From Malthusian War to Prosperous Peace

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Abstract: Over the past several centuries European wars became simultaneously less frequent, and initially more deadly before governments stopped fighting them altogether. We set up a unified growth model which endogenously replicates these trends, together with a takeoff from Malthusian stagnation to sustained growth in per-capita incomes. In environments with scarce resources – meaning high population density, and/or low levels of technology – governments are more prone to start wars to try to conquer new land. Technological progress mitigates resource scarcity, making war less likely; at the same time, if war breaks out it is deadlier if technologies are more advanced. Thus, the transition to sustained growth passes a very deadly intermediate phase, when wars have not yet become improbable events but new technologies have made them extremely lethal if and when they break out. We suggest that this is roughly what happened in Europe during the first half of the 20th century.

1 Introduction

"We need land on this earth [...] We must continue to receive what is necessary from future apportionments until such time as we are satiated to approximately the same degree as our neighbors."

German industrialist Walter Rathenau in 1913 (as quoted from Hardach 1977, p. 8)

"Our nation seems to be at a dead-lock, and there appears to be no solution for the important problems of population and food. The only way out, according to public opinion, is in the development of Manchuria and Mongolia."

Lieutenant-General Kanji Ishihara, justifying the Japanese military expansion in the 1930's (as quoted from Yasuba 1996, p. 553)

The transition from Malthusian stagnation to modern growth that began in Europe a couple of centuries ago was paralleled by certain time trends in war. First, wars became *less frequent*; since 1945 Western Europe has been completely peaceful. Second, wars became *more lethal*; in particular, the two world wars 1914-1945 were far more deadly than previous Great Power wars.

We set up a model where these time trends are driven by technological progress. New technologies make wars more lethal if they break out, which also makes governments less inclined to fight them. Moreover, by mitigating resource scarcity technological progress reduces resource competition, thus also making governments less inclined to start wars.

Wars in this model are Malthusian, in the sense that they are caused by competition for land and other resources. We do not suggest that resource competition has been the only factor behind every war, but as discussed at length later on, it seems to have been *one* important factor.

In this model, governments of two symmetric countries have an incentive to try to conquer land from eachother to alleviate domestic resource scarcity. No country gains from war in equilibrium. However, if one country attacks, the other's best response is to also be aggressive, because its people are killed anyhow; being peaceful only leads to land being lost. At low levels of technology, wars are not very lethal and resource scarcity is a grave problem. In this environment, the only equilibrium is one with war.

For sufficiently advanced technology, however, being peaceful becomes the best response if the other country is peaceful. Thus, the two countries can coordinate on a peaceful equilibrium. Peace in turn enables further growth in technology and living standards, thus making peace perpetuate itself over time.

The model generates a path which leads in the end to sustained peaceful prosperity, but also passes a very deadly war phase in the transition. Europe may have passed such a phase in the first half of the 20th century.

We also allow for a random element in governments' war decisions. This makes the take-off from stagnation to growth, and from war to peace, differently timed across countries. Over time the model generates an initial rise in the cross-country variance in war death rates, before permanent peace breaks out. This also seems consistent with the data.

An extended framework allows agents to choose the number of children and human capital investment in each child. Although more complex, this setting can replicate an initial joint rise in the growth rates of population and per-capita consumption, and a subsequent spurt in per-capita consumption growth and decline in population growth. This is consistent with both the three-stage development process described by e.g. Galor and Weil (1999, 2000), and the war trends described here. The model also replicates a "dent" in the population growth path, observed in Western European history (cf. Figure 7).

The rest of this paper continues in Section 2 by relating it to earlier literature. Thereafter Section 3 describes a number of facts about war, in particular in European history. Section 4 sets up a model where fertility is exogenous. Section 5 models fertility and human capital investment endogenously. Finally, Section 6 ends with a concluding discussion.

2 Existing literature

A large microeconomic literature on conflict examines how rational agents weigh appropriation (stealing) against production. (See e.g. Grossman 1991; Grossman and Kim 1995; Hirschleifer 1988, 2001.) However, these usually use static models and do not explain the time-trends of war discussed here.

Neither does this literature actually model any link from resource scarcity to violence. The one exception is Grossman and Mendoza (2003), who set up a model where competition for resources is induced by a desire for survival. They show that if the elasticity of the survival function is decreasing in consumption more scarcity leads to more violence.[1] Our survival function satisfies this Grossman-Mendoza condition.

