Supplementary Notes to From Malthusian War to Solovian Peace

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1 Introduction

These notes discuss how the results in Lagerlöf (2009) (referred to as "the paper" below) compare with those of Hansen and Prescott (2002) (HP below), and to data from England. We also discuss the distribution of the stochastic components of the conquest function.

2 HP and the model

To make the model in the paper match HP's setting more closely it is informative to first shut down all wars. Figure A shows the time paths for some of the more important variables in an economy without wars, and with all parameters at their baseline values. The paths can be compared to Figures 2 and 3 in the paper (i.e., our model with war) and various figures in HP. The broad outlines are very similar.

The time path for population growth (n_t) in Panel A of Figure A resembles that in Figure 8 in the working paper version of HP (see Hansen and Prescott 1999). Importantly, the timing is identical: population growth peaks in the third post-transitional period, and the demographic transition is completed in the sixth post-transitional period. The main difference is that population growth in our setting peaks at a lower level (1.5 instead of 2), which fits better with the English data, where the population growth factor over a 35-year period never exceeds 1.46; see column (G) in Table A and the discussion below.

Panel B shows wages rising by a factor of about 6-12 over 5-6 periods from the time of the transition, which is roughly in line with Figure 5 in HP. Different from HP, population increases by a much lower factor, but as discussed further below this is actually consistent with the English experience.

Panel C shows the fractions of the capital stock and the labor force allocated to the Solow sector. The time paths are almost identical to those in HP's Figure 4. The fractions rise to almost one over three periods and a larger fraction capital than labor is allocated to the Solow sector throughout the transition. Note also that in this no-war economy period zero is the last period before the Solow sector becomes active; when allowing for war the country that has lost land transits slightly earlier.

3 Data and the model

Figure B provides some information about how the model simulation (with baseline parameter values and without war) compares to data from England. The upper panel shows the fertility function used in the model, together with nine data points, as reported in columns (G) and (I) in Table A; the lower panel shows how wage and population levels generated by the model compare to the data. To make the data points comparable to the model requires some work.

3.1 Making data comparable to the model

Table A explains how the numbers used in Figure B are derived. Columns (A)-(D) are identical to HP's Table 1, which in turn relies on Maddison (1991). To make these numbers comparable to those generated in our model we first assign the growth rate of each period to its midpoint, i.e., the annual population growth rate of 0.47% over the period 1700-1760 is assigned to the year 1730, etc. Thus, the growth rates in column (F) [identical to HP's numbers in column (A)] are assigned to the years in column (E). To make the numbers in column (F) comparable to the variable n_t in the model, we then compute the implied growth factor over 35 years (a model period); see column (G). That is, if the annual population growth rate is $x\%$, the 35-year growth factor is computed as $(1 + x/100)^{35}$.

To generate comparable numbers for GDP/hour for the same years we use the HP data in column (A), and assume constant growth rates over the relevant period. Thus, to impute the value for GDP/hour in 1730 we multiply the number for 1700 [which equals 0.82; see column (A)] by the relevant 30-year growth factor, $1.0027^{30} \approx 1.084$ [since the annual growth rate in GDP/hour from 1700 to 1780 is 0.27%; see column (B)]. This gives the value 0.89, reported in column (H). Finally, the index in column (I) is derived by simply dividing the numbers in column (H) by the value for 1780, which we find in column (A) to be 1.02.

3.2 How the model outcome compares to the data

The upper panel in Figure B shows the nine data points given by columns (G) and (I) in Table A, together with the corresponding values given by the piece-wise linear function $n(w)$ in the model. There is obviously nothing about these particular data points that suggests that the relationship changes from positive to negative at some point, but if this feature is imposed the fit looks reasonable.

Note that we have chosen the parameters of $n(w)$ somewhat differently from HP, who fit $n(w)$ to data for countries all over the world, as detailed in the appendix to Lucas (2002, Ch. 5).[1] Lucas' data includes several later developers in e.g. Africa and South Asia. Here the focus is on countries that were early technological leaders, namely Great Power nations, which were dominated by European countries. It thus seems more reasonable to fit the data to observations from England.

