Supplementary Notes to Slavery and Other Property Rights

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1 Introduction

These notes consider some alternative formulations of the model set up in Lagerlöf (2006).[1] The disposition is as follows. Section 2 examines an equilibrium approach to the institutional choice problem. Section 3 shows how to interpret free labor and slavery in a principal-agent setting. Section 4 addresses the issue of guarding the guards. Section 5 presents a setting where the elite do not constitute an arbitrarily small fraction of the population. Finally, Section 6 derives the institutional borders in the framework where guarding costs increase with technology.

Throughout the analysis that follows all variables refer to the same time period. To facilitate the disposition somewhat, we therefore suppress all time-indexes, t.

[1] The 2006 version is available on my homepage and slightly different from that published in the Review of Economic Studies. In particular, the analysis in Section 2 is slightly less relevant to the published version, but left in these notes, since some other sections make references there.

2 The equilibrium institution

In the paper the institution is determined according to what maximizes each elite's payoff if all elites can coordinate (e.g., through voting) on the same institution.[2] Call this the optimal institution.[3] Now instead let the institution be chosen by each society's elite, taking as given the choices made simultaneously by the elites in other societies. This gives the equilibrium institution.

The equilibrium approach does not internalize market effects from institutional choices in each society. These effects may arise if equilibrium prices (of slaves and free workers) depend on how many societies choose slavery and free labor, respectively. Also, the payoff to the elite of choosing a particular institution depends on migration decisions made by agents, driven by wages and other factors which depend on what other elites choose.

Another difference is that the optimal institution is unique, but the equilibrium institution need not be. If there are multiple equilibria, then there are also mixed-strategy equilibria, where different elites choose different institutions.

The timing of events is as follows. First, P agents are born into each society. Then in each society the elite choose one of the three institutions. Agents may then migrate across societies, if the institution so allows. Finally, factor prices and payoffs to the elites are realized.

We begin by analyzing the elite's institutional choice when choosing between two institutions at a time, taking as given the choices made by other elites (Sections 2.1, 2.2, and 2.3). Once we know how these choices are made, it is easy to see the outcome when elites choose simultaneously between all

[2] We here abstract from the terminology "external" and "internal" elite. Like in the paper, when referring simply to the elite, we implicitly mean the internal elite when the institution is egalitarianism, and the internal and external elite collectively when the institution is free labor or slavery.

[3] By "optimal" we mean from the perspective of the elite, not in a Pareto sense. For example, slavery is always Pareto inefficient, because using workers as unproductive guards is a social waste, but may nevertheless maximize the income of the elite.

three institutions (Section 2.4).

2.1 Slavery and free labor

Consider first the choice between free labor and slavery. There are N societies, and P agents are born into each society. In each society, the elite choose whether to enslave all P agents, or set them free. As explained in the paper, keeping slaves requires γ guards per slave (the guards are slaves too), and every slave and guard consumes $\bar{c}$. Here we also explicitly allow for trade in slaves across societies (although it will be seen that no trade occurs in equilibrium). Under free labor, agents can migrate and work for other elites at an inter-society wage rate, w.

2.1.1 Free labor

Like in the paper, the elite's maximum profit if choosing free labor can be written:

$$\pi^F = \max_{L \geq 0} \left\{ A^\alpha L^{1-\alpha} - wL \right\}. \tag{1}$$

Solving this maximization problem, demand for free labor becomes:

$$L = \left(\frac{1-\alpha}{w} \right)^{\frac{1}{\alpha}} A. \tag{2}$$

Substituting back into (1) gives the profit as a function of the wage rate, w:

$$\pi^F = A\alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}}. \tag{3}$$

There are two cases to consider. First, if the wage rate is constrained to subsistence, $w = \bar{c}$, it is seen directly from (3) that the payoff becomes

$$\pi^F = \alpha A \left(\frac{1-\alpha}{\bar{c}} \right)^{\frac{1-\alpha}{\alpha}}. \tag{4}$$

If the wage rate exceeds subsistence consumption, $w > \bar{c}$, we need to find an expression for the equilibrium wage. Let $\theta \in [0, 1]$ denote the fraction societies whose elites choose free labor. Total supply of free labor across all N societies then equals $N\theta P$. Labor demand in each free labor society is given by (2) and there are $N\theta$ such societies. Thus, aggregate labor demand across all societies becomes $N\theta[(1-\alpha)/w]^{1/\alpha}A$. Equalizing supply and demand (that is, setting $N\theta P = N\theta[(1-\alpha)/w]^{1/\alpha}A$), the θ's cancel, and we get the wage rate as $w = (1-\alpha)(A/P)^{\alpha}$. Together with (3) this gives:

$$\pi^F = \alpha A^{\alpha} P^{1-\alpha}. \tag{5}$$

Using $w = (1-\alpha)(A/P)^{\alpha}$, it is seen that $w > (=)\bar{c}$ when $A > (\leq)[\bar{c}/(1-\alpha)]^{1/\alpha}P$. If $A > [\bar{c}/(1-\alpha)]^{1/\alpha}P$, π^F is given by (4); if $A \leq [\bar{c}/(1-\alpha)]^{1/\alpha}P$, π^F is given by (5). That is,

$$\pi^F = \begin{cases} \alpha A^{\alpha} P^{1-\alpha} & \text{if } A > \left[\frac{\bar{c}}{1-\alpha}\right]^{\frac{1}{\alpha}} P, \\ \alpha \left[\frac{1-\alpha}{\bar{c}}\right]^{\frac{1-\alpha}{\alpha}} A & \text{if } A \leq \left[\frac{\bar{c}}{1-\alpha}\right]^{\frac{1}{\alpha}} P, \end{cases} \tag{6}$$

which is identical to Eq. (11) in the paper (except for the suppressed time-indexes). We can now state the following.

Result 1 *Consider a set of societies whose elites choose between free labor and slavery. The payoff to one elite of choosing free labor, π^F, is given by (6).*

This payoff does not depend on the fraction of all other elites who choose free labor, θ. Intuitively, when the elite in one society let their agents free this increases the total labor supply from which other elites hire their workers. At the same time, when choosing the free labor institution the elite automatically enter the labor market as buyers, thus increasing demand. Since all societies are identical the net effect on prices and profits is zero.

2.1.2 Slavery

The payoff of choosing slavery can be derived in two ways. The first abstracts from trade in slaves between different slaveowning elites, so that the elite in each society use their endowment of P agents (all or some of them) as slaves and guards. That approach becomes identical to the one in the paper, and generates a payoff like Eq. (17) in the paper; see (14) below.

