as the corresponding per-capita income gap. As is well known, the per-capita income gap explodes in the wake of Europe's industrialization.

There is also a rise in the population gap after 1000: Europe's population, while always smaller, catches up a little with China's. The gap rises from around 43% in 1000 to 57% in 1500, followed by further large swings, peaking "globally" in the early 20th century a bit over 60%. Then Europe's

[1] War deaths were probably not quantitatively important. According to the Brecke data discussed in Section 1 above, there is little evidence of China being overall more war torn than Europe, at least after 1400, and events in China around the 1644-1662 Ming-Qing transition were far less deadly than the 1618-1648 Thirty Years' War in Europe. However, Maddison (2007) discusses famines and epidemics.

population drops to around 30% of China's, where it stands today. This last drop arrives as Europe becomes the first region to enter the Demographic Transition.

The most relevant observation from Figure 2 in these notes (and from Figure 1 in the paper) is that there was an initial rise in the relative levels of both population and per-capita income in Europe. Moreover, these changes begin prior to the Demographic Transition and the Industrial Revolution, suggesting that they should be explained within a Malthusian framework. How big the changes are depends on what exact year is chosen. Given the volatility of the population gap one could alternatively smooth the path (e.g. through a cubic spline) and then match the model to that smoothed path. The paper instead calibrates the model to fit actual data from 1000 up to the local peak in 1700.

3 Trends in the number of countries in Europe

The table below indicates the number of countries listed as sovereign ("S") in the online Periodis Historical Atlas of Europe (© Christos Nussli 2010), available here:

http://www.euratlas.net/history/europe/index.html

The region in question actually includes North Africa and big parts of today's Russia and Middle East, but most countries were located in what we today might call Europe. Here are the numbers:

Year	Number of countries
1000	57
1300	155
1500	126
1700	105

The declining trend from 1300 illustrates how groups of polities were consolidated into empires or nations, as seems well known from the state capacity literature.

The lower number for 1000 seems related to the fission of a couple of larger polities, like the Roman Empire (later the Holy Roman Empire) and West Francia. However, these were loose confederations with less centralized power.

4 The technology-maximizing level of fragmentation

Here we solve analytically for the degree of fragmentation (N) that maximizes steady-state technology (A^*). In the paper we saw that

$$A^* = Q_A \left[Z \left(\frac{\lambda + \theta \left[1 - \delta\sigma\right]}{N} \right)^{\beta} \right]^{\frac{\kappa}{1-\alpha\kappa}}, \tag{1}$$

where λ and σ are given

$$\begin{aligned} \sigma &= 1 - \beta + \Lambda\beta(1-\rho)\left(1 - \tfrac{1}{N}\right), \\ \lambda &= \alpha + \Lambda\beta\rho\left(1 - \tfrac{1}{N}\right), \end{aligned} \tag{2}$$

and Q_A depends on exogenous parameters other than Z, N and Λ. It follows that A^* depends (positively) on N only through the factor

$$\frac{\lambda + \theta \left[1 - \delta\sigma\right]}{N}. \tag{3}$$

Let N^* be the N that maximizes (3) subject to $N \geq 1$ and (2). Substituting (2) into (3) and letting $x = 1/N$, the first-order condition for an interior solution to the maximization problem can be written

$$\alpha + \theta\left[1 - \delta(1-\beta)\right] + \Lambda\beta\left[\rho - \theta\delta(1-\rho)\right] - 2\Lambda\beta\left[\rho - \theta\delta(1-\rho)\right]x = 0. \tag{4}$$

If the value of N implied by the interior candidate in (4) falls below one, then $N^* = 1$. Disregarding integer constraints on the number of countries (other than $N \geq 1$) this gives

$$N^* = \max\left\{1, \frac{2\beta\Lambda\left[\rho - \theta\delta(1-\rho)\right]}{\alpha + \theta[1 - \delta(1-\beta)] + \beta\Lambda\left[\rho - \theta\delta(1-\rho)\right]}\right\}. \tag{5}$$

If $\theta \geq \rho/[\delta(1-\rho)]$, then N^* is always one, and unified regions always dominate fragmented ones technologically: elites in fragmented regions invest less in technology if it reduces fertility a lot (θ is large), and/or if armies are important in resource competition, relative to technology (ρ is small). If instead $\theta < \rho/[\delta(1-\rho)]$, then fragmented regions may dominate unified. For example, when $\theta = 0$ (5) simplifies to $N^* = \max\{1, 2\beta\Lambda\rho/(\alpha + \beta\Lambda\rho)\}$, so that $N^* > 1$ if $\rho\beta\Lambda > \alpha$. That is, more intense resource competition (higher Λ), or technology being more important in resource competition (higher ρ), make fragmentation more technology promoting.