There is also work on how social conflicts within societies can hinder development, both theoretical (e.g., Benhabib and Rustichini 1996) and empirical (e.g., Collier and Hoeffler 1998, 2004; Easterly and Levine 1997). More applied papers include Martin et al. (2005), who examine the relationship between trade, war, and the geographical distances between belligerent countries. Iyigun (2007) documents the role played by the Ottoman wars in reducing fighting between Christian nations.

These papers do not explain the particular century-long time trends described here, or how these relate to industrialization.[2] In that sense, our contribution relates more closely to a literature trying to explain growth in population and per-capita income, not only over the last couple of decades, but several centuries (or millennia) back. See, among others, Cervellati and Sunde (2005), Galor and Moav (2002), Galor and Weil (2000), Hansen and Prescott (2002), Jones (2001), Lagerlöf (2003a,b; 2006), Lucas (2002), and Tamura (1996, 2002). However, none of these discuss war.[3]

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[1] Other papers where "harder times" lead to more conflict include Dal Bó and Powell (2006), who allow for asymmetric information about the size of the contested pie.

[2] However, Iyigun (2007) does put a time dummy in his regressions to control for the secular decline in warfare that we seek to explain. Also, quite consistent with our story, he suggests that this decline may have been due to rising living standards (ibid., Footnote 12).

[3] One possible exception is Brander and Taylor (1998), who discuss the (probably violent) downfall of the civilization on Easter Island. However, they do not explicitly model

We also differ methodologically from many of these papers. For example, Galor and Weil (2000) restrict themselves to using phase diagrams to derive the time paths generated by complicated non-linear and multi-dimensional dynamical systems. We use both phase diagrams and simulations.

Easterly, Gatti, and Kurlat (2006) examine empirically the link between mass killings (including genocides), per-capita income, and democracy. They do not look at war deaths as such, but do note that episodes of mass killings are worse and more common in times of war (see also Rummel 1997). Like we document here for war deaths, they find that mass killings are non-linearly related to economic development, and initially increase with per-capita income. The potential causes that they discuss indeed relate to the mechanisms we model: "Economic development brings advances in technology and social organization that lower the cost of mass killings" (ibid., p. 131).

Several other papers have a more indirect connection. Alesina and Spolaore (2003, Ch. 7) model defense spending and the optimal size of nations. Johnson et al. (2006) explain why death tolls in many insurgencies (e.g., Iraq and Colombia) tend to follow a power-law distribution. These studies are not directly relevant for understanding the longer-term war trends, or takeoffs from stagnation to growth, discussed here.

3 Background

3.1 The Malthusian causes of war

Resource competition is a common factor behind violent conflict in hunter-gatherer societies (Harris 1974, Ember 1982; see Lagerlöf 2007 for an overview). It is often suggested that scarcity brought a violent end to many ancient civilizations, e.g. the Anasazi and Easter Island (Diamond 2005).

The links from resource scarcity to conflict are today seen most clearly in poorer regions, which are more dependent on land and agriculture. The 1969 so-called Soccer War between Honduras and El Salvador was the outcome of depletion of agricultural lands in El Salvador, and subsequent migration into

violence or conflicts over resources.

Honduras (Durham 1979). André and Platteau (1998) document how over-population and competition for agricultural land worked as a factor behind the 1994 civil war and genocide in Rwanda. Miguel et al. (2004) find a strong negative effect of economic growth on the likelihood of outbreak of civil war among 41 African countries, using rainfall as an exogenous instrument.[4]

Rich countries tend to be less prone to war and violent conflict. Friedman (2005) lists numerous examples of how rising living standards have made Western societies more open, democratic, and peaceful. When Europe was poor, famine and high food prices contributed to many episodes of social unrest. One example is the French revolution in 1789. McNeill (1982, p. 185) writes that "the fundamental disturber of Old Regime patterns in France and England in the last years of the eighteenth century was population growth."

The European colonization of the rest of the world can also be interpreted as the outcome of European population pressures and a hardening competition for land (Pomeranz 2001). The English colonization of North America served partly to secure supply of wood for ship building, thus enabling British imperial expansion at a time when the British Isles had been largely deforested (Albion 1965). The Indian Wars fought in what is today the Dakotas and Montana in the United States were clearly about land for agriculture and mining.