With a market for slaves, there are potential externalities on slave prices from the institutional choices made by the elite in each society. To make sure we are not ruling such externalities out, consider now a different approach. Let there be a slave market, where the (endogenous) price of a slave is υ. Then the profit can be calculated in a manner analogous to the free labor case: the elite sell all P agents as slaves, receiving υP, and then buys S new slaves, each of whom requires γ guards. The resulting maximum profit can be written:

$$\pi^S = \upsilon P + \max_{S \geq 0} \left\{ A^\alpha S^{1-\alpha} - w^S S \right\}. \tag{7}$$

where we let $w^S = (\bar{c} + \upsilon)(1+\gamma)$. This gives demand for slaves as

$$S = \left(\frac{1-\alpha}{w^S} \right)^{\frac{1}{\alpha}} A. \tag{8}$$

This can be substituted back into (7) to give:

$$\pi^S = \upsilon P + A\alpha \left(\frac{1-\alpha}{w^S} \right)^{\frac{1-\alpha}{\alpha}}. \tag{9}$$

Under slavery, the elite dispose freely over their agents, and may kill some of them (e.g., by not giving them any food). Therefore, the slave price cannot be negative, so there are two cases to consider: $\upsilon = 0$, and $\upsilon > 0$. If $\upsilon = 0$, it follows from (9) and $w^S = \bar{c}(1+\gamma)$ that:

$$\pi^S = A\alpha \left(\frac{1-\alpha}{\bar{c}[1+\gamma]} \right)^{\frac{1-\alpha}{\alpha}}. \tag{10}$$

To determine when $\upsilon > 0$, or $\upsilon = 0$, we next examine the slave market. Because a fraction θ of all elites choose free labor, a fraction $1-\theta$ choose slavery, so the total supply of slaves across all N societies is $N(1-\theta)P$. Demand for slaves in each slave society is given by (8), so across all N

societies demand for slaves becomes $N(1-\theta)[(1-\alpha)/w^S]^{1/\alpha}A$. Equalizing supply and demand (setting $N[1-\theta]P = N[1-\theta][(1-\alpha)/w^S]^{1/\alpha}A$) gives $w^S = (1-\alpha)(A/P)^\alpha$. Recalling that $w^S = (\bar{c}+\upsilon)(1+\gamma)$, and that slave prices cannot be negative, we get the slave price:

$$\upsilon = \max\left\{0, \left(\frac{1-\alpha}{1+\gamma}\right)A^\alpha\left(\frac{P}{1+\gamma}\right)^{-\alpha} - \bar{c}\right\}. \tag{11}$$

Thus, $\upsilon > (=)0$ is equivalent to $A > (\leq)\Gamma(P;\gamma)$, where

$$\Gamma(P;\gamma) = \left(\frac{1}{1+\gamma}\right)\left(\frac{\bar{c}(1+\gamma)}{1-\alpha}\right)^{\frac{1}{\alpha}}, \tag{12}$$

which is also found in the paper. Recall from the paper that, in a setting where there is no slave market, $A \leq \Gamma(P;\gamma)$ implies that the elite choose to use only some of their P agents as slaves, killing the remainder. Intuitively, A is too low to make it profitable to feed and guard all P agents. When there is a slave market, this is equivalent to a zero slave price.[4]

If $\upsilon > 0$, from (9), (11), and $w^S = (\bar{c}+\upsilon)(1+\gamma)$ it follows that

$$\begin{aligned}\pi^S &= \overbrace{(1-\alpha)A^\alpha\left(\frac{P}{1+\gamma}\right)^{1-\alpha} - \bar{c}P}^{\upsilon P} + \overbrace{\alpha A^\alpha\left(\frac{P}{1+\gamma}\right)^{1-\alpha}}^{A\alpha\left([1-\alpha]/w^S\right)^{\frac{1-\alpha}{\alpha}}} \\ &= A^\alpha\left(\frac{P}{1+\gamma}\right)^{1-\alpha} - \bar{c}P.\end{aligned} \tag{13}$$

To sum up, if $A > \Gamma(P;\gamma)$, and thus $\upsilon > 0$, π^S is given by (13); if $A \leq \Gamma(P;\gamma)$, and thus $\upsilon = 0$, π^S is given by (10). That is,

$$\pi^S = \begin{cases} A^\alpha\left(\frac{P}{1+\gamma}\right)^{1-\alpha} - \bar{c}P & \text{if } A > \Gamma(P;\gamma), \\ \alpha\left[\frac{1-\alpha}{(1+\gamma)\bar{c}}\right]^{\frac{1-\alpha}{\alpha}} A & \text{if } A \leq \Gamma(P;\gamma), \end{cases} \tag{14}$$

[4] In this case, the elite may choose to let those agents who are not enslaved free (rather than killing them). That would raise the supply of free labor, and thus give a higher payoff to elites who choose free labor. However, it turns out that free labor is the dominant choice anyhow when $A \leq \Gamma(P;\gamma)$, so allowing for such an extra supply of free labor would not change the equilibrium choices of the elites.

which is identical to Eq. (17) in the paper (except for the suppressed time-indexes). We can now state the following.

Result 2 *Consider a set of societies whose elites choose between free labor and slavery. The payoff to one elite of choosing slavery, π^S, is given by (14).*

It is interesting to note that the profit from slavery is the same whether the elite use the P domestic agents as slaves and guards, or trade in slaves on a market. Intuitively, because societies are identical, the slave price adjusts so that no trade occurs in equilibrium.

Moreover, because the payoffs to the elites are identical to the payoffs used in the paper [Eqs. (11) and (17)], the conditions under which $\pi^S > \pi^F$, and vice versa, are the same as those derived in the paper. Thus, the border separating free labor and slavery, $A = \Psi(P)$, is the same whether we take the optimal or the equilibrium approach.

2.2 Free labor and egalitarianism

Consider next the choice between free labor and egalitarianism. Like in Section 2.1, there are N societies and all societies start with P agents. Under both institutions, all P agents are free to work for any landowning elite at the inter-society wage rate, w. Agents may, or may not, be allowed to migrate to egalitarian societies; it is argued below that there are good reasons to assume that they are not. However, to facilitate the presentation, we begin by considering the case when such migration is allowed.

2.2.1 Free migration

Let η denote the fraction societies whose elites choose the free labor institution, and let ε denote the fraction agents (across all societies) who choose to live in free labor societies. With εP agents choosing to live and work in free labor societies, total labor supply across all N societies equals $N\varepsilon P$. Labor demand in each free society is given by (2) and there are $N\eta$

free labor societies. Thus, aggregate labor demand across all societies becomes $N\eta[(1-\alpha)/w]^{1/\alpha}A$. Equalizing supply and demand (setting $N\varepsilon P = N\eta[(1-\alpha)/w]^{1/\alpha}A$) gives the wage rate as follows:[5]

$$w = (1-\alpha)\left(\frac{\eta A}{\varepsilon P}\right)^{\alpha}, \tag{15}$$

which can be substituted into the profit function in (3). After some algebra we get:

$$\pi^F = \alpha A^{\alpha} P^{1-\alpha}\left(\frac{\varepsilon}{\eta}\right)^{1-\alpha}. \tag{16}$$

Note that, the smaller is the fraction agents who choose to live in free labor societies (the smaller is ε), the lower is the payoff to the elites in free labor societies.

Next we calculate the payoff to the elite of choosing egalitarianism. There are $N(1-\eta)$ egalitarian societies, and $N(1-\varepsilon)P$ agents who choose to live in egalitarian societies. This makes $(1-\varepsilon)P/(1-\eta)$ agents per egalitarian society. The payoff to each agent in such a society, as well as to the elite, is thus:

$$\pi^E = \left(\frac{[1-\eta]A}{[1-\varepsilon]P}\right)^{\alpha}. \tag{17}$$

Note that the larger is the fraction agents who choose to live in egalitarian societies (the larger is $1-\varepsilon$), the lower is income of all agents (including the elite) in egalitarian societies.