We can also get a sense for how good this fit is by comparing the model's time paths for population and wage levels to the numbers in the English data in columns (I) and (C) in Table A, where the population numbers are also indexed to equal one in 1780. We translate model periods to years by setting the transition period (the last period before the Solow sector becomes active) to 1780, and (recall) each period to 35 years.

The lower panel in Figure B shows a decent fit between model and data. By comparison, HP seem to do worse (although they do not explicitly show model and data outcomes in levels), in the sense that their calibration generates a rise in both population and per-capita income by approximately the same factor (about 9) over 5 periods from the transition (see HP, Figure 5); in the data for England (and probably most rich countries), population rises by a much lower factor than per-capita income over the same post-transitional period.

4 The distribution of the shocks

Recall that the attacking country's land conquest depends on the product of two independent random variables, x_t^k and x_t^u. The government knows x_t^k before going to war, whereas x_t^u is unknown. Both take the values $\{0, 0.1, ..., 1\}$ with equal probability, 1/11, for all outcomes. Thus, conditional on some realization of x_t^k the product $x_t^k x_t^u$ is distributed on $\{0, x_t^k/10, 2x_t^k/10, ..., 9x_t^k/10, x_t^k\}$, with all outcomes having probability 1/11. A large realization of x_t^k thus expands the distribution of $x_t^k x_t^u$ in an accordion-

[1] HP cite Lucas' (2002) book as an unpublished manuscript, namely Lucas (1999).

like fashion.

The left panel of Figure C shows a histogram over the unconditional distribution of $x_t^k x_t^u$ generated by a simulation where both are drawn 50,000 times. The distribution has more weight around 0 and less around 1, similar to a power-law distribution but with support [0,1].[2] In that sense, the outcome from going to war has a sort of status-quo bias.

Note, however, that a country goes to war only if the realization of x_t^k is large enough. The right panel of Figure C shows a histogram over $x_t^k x_t^u$ for outcomes where $x_t^k \geq 0.4$. If war is worthwhile only if $x_t^k \geq 0.4$ and if the two countries are roughly symmetrical [so that $H(\cdot)$ is close to 1], then this shows the distribution over possible outcomes of war (the share of the losing country's land that is lost), conditional on there being war. As seen, the conditional distribution is "flatter" than the unconditional, but still with higher weight around zero than the uniform distribution.

References

[1] Hansen, G.D., and E.C. Prescott, 1999, Malthus to Solow, NBER Working Paper 6858.

[2] – – –, 2002, Malthus to Solow, American Economic Review 92, 1205-1217.

[3] Lagerlöf, N.-P., 2009, From Malthusian war to Solovian Peace, Review of Economic Dynamics, forthcoming.

[4] Lucas, R.E., 2002, Lectures on Economic Growth, Harvard University Press, Cambridge.

[5] Maddison, A., 1991, Dynamic Forces in Capitalist Development: A Long–Run Comparative View, Oxford. University Press, Oxford.

[2] The product of two independent random variables, which are both continuous and distributed uniformly on $[0, 1]$, has the probability density function $f(x) = -\ln(x)$.

Table A: Population and GDP/hour

	Numbers from Hansen and Prescott				Own calculations				
	GDP/hour		Population		Population			GDP/hour	
Year	1985 US$	Growth rate (% per year)	Millions	Growth rate (% per year)	Year (midpoint)	Growth rate (% per year)	35-year growth factor	Imputed	Index (1780=1)
	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)
1700	0.82		8.4						
1760			11.1	0.47	1730	0.47	1.18	0.89	0.87
1780	1.02	0.27							
1820	1.21	0.43	21.2	1.08	1790	1.08	1.46	1.06	1.04
1870	2.15	1.16	31.4	0.79	1845	0.79	1.32	1.61	1.58
1890	2.86	1.44	37.5	0.89	1880	0.89	1.36	2.48	2.43
1913	3.63	1.04	45.6	0.85	1902	0.85	1.35	3.22	3.16
1929	4.58	1.46	45.7	0.01	1921	0.01	1.00	4.08	4.00
1938	4.97	0.91	47.5	0.43	1934	0.43	1.16	4.77	4.68
1960	8.15	2.27	52.4	0.45	1949	0.45	1.17	6.36	6.24
1989	18.55	2.88	57.2	0.30	1975	0.30	1.11	12.30	12.05