Next let

$$\overline{\theta} = \frac{\beta\rho - \alpha}{1 - \delta + \beta\delta(2-\rho)}. \tag{6}$$

We can now state the following:

Proposition 1 *Assume that $\alpha < \beta\rho$ and $\theta < \overline{\theta}$. Then there exists a*

$$\widehat{\Lambda} = \frac{\alpha + \theta[1 - \delta(1-\beta)]}{\beta\left[\rho - \theta\delta(1-\rho)\right]} < 1, \tag{7}$$

such that if, and only if, $\Lambda > \widehat{\Lambda}$, then $N^* > 1$.

Proof: For $N^* > 1$ in (5) it must hold that

$$\frac{2\beta\Lambda\left[\rho - \theta\delta(1-\rho)\right]}{\alpha + \theta[1 - \delta(1-\beta)] + \beta\Lambda\left[\rho - \theta\delta(1-\rho)\right]} > 1, \tag{8}$$

which can be seen to give $\Lambda > \widehat{\Lambda}$, where $\widehat{\Lambda}$ is defined in (7). From (6) and (7) some algebra shows that $\widehat{\Lambda} < 1$ holds for positive θ if, and only if, $\beta\rho > \alpha$ and $\theta < \overline{\theta}$. Q.E.D.

5 Asymmetric equilibria

The paper analyzes a symmetric equilibrium in which all countries in fragmented regions are inherently identical in both endowments and behavior. Here we sketch a formulation in which no such symmetry is imposed. We are not able to solve this model here. Rather this presentation serves to illustrate how difficult it is to solve.

Let the subindex i denote the country, where $i \in \{1, ..., N\}$. The dynamics for country i can be written as in the paper, except that variables which differ across countries carry a subindex i. Population and technology for country i thus evolve according to:

$$\begin{aligned} A_{i,t+1} &= BA_{i,t}^{\phi}(s_{i,t}P_{i,t})^{\omega}, \\ P_{i,t+1} &= \left(\tfrac{\gamma Z}{\upsilon}\right)(1-\tau_{i,t})A_{i,t}^{\alpha}R_{i,t}^{\beta}[P_{i,t}(1-s_{i,t}-m_{i,t})]^{1-\beta}. \end{aligned} \tag{9}$$

Let $\chi_{i,t}$ be the resource share that belongs to country i regardless of its inputs into the contest function, and $1-\sum_{i=1}^{N}\chi_{i,t} = \Lambda_t \in [0,1]$ the share of the total resource that is contested. Country i's resource, $R_{i,t}$, is now determined by

$$R_{i,t} = \chi_{i,t} + \Lambda_t \left(\frac{A_{i,t}}{A_{i,t}+(N-1)\overline{A}_{i,t}}\right)^{\rho}\left(\frac{m_{i,t}P_{i,t}}{m_{i,t}P_{i,t}+(N-1)\overline{m}_{i,t}\overline{P}_{i,t}}\right)^{1-\rho}, \tag{10}$$

where bars denote averages among the $N-1$ countries other than i:

$$\begin{aligned} \overline{m}_{i,t} &= \textstyle\sum_{j=1,j\neq i}^{N} m_{j,t}, \\ \overline{P}_{i,t} &= \textstyle\sum_{j=1,j\neq i}^{N} P_{j,t}, \\ \overline{A}_{i,t} &= \textstyle\sum_{j=1,j\neq i}^{N} A_{j,t}. \end{aligned} \tag{11}$$

The Bellman equation becomes

$$V(A_{i,t}, P_{i,t}; \overline{A}_{i,t}, \overline{P}_t) = \max_{(m_{i,t},s_{i,t},\tau_{i,t})\in[0,1]^3} \ln(\tau_{i,t}Y_{i,t}) + \delta V(A_{i,t+1}, P_{i,t+1}; \overline{A}_{i,t+1}, \overline{P}_{i,t+1}), \tag{12}$$

subject to (9), (10), and

$$Y_{i,t} = ZA_{i,t}^{\alpha+\theta}R_{i,t}^{\beta}[P_{i,t}(1-s_{i,t}-m_{i,t})]^{1-\beta}. \tag{13}$$

In the general case, the equilibrium is defined as N separate sequences, one for each and $i \in \{1, 2, ..., N\}$:

$$\{(A_{i,t}, P_{i,t}, R_{i,t}, m_{i,t}, s_{i,t}, \tau_{i,t})\}_{t=0}^{\infty}, \tag{14}$$

such that (9), (10), and (11) hold for all $t \geq 0$, and where the sequences for $m_{i,t}$, $s_{i,t}$ and $\tau_{i,t}$ solve (12).