With free migration, agents must be indifferent between living in free labor societies and egalitarian societies, implying that $\pi^E = w$. Using (15) and (17) this gives the equilibrium fraction agents who choose to live in egalitarian societies:

$$\varepsilon = \frac{\eta(1-\alpha)^{\frac{1}{\alpha}}}{1-\eta+\eta(1-\alpha)^{\frac{1}{\alpha}}}, \tag{18}$$

[5]This implicitly assumes that η and ε are such that $w > \bar{c}$. As long as $\eta < 1$, under free migration this must hold; otherwise workers would migrate to egalitarian societies. However, there may exist equilibria under free migration where $w = \bar{c}$ and $\eta = 1$, because workers in free labor societies then have nowhere else to migrate. See the Section A of Appendix following these notes.

which can be seen to imply that $\varepsilon < \eta$ for $\eta \in (0, 1)$.

2.2.2 No migration to egalitarian societies

Because land is shared equally in egalitarian societies, all agents in such societies (including the elite) have an interest in imposing a law (or policy) which keeps agents from other societies out, although allowing their own agents to emigrate. Likewise, the elite in free labor societies have an interest in preventing their workers from migrating, but would not object to immigration.

We now assume that such a law exists (and is implemented). This implies that only the P agents who were born in each egalitarian society are allowed to live there. However, anyone of those P agents is allowed to migrate to free labor societies. The law would thus affect migration if, and only if, in the absence of migration restrictions, net migration flows into, rather than out of, egalitarian societies. As shown by the following proposition, it does.

Proposition 1 *Under free migration, net migration flows into egalitarian societies and out of free labor societies.*

Proof. Under free migration, $N(1-\varepsilon)P$ agents choose to live in egalitarian societies, and there are $N(1-\eta)$ egalitarian societies. This makes $(1-\varepsilon)P/(1-\eta)$ agents per egalitarian society. The free-migration equilibrium level of ε is given in (18), which shows that

$$\left(\frac{1-\varepsilon}{1-\eta}\right) P = \left(\frac{1}{1-\eta[1-(1-\alpha)^{\frac{1}{\alpha}}]}\right) P > P.$$

Since population per egalitarian society exceeds the P agents who were born there, there is net migration into egalitarian societies. ■

To see the intuition note that, absent migration, $\pi^E = (A/P)^\alpha$, and $w = (1-\alpha)(A/P)^\alpha < \pi^E$. Under free labor, workers split total output minus the landowners' rents, whereas under egalitarianism they just split total output. Put another way, workers are better off not sharing output with landowners. A related point was made by Samuelson (1974).

Proposition 1 implies that a law stopping migration into, but not out of, egalitarian societies would stop migration altogether; no agent wants to migrate from egalitarian to free labor societies anyhow.

The payoffs to the elite from choosing free labor or egalitarianism thus become identical to the expressions used in the paper. More precisely, in a society where the elite choose the egalitarian institution, all P agents stay and share output. The payoff to the elite of choosing egalitarianism thus equals

$$\pi^E = \left(\frac{A}{P}\right)^\alpha . \tag{19}$$

We can now state the following.

Result 3 *Consider a set of societies whose elites choose between free labor and egalitarianism. If migration to egalitarian societies is not allowed, the payoff to one elite of choosing egalitarianism, π^E, is given by (19).*

Likewise, absent migration the fraction agents living in free labor societies must equal the fraction societies choosing the free labor institution: $\varepsilon = \eta$. If the wage rate exceeds subsistence ($w > \overline{c}$), all P agents in free societies can survive on their wage. Then (15) gives the wage rate: $w = (1-\alpha)(A/P)^\alpha$. Thus, $w > \overline{c}$ is equivalent to $A > [\overline{c}/(1-\alpha)]^{1/\alpha} P$. If this holds, (16) and $\varepsilon = \eta$ generate $\pi^F = \alpha A^\alpha P^{1-\alpha}$; otherwise, (3) and $w = \overline{c}$, give $\pi^F = \alpha A[(1-\alpha)/\overline{c}]^{(1-\alpha)/\alpha}$. The payoff is thus the same as in (6) above. This can be summed up as follows.

Result 4 *Consider a set of societies whose elites choose between free labor and egalitarianism. If migration to egalitarian societies is not allowed, the payoff to one elite of choosing free labor, π^F, is given by (6).*

We also note that, because the payoffs are the same as in the paper, so are the conditions under which $\pi^E > \pi^F$, and vice versa. That is, the borders separating free labor and egalitarianism, $A = \Phi(P)$ and $P = 1/\alpha$, are the same whether we take the optimal or the equilibrium approach.

2.3 Slavery and egalitarianism

Consider finally the choice between egalitarianism and slavery. This case becomes rather trivial, since there can be no migration across societies. Even if egalitarian societies were to allow slaves to immigrate, by assumption slaves cannot run away, because guards watch them. Likewise, agents in egalitarian societies would not be better off by migrating to become slaves.

Since there is no migration, the payoff to the elite from choosing egalitarianism equals average output across the P agents, $(A/P)^{\alpha}$, as given in (19). To sum up:

Result 5 *Consider a set of societies whose elites choose between egalitarianism and slavery. The payoff to one elite of choosing egalitarianism, π^E, is given by (19).*

The payoff of choosing slavery is derived just like when comparing free labor and slavery (see Section 2.1 above), and generates the same expression. There are N societies and each has P agents. Let χ denote the fraction societies whose elites choose slavery, so that total supply of slaves equals $N\chi P$. Demand for slaves in each slave society is given by (8), so across all N societies demand becomes $N\chi[(1-\alpha)/w^S]^{1/\alpha}A$, where $w^S = (\bar{c}+\upsilon)(1+\gamma)$. Equalizing supply and demand (setting $N\chi P = N\chi[(1-\alpha)/w^S]^{1/\alpha}A$) gives $w^S = (1-\alpha)(A/P)^{\alpha}$. Recalling that $w^S = (\bar{c}+\upsilon)(1+\gamma)$, and that slave prices cannot be negative, gives the same slave price as in (11). Using the condition for when $\upsilon > 0$, and $\upsilon = 0$, then gives the payoff to choosing slavery as in (14). To sum up:

Result 6 *Consider a set of societies whose elites choose between egalitarianism and slavery. The payoff to one elite of choosing slavery, π^S, is given by (14).*

Like in the previous cases, the payoffs that are compared are the same as in the paper. Thus, so are the conditions under which $\pi^E > \pi^S$, and vice versa. That is, the border separating slavery and egalitarianism, $A = \Omega(P)$, is the same whether we take the optimal or the equilibrium approach.

2.4 Summary

Results 1 to 6 demonstrate that (under certain plausible assumptions) the payoff of choosing anyone particular institution does not depend on which other institution it is compared to, or potentially coexists with. For example, the payoff to choosing slavery is given by (6), when compared both to free labor and egalitarianism. Moreover, the payoffs compared are identical to those used to derive Proposition 1 in the paper. Thus, Proposition 1 also gives the institution that elites choose in equilibrium. For example, for any point (A, P) in the subset $\mathcal{S}^S$ of the state space, it holds that slavery is the unique equilibrium institution, because π^S is greater than both π^E and π^F.

One assumption driving this result is that migration is not allowed into egalitarian societies by agents from free labor societies; however, migration to free societies is allowed. This seems plausible from a political-economy perspective, because land (or, equivalently, output) in egalitarian societies is shared equally. Therefore, all agents in such societies (including the elite) have an interest in keeping agents from other societies out, while allowing their own agents to emigrate. Likewise, the elite in free labor societies have an interest in preventing their workers from migrating, but would not object to immigration. (The losers with such a migration restriction are the workers in free labor societies.)