Figure A: the no-war economy, baseline parameters

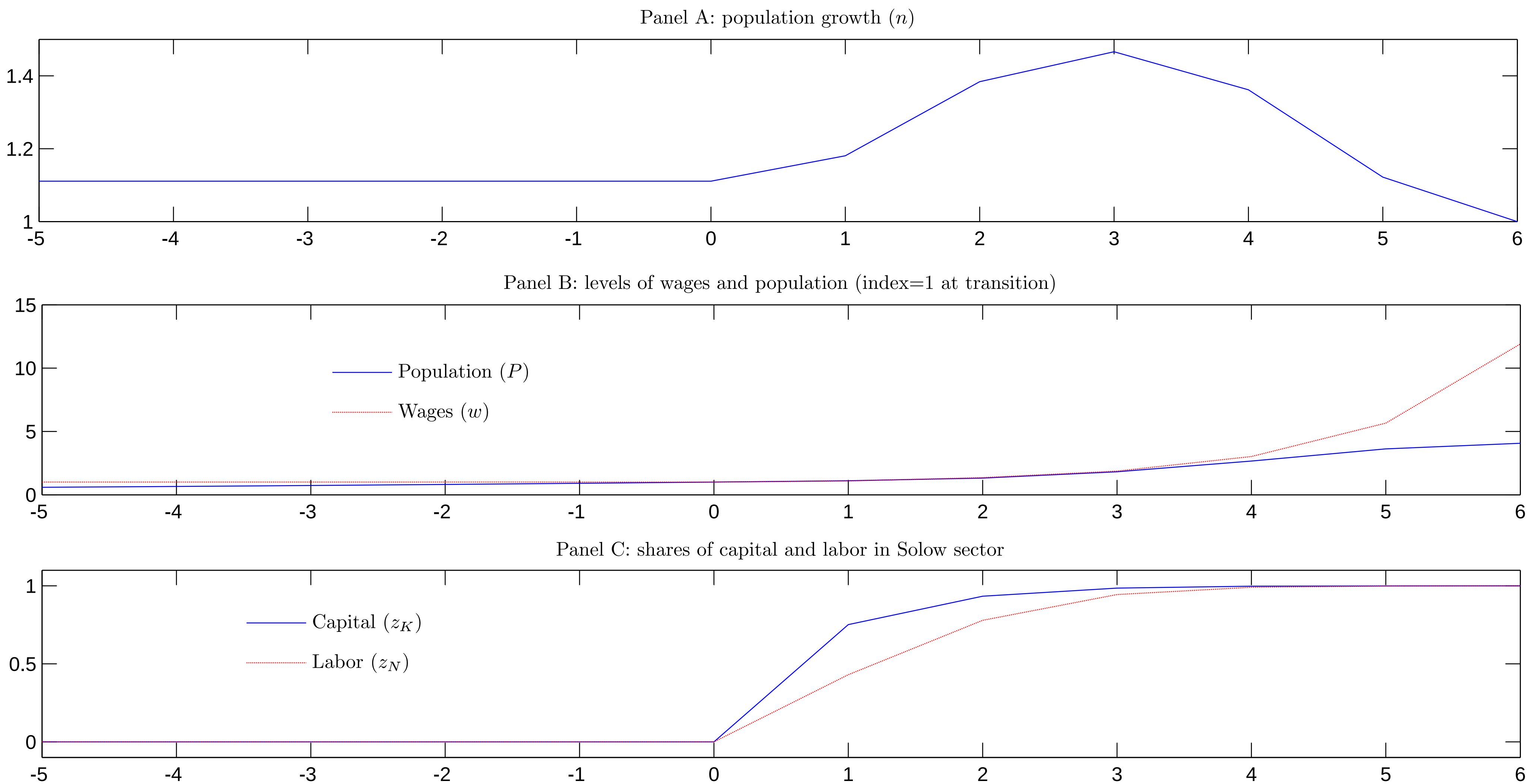