WWI may to some extent be linked to struggles over land and resources, in particular coal and iron ore (Choucri and North 1975; McNeill 1982, pp. 310-316). Scarcity was particularly strongly felt in Germany, which lacked the colonial assets of its Atlantic neighbors.[5] From the late 19th century, Germany's population and energy consumption rose more rapidly than that of e.g. the United Kingdom and France.[6] This was paralleled by Germany's

[4]Some seek to attribute almost all conflicts in developing countries to resource competition. Homer-Dixon (2001) proposes that it is one factor behind e.g. the guerrilla insurgencies in Peru and the Philippines.

[5]Acemoglu et al. (2005) document a rise of the Atlantic regions and cities of Europe following the discovery of the Americas, suggesting that Germany's geographical location may have mattered for its lack of colonies.

[6]Germany's population rose from about 40 million in 1870 to about 67 million in 1913. The corresponding population numbers for France were 37 million in 1870 and 40 million in 1913 (Mitchell 2003, Table A5; Browning 2002, Table 9). From 1890 to 1913 France's

emergence as a Great Power, and its search for "a place in the sun." Before WWI, the so-called the Pan-German League had formulated a war-aims program containing explicit demands for territorial conquests (Hardach 1977, Ch. 8). Germany's land-owning elite (the Junkers) wanted more agrarian lands; industrialists called for the annexation of territories rich in coal and iron ore; see the words of Walter Rathenau in the introduction. Indicatively, the most coal rich regions of Europe, such as Saar and Silesia, were highly contested in the Paris peace talks after WWI.

It is well known that pre-WWII Nazi agitation held that Germany suffered from a shortage "living space" (*lebensraum*). In the early phases of WWII, living standards in Germany were in fact markedly improved due to war conquests, for example through loot sent by soldiers in the occupied areas (Aly 2005).

Likewise, competition for natural resources and land mattered for Japan's conquests in East Asia, and its occupation of Manchuria (Ferguson 2006, pp. 285-297); see the words of Kanji Ishihara in the introduction.

Today each unit of land can feed many more people than in the 1930's, and most resources are traded internationally; going to war for resources seems irrational. However, as discussed at length by Ferguson (2006, pp. 281-297), resource competition may have played a role even as late as in the 1930's. Indicatively, the most densely populated countries, with the lowest land-to-labor ratios in agriculture, and with the fewest colonies (in particular after WWI), were the three axis powers: Germany, Japan, and Italy. (Great Britain's population was denser but they also had a huge empire.) The axis powers also lacked raw materials, such as ore, coal, rubber, oil, and crucial metals (ibid., pp. 283-284). In between them, the United States, the Soviet Union, and the British Empire controlled most of these supplies. As Ferguson (2006, p. 284) sums it up, "the case that Germany, Italy, and Japan lacked living space was therefore far from weak."

Obviously, resource competition has not been the *only* factor behind *every* war, or even the most important. We merely suggest it was *one* factor, and

energy consumption rose by 80%, whereas that of Germany rose by 224% (Browning 2002, Table 10).

that it became less important with time.[7]

3.2 The frequency of war

Figure 1 shows a declining trend in the number of ongoing wars, starting in 1500. The two indices shown are based on data from Levy (1983), and Wikipedia, respectively. The Levy data consist of wars fought by Great Power nations (mostly Western European and technological leaders), from which we have selected wars involving two or more Great Powers.[8] From the Wikipedia data we selected wars involving Western European countries. For further details, see Section A.1 in the Appendix.

This downward trend may not be very surprising. It is well known that earlier wars lasted longer (as emphasized by Levy 1983, p. 123). Indicatively, these are often known to us by names such as the Thirty Years' War.[9]

One may note in Figure 1 that war frequency rose before the 17th century. One possible explanation, consistent with our story, is that land was still relatively abundant in the wake of the Black Death of the 14th century. Clark (2006) finds that agricultural real wages in England peaked in 1450, then declined to reach a trough in the 1630's, and then rose again as agricultural productivity began to increase. Thus, Malthusian pressures may have driven the temporary rise in war frequency before the mid 17th century in Figure 1.[10]

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[7] The fact that the most conflict-ridden region of the world today is the oil-rich Middle East suggests that *abundance* of resources, rather than scarcity, now makes a region more war prone. However, scarcity may matter in the sense that competition for natural resources is more intense in regions where living standards are low, e.g. the Middle East and Nigeria. Oil abundance is not a curse in regions where living standards are higher, like Norway and Alaska.