Section A in the Appendix following these notes analyzes the case when migration is free. The analysis is more cumbersome, but the results resemble those in the no-migration case. More precisely, as shown in Propositions A3 and A4, in the region of the state space where egalitarianism dominates with no migration, there exists an equilibrium under free migration where all agents choose egalitarianism; the corresponding holds for the free labor region. However, this equilibrium need not be unique.

Other assumptions are more implicit. For example, the elite cannot choose more than one institution. That is, we do not allow the elite to divide land between slave based production and free labor production, or sell (or use) some of their agents as slaves and set others free. However, such assumptions are arguably not restrictive, but rather serve to put some structure on the institutional choice problem.

3 A principal-agent model

Here we derive the elite's payoff under slavery and free labor from a principal-agent setting. The general idea is that free workers are paid more because they can walk away. A participation constraint thus requires that they are paid as much as any other principal would pay them, i.e., their marginal product. If workers are surrounded by (sufficiently many) guards they cannot run away (or fail trying with probability one), and effectively become slaves. Moreover, an incentive compatibility constraint requires that agents (guards and workers) choose to exert effort, which is costly to the agent.

The setting and notation relate to Banerjee, Gertler and Ghatak (2002).[6] Each agent has one unit of time, a fraction of which he spends exerting effort, denoted $e \in [0, 1]$. The principal pays h when the agent exerts effort, and l when he does not, so the agent's income is $eh + (1 - e)l$. Like in the paper, income is spent on consumption, c, and children, n. The cost per child is q units of the consumption good.

The agent's utility function is given by:

$$U = \begin{cases} (1-\beta)\ln c + \beta \ln n & \text{if } c \geq e\bar{c}, \\ -\infty & \text{if } c < e\bar{c}, \end{cases} \tag{20}$$

where the budget constraint can be written $c = eh + (1 - e)l - qn$.

Here $\bar{c}$ can be interpreted as metabolic requirements resulting from effort (or physical activity): if the agent does not work he does not need to eat; or, equivalently, he cannot work without sufficient food. Note that if $e = 1$ (as will be the case in equilibrium) the utility function in (20) boils down to that in the paper.

[6] One important difference compared to Banerjee et al. (2002) is that there are here many societies, and thus many landowning elites, so that the best outside option for an agent equals the marginal product of his labor.

If $h - l$ exceeds $\bar{c}$ an increase in e raises income more than metabolic needs, generating a "surplus" which can be spent on consumption and/or fertility. Therefore, an agent's choice of e can be thought of a maximizing this surplus, i.e., $eh + (1 - e)l - \bar{c}e$. Because the surplus is linear in e, the optimal choice is either $e = 1$ or $e = 0$. (We assume the agent chooses $e = 1$ if he is indifferent, $h - l = \bar{c}$.) That is:

$$e = \begin{cases} 1 & \text{if } h - l \geq \bar{c}, \\ 0 & \text{if } h - l < \bar{c}. \end{cases} \tag{21}$$

The principal is here the (internal and external) elite (i.e., the landowner). There are P agents, L of whom are hired as workers and G as guards. (The purpose of these guards is to be explained within this principal-agent framework.)

Workers are productive only if they exert effort, so with L workers and the same production function as in the paper, output is given by $A^{\alpha}(eL)^{1-\alpha}$. The principal's profit can then be written

$$\pi = A^{\alpha}(eL)^{1-\alpha} - [eh + (1 - e)l]\,(L + G). \tag{22}$$

Recalling from (21) that optimal e equals either 1 or 0, (22) can be written

$$\pi = \begin{cases} A^{\alpha}L^{1-\alpha} - h(L + G) & \text{if } e = 1, \\ -l(L + G) & \text{if } e = 0. \end{cases} \tag{23}$$

The principal chooses h, l, L, and G to maximize π subject to $L + G \leq P$ and three other constraints. First, the *incentive compatibility constraint* (ICC) states that the agent chooses e to maximize his payoff in (20). That is, e is given by (21).

Second, the *limited liability constraint* (LLC) says that the principal cannot confiscate resources from the worker:

$$h \geq 0,\, l \geq 0. \tag{24}$$

Finally, the *participation constraint* (PC) states that the agent does not (try to) run away. A run-way agent earns an income m from whatever other principal wants to employ him. If he fails (and is captured by the guards) he is simply thrown back into work for the principal. The probability of a successful escape is decreasing in the (efficient) number of guards, eG; and increasing in the number of workers they are guarding, L. Denoting this probability $R(eG, L)$ the PC can be written:

$$R(eG, L)\left[\left(\max_{e\in[0,1]} eh + (1-e)l\right) - m\right] \geq 0. \tag{25}$$

One can think of many functional forms for $R(eG, L)$. Here we choose one which generates the same profit functions as those postulated in the paper:[7]

$$R(eG, L) = \begin{cases} 0 & \text{if } eG \geq \gamma L, \\ 1 & \text{if } eG < \gamma L. \end{cases} \tag{26}$$

The PC thus implies that the principal would either keep exactly γ guards per worker ($G = \gamma L$), which amounts to slavery; or no guards at all ($G = 0$), which amounts to free labor. In the former case, if all γL guards exert effort ($e = 1$) $R(eG, L) = 0$ and the PC in (25) holds for any h, l, and m. To induce workers and guards to choose $e = 1$ the principal must set h and l to satisfy the ICC in (21). Together with the LLC in (24) this gives $l = 0$ and $h = \bar{c}$. Substituting back into (23) the principal's profit if choosing this option can be written:

$$\pi = A^{\alpha} L^{1-\alpha} - \bar{c}(L + G) = A^{\alpha}\left(\frac{P}{1+\gamma}\right)^{1-\alpha} - \bar{c}(1+\gamma)P, \tag{27}$$

where the second equality uses $G + L = (1+\gamma)L = P$. This is π^S in the paper (in the case where the principal chooses to hire all P agents, i.e., if $G + L \leq P$ binds).

The other option is to keep no guards and thus have to set h and l so that $\max_{e\in[0,1]} eh + (1-e)l \geq m$ to ensure that the PC in (25) holds. Using

[7] Letting probabilities take values 0 or 1 also avoids issues of risk aversion. Note that we cannot simply assume risk neutrality since we have already specified preferences in (20), and these are non-linear.

the LLC in (24) the profit maximizing choice is $l = 0$ and $h = m$; this assumes $m > \bar{c}$ so that the ICC is not binding. Using (23) the profit becomes $A^{\alpha}L^{1-\alpha} - mL$. Taking m as given, the principal in each society chooses L to maximize profits, which gives $m = (1-\alpha)(A/L)^{\alpha}$. Substituting back into $A^{\alpha}L^{1-\alpha} - mL$ and evaluating at $P = L$ (since $G = 0$ and there are P agents per society) gives

$$\pi = \alpha A^{\alpha} P^{1-\alpha}. \tag{28}$$

This is one of the expressions for π^F in the paper. The other arises if $(1-\alpha)(A/P)^{\alpha} < \bar{c}$ so that the ICC in (21) is binding in equilibrium. The principal does not hire all P agents but sets L so that $\bar{c} = (1-\alpha)(A/L)^{\alpha} = m$, and the ICC and PC become identical. As detailed in the paper the profit then becomes $A^{\alpha}L^{1-\alpha} - \bar{c}L = \alpha A[(1-\alpha)/\bar{c}]^{(1-\alpha)/\alpha}$.