To study this equilibrium analytically becomes very involved. Gaps in resources between countries in period t depend on both the exogenously given component $(\chi_{i,t})$, and on endogenous differences in inputs ($A_{i,t}$ and $P_{i,t}$), both of which evolve over time endogenously, and depend on the choices of $m_{i,t}$, $s_{i,t}$ and $\tau_{i,t}$, which in turn are difficult to solve for without imposing a symmetric equilibrium on the first-order conditions.

One special case is when $\chi_{i,t} = \chi_i$ for all $t \geq 0$, and Λ_t is constant at $\Lambda = 1 - \sum_{i=1}^{N} \chi_i$. That is, each country i controls some constant share χ_i and may conquer some fraction of the remaining contested resource Λ. This corresponds to the setting analyzed in Section 5 of the paper, where there is no change over time in the intensity of resource competition, except here the countries may have different χ_i.

Even if we consider this simpler setting, and focus on steady-state outcomes, things get difficult: gaps in resources are constant but still depend on both the exogenously given component (χ_i), and on differences in steady-state inputs (A_i^* and P_i^*), both of which depend on the endogenous steady-state choices of m_i^*, s_i^* and τ_i^*.

References

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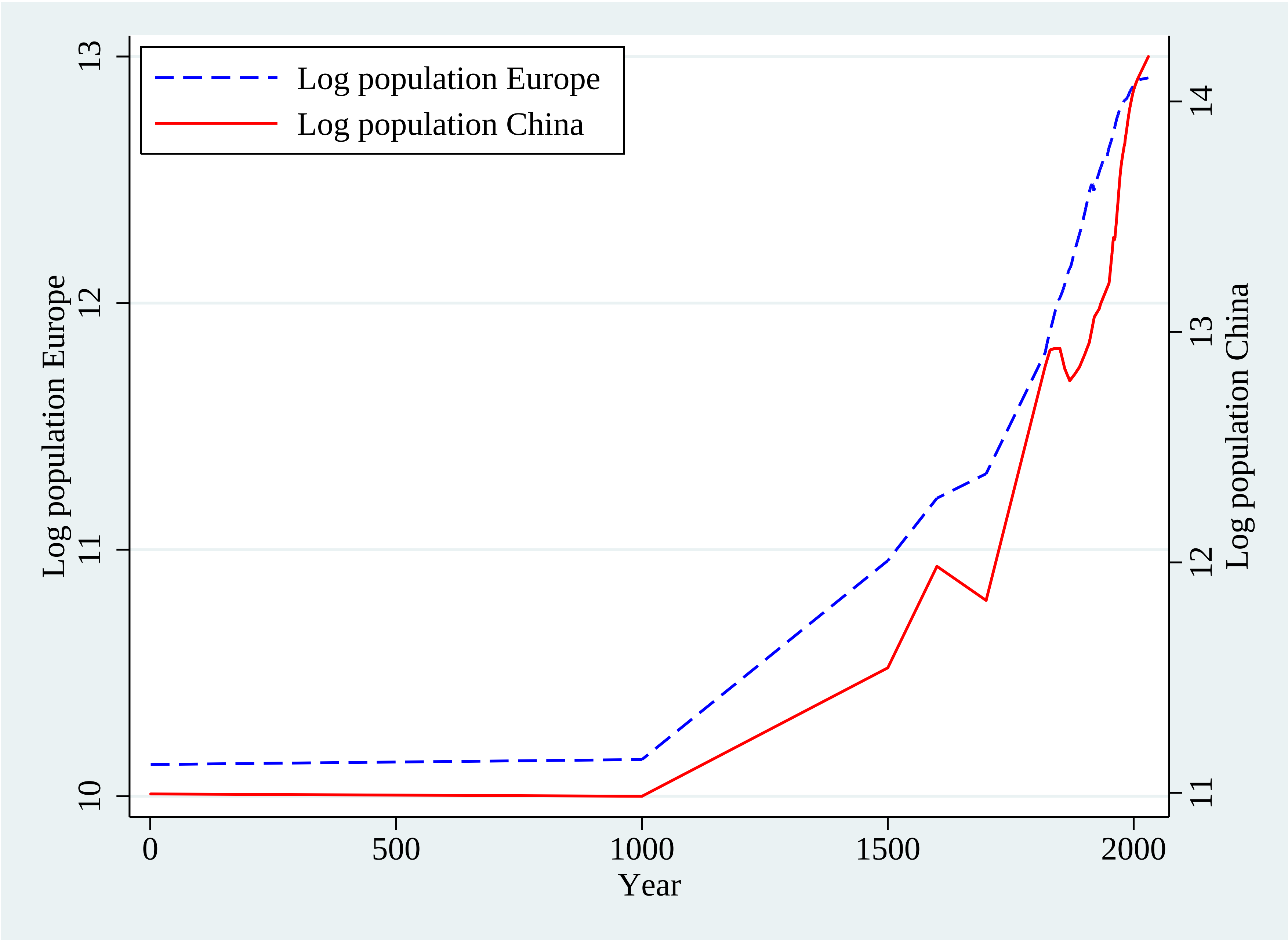

Figure 1: Population levels in China and Europe from year 1 to today.