Figure B: comparing model and data

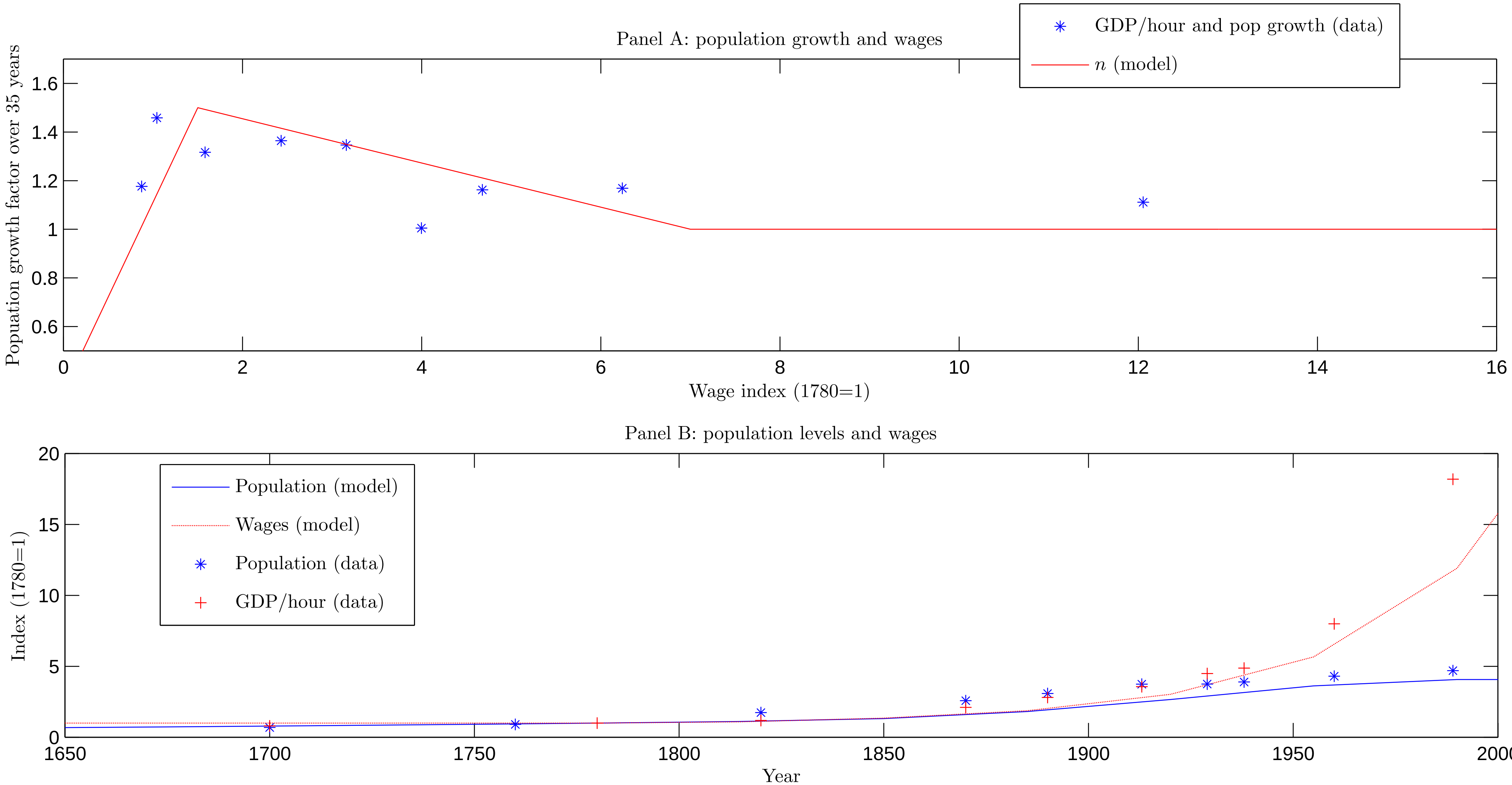

Figure C: the distribution of $x_t^k x_t^u$