4 Guarding the guards

4.1 Guards as slaves

In the paper, guards are slaves and paid subsistence, $\bar{c}$, which raises the question who guards the guards. There are two ways to address this question. First, we can let guards be paid $\bar{c}$, but assume that they, like slaves, must be watched over by other guards, who are watched by other guards, and so on. Formally, let $\widetilde{\gamma} \in (0,1)$ denote the number of agents needed to watch over each slave or guard. To watch S slaves, the elite needs guards in an infinite number of hierarchies: $\widetilde{\gamma}S$ guards on the first level (to watch the S slaves); $\widetilde{\gamma}^2 S$ guards on the second level (to watch the first level of guards); $\widetilde{\gamma}^3 S$ on the third level (to watch the second level); and so on. Summing up slaves and guards across hierarchial levels gives $S(\widetilde{\gamma}+\widetilde{\gamma}^2+\widetilde{\gamma}^3+...) = S/(1-\widetilde{\gamma}) = S(1+\gamma)$, where $\gamma \equiv \widetilde{\gamma}/(1-\widetilde{\gamma})$. This redefinition of γ generates exactly the same model as in the paper. The setting in the paper can thus be interpreted as derived from a setting with hierarchies of guards.

4.2 Free guards

The other way to address to the guarding-the-guards issue is to let guards be free and hired on a labor market at the inter-society wage rate w^G. Each slave requires $\bar{c}$ units of the consumption good, and γ guards, so S slaves cost $\left(\bar{c} + \gamma w^G\right) S$. The maximum payoff to the elite with a slavery institution can thus be written:

$$\pi^S = \max_{S \geq 0} \{A^\alpha S^{1-\alpha} - \left(\bar{c} + \gamma w^G\right) S\}. \tag{29}$$

The number of agents enslaved is given by:[8]

$$S^* = \left(\frac{1-\alpha}{\bar{c} + \gamma w^G}\right)^{\frac{1}{\alpha}} A. \tag{30}$$

We now need to consider two cases: when the equilibrium wage rate exceeds subsistence consumption ($w^G > \bar{c}$), and when the wage rate is constrained to subsistence ($w^G = \bar{c}$). In the former case, the supply of guards is given by the number of agents who are not enslaved, $P - S^*$ per society; demand is γS^* per society. Equalizing supply and demand on the guards market (setting $P - S^* = \gamma S^*$) and using (30) gives the guard wage rate as

$$w^G = \frac{1}{\gamma}\left[(1-\alpha)(1+\gamma)^\alpha \left(\frac{A}{P}\right)^\alpha - \bar{c}\right]. \tag{31}$$

The condition for $w^G > \bar{c}$ can thus be written $A > \Gamma(P;\gamma)$, where $\Gamma(P;\gamma)$ is given by (12) in these notes, and Eq. (14) in the paper. We restate it here for convenience:

$$\Gamma(P;\gamma) = \left(\frac{1}{1+\gamma}\right)^{1-\alpha} \left[\frac{\bar{c}}{1-\alpha}\right]^{\frac{1}{\alpha}}.$$

Vice versa, the guard wage is constrained to subsistence ($w^G = \bar{c}$) when $A \leq \Gamma(P;\gamma)$.

[8] The elite cannot enslave more than P agents, so one may impose the restriction that $S \leq P$. However, this can never bind in equilibrium since it would imply zero guards and an infinite wage rate for guards.

In the case when $w^G > \bar{c}$, (29), (30), and (31), give the elite's payoff with slavery as

$$\pi^S = \left(\frac{1}{1+\gamma}\right)^{1-\alpha} \alpha A^\alpha P^{1-\alpha}. \tag{32}$$

When $w = \bar{c}$, guards are paid the same as slaves. The model thus boils down to the same guards-as-slaves setting used in the paper, with the payoff to the elite given by $\pi^S = \max_{S \geq 0}\{A^\alpha S^{1-\alpha} - \bar{c}(1+\gamma)S\}$. As shown in the paper, when $A \leq \Gamma(P;\gamma)$ the demand for slaves is unconstrained, so the maximization problem gives the optimal number of slaves as $S^* = [(1-\alpha)/\{\bar{c}(1+\gamma)\}]^{1/\alpha} A$. Substituting back into the maximization expression gives the elite's payoff with slavery:

$$\pi^S = \alpha A \left[\frac{1-\alpha}{\bar{c}(1+\gamma)}\right]^{\frac{1-\alpha}{\alpha}}.$$

To sum up, if guards are free and hired on a labor market, the payoff to the elite with a slavery institution is:

$$\pi^S = \begin{cases} \left(\frac{1}{1+\gamma}\right)^{1-\alpha} \alpha A^\alpha P^{1-\alpha} & \text{if } A > \Gamma(P;\gamma), \\ \alpha A \left[\frac{1-\alpha}{\bar{c}(1+\gamma)}\right]^{\frac{1-\alpha}{\alpha}} & \text{if } A \leq \Gamma(P;\gamma). \end{cases}$$

The payoff with slavery thus differs from the guards-as-slaves setting analyzed in the paper only when $A > \Gamma(P;\gamma)$. In this case, the payoff under a free labor institution is $\pi^F = \alpha A^\alpha P^{1-\alpha}$, which is clearly greater than the payoff with slavery. In the case when $A \leq \Gamma(P;\gamma)$, the payoff under slavery is identical to that analyzed in the paper; in this case, the payoff under slavery is shown to be lower than that under free labor. We can sum this up as follows:

Result 7 *If guards are hired on a free labor market, the payoff to the elite under slavery is always lower than the payoff under free labor.*

The intuition is that, when labor is abundant the elite pay guards a subsistence wage, but then the wage the elite would be paying free workers is low as well: either at, or close to, subsistence. This makes the payoff under free labor higher than under slavery. When the guard wage is above subsistence, slavery amounts to keeping the same share, α, of total output, as under free labor, but total output is lower because guards are not contributing to production; thus free labor generates a higher payoff to the elite.

5 Unit mass of the elite

In the paper the external elite are a continuum of agents of mass one and the internal elite consist of a finite number of agents, and are thus of mass zero. Now consider a variation of this model where there is only one elite carrying mass one. As in the paper, P denotes the mass of non-elite agents. Thus, total population equals $1 + P$.

Under slavery and free labor, the elite do not work (which serves to abstract from the subsistence consumption constraint for the elite), so the labor force equals P, and the payoffs are the same as in the paper. However, under an egalitarian regime we let the elite work, so the labor force has size $1 + P$ and total output equals $A^{\alpha}(1 + P)^{1-\alpha}$. Thus, under egalitarianism, an elite agent's payoff (which is the same as that of all other agents) is given by

$$\pi^E = A^{\alpha}(1 + P)^{-\alpha}. \tag{33}$$

This implicitly assumes that every agent is able to work earning the average product $[A^{\alpha}(1 + P)^{-\alpha} \geq \bar{c}]$, and we here restrict attention to combinations of A and P where this holds.