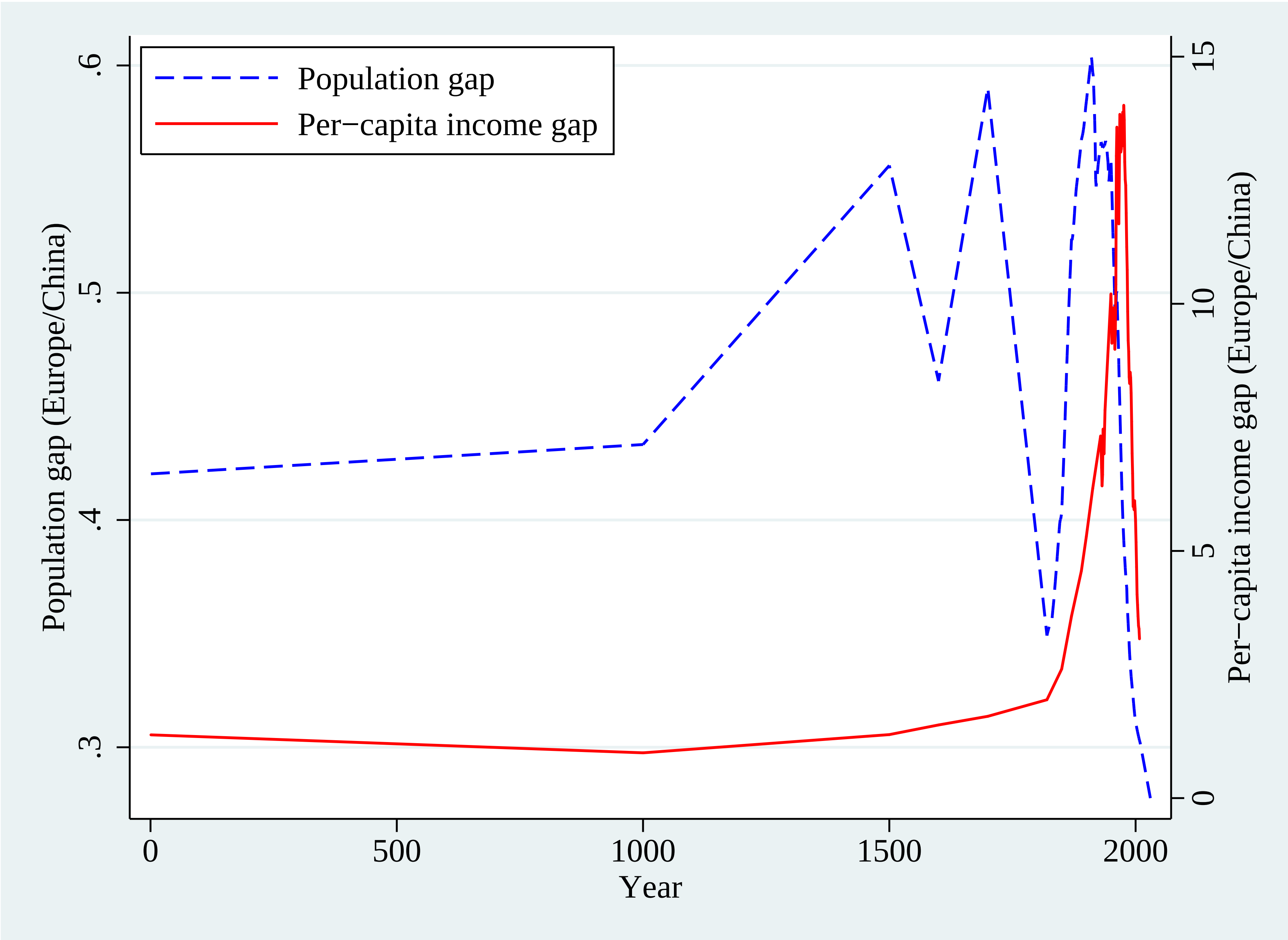

Figure 2: Gaps in population and per-capita incomes between China and Europe from year 1 to today.