The payoffs under the other two institutions (which are the same as in the paper) are restated here for convenience:

$$\pi^F = \begin{cases} \alpha A^{\alpha} P^{1-\alpha} & \text{if } A \geq \left[\frac{\bar{c}}{1-\alpha}\right]^{\frac{1}{\alpha}} P, \\ \alpha \left[\frac{1-\alpha}{\bar{c}}\right]^{\frac{1-\alpha}{\alpha}} A & \text{if } A < \left[\frac{\bar{c}}{1-\alpha}\right]^{\frac{1}{\alpha}} P, \end{cases} \tag{34}$$

$$\pi^S = \begin{cases} A^{\alpha}\left(\frac{P_t}{1+\gamma}\right)^{1-\alpha} - \bar{c}P & \text{if } A > \Gamma(P;\gamma), \\ \alpha \left[\frac{1-\alpha}{(1+\gamma)\bar{c}}\right]^{\frac{1-\alpha}{\alpha}} A & \text{if } A \leq \Gamma(P;\gamma), \end{cases} \tag{35}$$

where we also recall that

$$\Gamma(P;\gamma) = \left(\frac{1}{1+\gamma}\right)\left[\frac{\overline{c}(1+\gamma)}{1-\alpha}\right]^{\frac{1}{\alpha}} P. \tag{36}$$

Following the analysis in the paper, the next step is thus to examine which of the payoffs in (33), (34), and (35), is larger. The result is very similar to that in the paper, except that two of the functions which separate the institutional regions differ slightly. First, the function

$$\Psi(P) = \left[\frac{\overline{c}(1+\gamma)^{1-\alpha}}{1-\alpha(1+\gamma)^{1-\alpha}}\right]^{\frac{1}{\alpha}} P, \tag{37}$$

is the same as in the paper; recall that it is assumed that $1-\alpha(1+\gamma)^{1-\alpha} > 0$.

The functions $\Omega(P_t)$ and $\Phi(P_t)$, however, are different and now defined as follows:

$$\Omega(P) = \left[\frac{\overline{c}(1+\gamma)^{1-\alpha}P(1+P)^{\alpha}}{P^{1-\alpha}(1+P)^{\alpha}-\widehat{P}}\right]^{\frac{1}{\alpha}}, \tag{38}$$

$$\Phi(P) = \left(\frac{1}{\alpha}\right)^{\frac{1}{1-\alpha}}\left[\frac{\overline{c}}{1-\alpha}\right]^{\frac{1}{\alpha}}(1+P)^{-\left(\frac{\alpha}{1-\alpha}\right)}. \tag{39}$$

Also, let P^* and $\widehat{P}$ be defined from $G(P^*) \equiv 1$ and $G(\widehat{P}) \equiv \alpha(1+\gamma)^{1-\alpha}$, where

$$G(P) = \alpha P^{1-\alpha}(1+P)^{\alpha}. \tag{40}$$

Like in the paper, the state space can be separated into three sets where each respective institution dominates:

$$\begin{aligned} \mathcal{S}^{\mathrm{S}} &= \{(A,P) \in \mathbb{R}_+^2 : A \geq \max\{\Psi(P), \Omega(P)\} \text{ and } P > \widehat{P}\}, \\ \mathcal{S}^{\mathrm{F}} &= \{(A,P) \in \mathbb{R}_+^2 : P \geq P^* \text{ and } \Phi(P) \leq A \leq \Psi(P)\}, \\ \mathcal{S}^{\mathrm{E}} &= \{(A,P) \in \mathbb{R}_+^2 : (A,P) \notin \mathcal{S}^{\mathrm{S}} \cup \mathcal{S}^{\mathrm{F}}\}. \end{aligned} \tag{41}$$

There are two differences compared to the paper. First, one of the conditions for free labor dominating egalitarianism is now $P \geq P^*$, rather than $P \geq 1/\alpha$ in the paper. Second, one of the conditions for slavery dominating egalitarianism is $P > \widehat{P}$, instead of $P > (1+\gamma)^{1-\alpha}$ in the paper.

Using the slightly redefined subsets of the state space in (41), Proposition 1 in the paper still holds, restated here:

Proposition 2 *The payoffs associated with slavery, egalitarianism, and free labor are ordered as follows:*

1. *Slavery (weakly) dominates when*

$$\pi^S \geq \max\{\pi^F, \pi^E\} \Longleftrightarrow (A, P) \in \mathcal{S}^S. \tag{42}$$

2. *Freedom (weakly) dominates when*

$$\pi^F \geq \max\{\pi^S, \pi^E\} \Longleftrightarrow (A, P) \in \mathcal{S}^F. \tag{43}$$

3. *Egalitarianism (strictly) dominates otherwise, i.e., if* $(A, P) \in \mathcal{S}^E$.

This is proven in the Section B in the Appendix.

6 Deriving $\widetilde{\Omega}(P)$ and $\widetilde{\Psi}(A)$

In the setting where the number of guards per slave is increasing in technology, we write the number of guards as

$$\gamma(A) = \begin{cases} \overline{\gamma} & \text{if } A \leq \widetilde{A}, \\ (1+\overline{\gamma})\left[A/\widetilde{A}\right]^{\theta} - 1 & \text{if } A \geq \widetilde{A}. \end{cases} \tag{44}$$

We restrict attention to the relevant payoff pairs that form the institutional borders. For $A \geq \widetilde{A}$, replacing γ by $\gamma(A)$ in (44), using (35), the payoff under slavery now becomes:

$$\pi^S = A^{\alpha}\left(\frac{P_t}{1+\gamma(A)}\right)^{1-\alpha} - \overline{c}P = ZA^{\alpha-\theta(1-\alpha)}P^{1-\alpha} - \overline{c}P, \tag{45}$$

where we have used (44) and denote $Z = \left[\widetilde{A}^{\theta}/(1+\overline{\gamma})\right]^{1-\alpha}$. For free labor and egalitarianism the relevant payoffs are the same as before: $\pi^F = \alpha A^{\alpha} P^{1-\alpha}$, $\pi^E = (A/P)^{\alpha}$ [see (19) and (34)].

We see that $\pi^F \geq \pi^S$ when $\alpha A^\alpha P^{1-\alpha} \geq Z A^{\alpha-\theta(1-\alpha)} P^{1-\alpha} - \bar{c}P$, or:

$$P \geq A \left[\frac{Z A^{-\theta(1-\alpha)} - \alpha}{\bar{c}} \right]^{\frac{1}{\alpha}} \equiv \widetilde{\Psi}^{-1}(A), \tag{46}$$

the inverse of which is graphed in Figure 5 in the paper. That is, in a diagram with P on the vertical axis the graph of $\widetilde{\Psi}^{-1}(A)$ is hump-shaped. Note that when $\theta = 0$ we have the standard case of non-increasing guarding costs, and (46) becomes identical to the inverse of $\Psi(P)$ in (37).

We next see that $\pi^E \geq \pi^S$ corresponds to $A^\alpha P^{-\alpha} \geq Z A^{\alpha-\theta(1-\alpha)} P^{1-\alpha} - \bar{c}P$, which does not transpire to any simple closed-form expression, but amounts to $A \geq \widetilde{\Omega}(P)$, where $\widetilde{\Omega}(P)$ is defined from

$$P^{\frac{1}{\alpha}} \left[Z \left\{ \widetilde{\Omega}(P) \right\}^{\alpha-\theta(1-\alpha)} - \bar{c}P^\alpha \right]^{\frac{1}{\alpha}} \equiv \widetilde{\Omega}(P). \tag{47}$$

In the special case when $\theta = \alpha/(1-\alpha)$ we can write an explicit expression for $\widetilde{\Omega}(P)$. In this case, the payoff under slavery is independent of A, i.e., the rise in income from higher A is exactly neutralized by the rise in guarding costs.

The precise shapes of $\widetilde{\Omega}(P_t)$ and $\widetilde{\Psi}(P_t)$ are not easily deduced from (46) and (47), but we can see that they meet at $P = 1/\alpha$, as drawn in Figure 5 in the paper. To see this set $P = \widetilde{\Psi}^{-1}\left\{ \widetilde{\Omega}(P) \right\}$ and solve for P. We can also confirm this by simply setting $\pi^F = \pi^E$, i.e., $\alpha A^\alpha P^{1-\alpha} = A^\alpha P^{-\alpha}$. Denote the corresponding coordinate on the A axis by $\widehat{A}$, as defined from $1/\alpha = \widetilde{\Psi}^{-1}(\widehat{A})$. Using (46) and some algebra this definition of $\widehat{A}$ is equivalent to

$$\widehat{A} = N(\widehat{A}) \equiv \frac{1}{\alpha} \left[\frac{\bar{c}(1+\bar{\gamma})^{1-\alpha}}{\left(\frac{\widetilde{A}}{\widehat{A}}\right)^\theta - \alpha(1+\bar{\gamma})^{1-\alpha}} \right]^{\frac{1}{\alpha}}, \tag{48}$$

where we note that $N(0) = 0$ and $N'(\widehat{A}) > 0$. For there to exist a slavery region we must ensure that $\widehat{A} > \widetilde{A}$, which upon applying (48) and $N'(\widehat{A}) > 0$ is equivalent to $N(\widetilde{A}) < \widetilde{A}$, or

$$\widetilde{A} > \frac{1}{\alpha} \left[\frac{\bar{c}(1+\bar{\gamma})^{1-\alpha}}{1-\alpha(1+\bar{\gamma})^{1-\alpha}} \right]^{\frac{1}{\alpha}} = \Psi(1/\alpha), \tag{49}$$

where the second equality comes from (37). That is, if (49) holds, there exists a slavery region.

APPENDIX

A Equilibria with free labor and egalitarianism under free migration

As argued, there is a strong case for assuming restrictions on migration into egalitarian societies. For completeness, it may still be useful to see what type of equilibria arise in the free-migration case. It turns out that the results are not too different.

Using (16) and (18), we can rewrite π^F under free migration as a function of the fraction societies choosing free labor, η:

$$\pi^F = \alpha A^\alpha P^{1-\alpha} \left[\frac{(1-\alpha)^{\frac{1}{\alpha}}}{1-\eta+\eta\,(1-\alpha)^{\frac{1}{\alpha}}} \right]^{1-\alpha}, \tag{A1}$$

which is increasing in η. Likewise, from (17) and (18), we can write π^E and w (which, recall, are equal under free migration) as a function of η:

$$\pi^E = w = \left(\frac{A\left[1-\eta+\eta(1-\alpha)^{\frac{1}{\alpha}}\right]}{P} \right)^{\alpha}, \tag{A2}$$

which is decreasing in η. Next define

$$\begin{aligned} \overline{P} &= \tfrac{1}{\alpha}(1-\alpha)^{\frac{\alpha-1}{\alpha}}, \\ \underline{P} &= \tfrac{1-\alpha}{\alpha}. \end{aligned} \tag{A3}$$

where we note that $\overline{P} > \underline{P}$, and let

$$\eta^* = \frac{1-\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}P}{1-(1-\alpha)^{\frac{1}{\alpha}}}. \tag{A4}$$

We can now state the following.

Proposition A3 *Consider a set of societies whose elites choose between free labor and egalitarianism. Assume* $A > [\bar{c}/(1-\alpha)]^{1/\alpha} P$. *Under free migration, the equilibrium fraction societies whose elites choose free labor and egalitarianism is given as follows:*
(a) If $P \leq \overline{P}$, *there exists an equilibrium where all elites choose egalitarianism* ($\eta = 0$).
(b) If $P \geq \underline{P}$, *there exists an equilibrium where all elites choose free labor* ($\eta = 1$).
(c) If $P \in (\underline{P}, \overline{P})$, *there exists a mixed equilibrium where the fraction elites choosing free labor is given by* η^* *in (A4).*
(d) The mixed equilibrium ($\eta = \eta^*$) *generates a payoff which is lower than the payoffs in the equilibria where all elites choose free labor* ($\eta = 1$) *or all choose egalitarianism* ($\eta = 0$).

Proof. First note that $A > [\bar{c}/(1-\alpha)]^{1/\alpha} P$ and (A2) imply that $\pi^E = w > \bar{c}$ for all $\eta \in [0,1]$. Thus all workers can survive in an equilibrium where $\eta = 1$. Part (a) follows from setting $\eta = 0$ in (A1) and (A2) to get $\pi^E = (A/P)^\alpha$ and $\pi^F = \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} A^\alpha P^{1-\alpha}$. From (A3) then follows that $P \leq \overline{P}$ implies $\pi^E \geq \pi^F$, so no elite prefers to switch from egalitarianism to free labor. Similarly, part (b) follows from setting $\eta = 1$ in (A1) and (A2) to get $\pi^E = (1-\alpha)(A/P)^\alpha$ and $\pi^F = \alpha A^\alpha P^{1-\alpha}$. From (A3) then follows that $P \geq \underline{P}$ implies $\pi^F \geq \pi^E$, so no elite prefers to switch from free labor to egalitarianism. Part (c) follows from setting η as in (A4) to see that this implies $\pi^F = \pi^E$, so no elite has any incentive for a one-sided deviation. Part (d) follows from the fact that π^F is increasing in η and π^E decreasing in η. ■

Recall that $1/\alpha$ is the threshold level of P, above (below) which the equilibrium institution is free labor (egalitarianism), when migration to egalitarian societies is not allowed. Note that $1/\alpha \in (\underline{P}, \overline{P})$; that is, this threshold lies on the interval where both egalitarian and free labor equilibria exist (as well as the mixed equilibrium).

Because the mixed equilibrium gives a lower payoff to all elites than both non-mixed types of equilibrium it seems reasonable to restrict attention to non-mixed equilibria. Moreover, selecting between those two equilibria ($\eta = 1$ and $\eta = 0$) it makes sense to focus on the one which generates the highest payoff, as is done in the paper.

To analyze the case when $A \leq [\bar{c}/(1-\alpha)]^{1/\alpha} P$ is slightly more complicated. First recall the following function, also defined in the paper:

$$\Phi(P) = \left(\frac{1}{\alpha}\right)^{\frac{1}{1-\alpha}} \left[\frac{\bar{c}}{1-\alpha}\right]^{\frac{1}{\alpha}} P^{-\left(\frac{\alpha}{1-\alpha}\right)}. \tag{A5}$$

Now we can state the following.

Proposition A4 *Consider a set of societies whose elites choose between free labor and egalitarianism. Assume that $A \leq [\bar{c}/(1-\alpha)]^{1/\alpha} P$. Under free migration, the equilibrium fraction societies whose elites choose free labor and egalitarianism is given as follows:*
(a) If $A > \Phi(P)$, there exists an equilibrium where all elites choose free labor ($\eta = 1$).
(b) If $A < \Phi(P)$, there exists an equilibrium where all elites choose egalitarianism ($\eta = 1$).

Proof. We first prove part (a). If all elites use free labor ($\eta = 1$) and $A \leq [\bar{c}/(1-\alpha)]^{1/\alpha} P$, the wage rate is constrained to $\bar{c}$. Using (3), the payoff to a single elite of also choosing free labor then equals $\pi^F = \alpha A[(1-\alpha)/\bar{c}]^{(1-\alpha)/\alpha}$. The payoff to the same single elite of instead choosing egalitarianism cannot exceed what they would get if they could prevent immigration of workers from other societies, i.e., $\pi^E = (A/P)^\alpha$. It is then seen that $A > \Phi(P)$ ensures that the payoff to choosing free labor exceeds the payoff from choosing egalitarianism, which establishes the existence of an equilibrium where $\eta = 1$. We next prove part (b). If all elites use egalitarianism ($\eta = 0$) the payoff to one single elite of also choosing egalitarianism is $\pi^E = (A/P)^\alpha$. If instead choosing free labor, the elite will be hiring workers at no lower wage than $\bar{c}$. Thus, the payoff from choosing free labor cannot exceed $\pi^F = \alpha A[(1 -$

$\alpha)/\bar{c}]^{(1-\alpha)/\alpha}$ [as in part (a), use (3)]. It follows that $A < \Phi(P)$ ensures that the payoff of choosing egalitarianism exceeds the payoff of choosing free labor, which establishes the existence of an equilibrium where $\eta = 0$. ■

Propositions A3 and A4 establish that, even when free migration is allowed, the borders separating free labor and egalitarianism are not too different using the equilibrium approach, compared to the optimal approach. The difference here is that we only establish that each respective equilibrium exists on the relevant side of the border, not that any of them is unique.

B Proof of Proposition 2

Like in the paper we compare each of the three pairs of payoffs at a time.

Comparing π^S and π^E: Here we need to distinguish between the two cases for calculating π^S. Consider first Case 1, which can be written as $A \leq \Gamma(P;\gamma)$. Using (33) and the first line of (35), we see that $\pi^S \geq \pi^E$ when $\alpha \left[\frac{1-\alpha}{(1+\gamma)\bar{c}}\right]^{\frac{1-\alpha}{\alpha}} A \geq A^{\alpha}(1+P)^{-\alpha}$, or

$$A_t \geq \left(\frac{1}{\alpha}\right)^{\frac{1}{1-\alpha}} \left[\frac{\bar{c}(1+\gamma)}{1-\alpha}\right]^{\frac{1}{\alpha}} (1+P)^{-\left(\frac{\alpha}{1-\alpha}\right)} \equiv \Lambda(P). \tag{A6}$$

Consider next Case 2: $A > \Gamma(P;\gamma)$. Using (33) and the first line of (35), we see that $\pi^S \geq \pi^E$ when $A^{\alpha}\left(\frac{P}{1+\gamma}\right)^{1-\alpha} - \bar{c}P \geq A^{\alpha}(1+P)^{-\alpha}$. This requires both that $P > \widehat{P}$ and $A \geq \Omega(P)$, where $\Omega(P)$ is defined in (38), and $\widehat{P}$ is defined from $G(\widehat{P}) \equiv 1$, where $G(P)$ is defined in (40).

Considering both cases together we thus conclude:

$$\pi^S \geq \pi^E \iff \text{either } \Gamma(P;\gamma) \geq A_t \geq \Lambda(P) \text{ or } \left\{ \begin{array}{c} A \geq \max\{\Omega(P), \Gamma(P;\gamma)\} \\ \text{and} \\ P > \widehat{P} \end{array} \right\}. \tag{A7}$$

Comparing π^F and π^E: Here we distinguish between the two cases for calculating π^F. Consider first Case A: $A > \left[\frac{\bar{c}}{1-\alpha}\right]^{\frac{1}{\alpha}} P$. Using (33) and the first line in (34) we see that $\pi^F \geq \pi^E$ when $\alpha A^\alpha P^{1-\alpha} \geq A^\alpha(1+P)^{-\alpha}$, or $G(P) = \alpha P^{1-\alpha}(1+P)^\alpha \geq 1$; since $G'(P) > 0$ and $G(P^*) \equiv 1$ this requires that

$$P \geq P^*. \tag{A8}$$

Consider next Case B: $A \leq \left[\frac{\bar{c}}{1-\alpha}\right]^{\frac{1}{\alpha}} P$. Using (33) and the second line in (34) we see that $\pi^F \geq \pi^E$ when $\alpha \left[\frac{1-\alpha}{\bar{c}}\right]^{\frac{1-\alpha}{\alpha}} A \geq A^\alpha(1+P)^{-\alpha}$, or $A \geq \Phi(P)$, where $\Phi(P)$ is defined in (39).

Using (39) it can also be seen that $\left[\frac{\bar{c}}{1-\alpha}\right]^{\frac{1}{\alpha}} P > (=,<)\Phi(P)$ if, and only if, $G(P) = \alpha P^{1-\alpha}(1+P)^\alpha > (=,<)1$, which in turn is equivalent to $P > (=,<)P^*$. Thus, in a diagram with P on the horizontal axis, $\left[\frac{\bar{c}}{1-\alpha}\right]^{\frac{1}{\alpha}} P$ intersects $\Phi(P)$ at $P = P^*$.

Considering both cases together we thus conclude:

$$\pi^F \geq \pi^E \iff P \geq P^* \text{ and } A \geq \Phi(P). \tag{A9}$$

Comparing π^F and π^S: Here the payoffs being compared are the same as those in the paper. Thus, the result holds that

$$\pi^F \geq \pi^S \iff A \leq \Psi(P). \tag{A10}$$

Conditions for $\pi^S \geq \max\left\{\pi^F, \pi^E\right\}$: this holds when $\pi^S \geq \pi^F$ and $\pi^S \geq \pi^E$. As seen from (A10), $\pi^S \geq \pi^F$ requires that $A \geq \Psi(P)$. The second condition on the right-hand side of the implication arrow in (A7) shows the condition for $\pi^S \geq \pi^E$. As long as $\pi^S \geq \pi^F$ and thus $A \geq \Psi(P)$, it must always hold that $A > \Gamma(P;\gamma)$ since $\Psi(P) > \Gamma(P;\gamma)$ [see (36) and (37)]. It is then straightforward to use (A7) to see that $\pi^S \geq \max\left\{\pi^E, \pi^F\right\}$ when A is greater than both $\Psi(P)$ and $\Omega(P)$, and $P > \widehat{P}$, i.e.,

$$P > \widehat{P} \text{ and } A \geq \max\left\{\Psi(P), \Omega(P)\right\}. \tag{A11}$$

Conditions for $\pi^F \geq \max\left\{\pi^E, \pi^S\right\}$: this holds when $\pi^F \geq \pi^E$ and $\pi^F \geq \pi^S$. As seen from (A10), $\pi^F \geq \pi^S$ requires that $A \leq \Psi(P)$. The condition for $\pi^F \geq \pi^E$ is given in (A9): both $P \geq \frac{1}{\alpha}$ and $A \geq \Phi(P)$ must hold. Thus, $\pi^F \geq \max\left\{\pi^E, \pi^S\right\}$ holds when

$$\Phi(P) \leq A \leq \Psi(P) \text{ and } P \geq P^*. \tag{A12}$$

Q.E.D.

References

[1] Banerjee, A.V., P.J. Gertler, and M. Ghatak, 2002, Empowerment and efficiency: tenancy reform in West Bengal, Journal of Political Economy 110, 239-280.

[2] Lagerlöf, N.-P., 2006, Slavery and other property rights, manuscript, York University.

[3] Samuelson, P., 1974, Is the rent-collector worthy of his full hire?, Eastern Economic Journal 1, 7-10.