[8] In the model the belligerent nations will be technologically symmetric, so it makes sense to focus on wars between different Great Powers.

[9] Note, however, that the Thirty Years' War (1608-1648) is actually categorized as four shorter wars in the Levy (1983) data; see Table A.1 and Section A.1 in the Appendix.

[10] Another possibility is that Europeans up until about 1650 were too busy fighting the Ottomans to engage in wars against eachother (Iyigun 2007).

3.3 The lethality of war

Despite the downward trend in war frequency, Figure 2 shows an upward trend in the deadliness of wars involving two or more Great Powers. Notably, WWI and WWII, the last Great Power wars fought in Western Europe, were more lethal than all wars of the preceding centuries. However, Figure 2 does not include those two outliers: the upward trend is still clearly present; including WWI and WWII would strengthen the trend further.

Figure 2 also reveals a rise in the *variance* in war death rates. This would show up more clearly if we were to include wars not involving two or more Great Powers, which do not display the same rise in lethality (cf. Table A.1). This suggests, consistent with our story, that the level of technological (and/or economic/institutional) development of the belligerent powers positively impacts the deadliness of war (cf. Easterly et al. 2006).

4 Model A: exogenous fertility

Consider now a two-period overlapping-generations model, where agents live as children and adults. Adults rear children, some of whom die before reaching adulthood. There are two sources of death (other than old age): starvation and war. Those children who survive both war and starvation become adults in the next period.

There are two countries whose governments may fight wars to gain territory, with the aim to maximize the agents' survival rates. The two countries are indexed I and II. Each starts off with identical population, technology, and unit land endowments in the first period. This symmetry will later be seen to imply that, regardless of whether there is war or peace, no country ever gains any territory from the other in equilibrium, making the two countries identical in all subsequent periods.

The number of children born by each adult agent is here exogenous and denoted $\overline{n}$. (Section 5 endogenizes fertility.) In country j ($j =$I,II), a fraction $1 - s_{j,t}$ of these children die from purely non-violent causes, such as undernutrition and childhood disease; we call it starvation, for short. This mortality rate depends on time spent nurturing the children, keeping them clean, etc., and on the amount of food the parent can collect, which in turn depends on her time spent competing for food.[11] The parent has a unit time endowment, and the fraction of her $\overline{n}$ children surviving starvation is given by

$$s_{j,t} = q(c_{j,t})(1 - r_{j,t}), \tag{1}$$

where $r_{j,t}$ denotes the parent's time in resource competition, and $1 - r_{j,t}$ is the time spent nurturing the children. The amount of food procured, $c_{j,t}$, determines survival from starvation through the following function:

$$q(c) = \max\left\{0, \frac{c - \overline{c}}{c}\right\}, \tag{2}$$

where $\overline{c}$ will be called subsistence consumption. That is, the parent must procure more than $\overline{c}$ for any of the children to have a chance of survival. If, in any period, available food per agent falls below $\overline{c}$ the whole population dies out. Note also that $q(c)$ goes to one as c goes to infinity.

4.1 Available resources

For simplicity we abstract from production, and let agents take as given the amount of food generated per unit of land. More precisely, agents in country j compete over a land area equal to $m_{j,t}$, and each unit of land generates $A_{j,t}$ units of food, where $A_{j,t}$ measures the productivity of the available technology.

Country j has a continuum of $P_{j,t}$ (adult) agents. Let the average time spent in resource competition across all $P_{j,t}$ agents in country j be denoted $R_{j,t}$. An agent who fights $r_{j,t}$ units of time then acquires

$$c_{j,t} = \frac{r_{j,t}}{R_{j,t}} \frac{m_{j,t} A_{j,t}}{P_{j,t}} \tag{3}$$

units of food. In a symmetric equilibrium food per agent thus equals $m_{j,t}A_{j,t}/P_{j,t}$.

[11] The food procured here influence the survival probabilities of the $\overline{n}$ children. It can alternatively be thought of as being eaten by the mother, thus prolonging her life, making her able to rear more offspring. The point is that the larger is the total amount of food collected, the more agents enter as adults in the next period.

Section A.2 in the Appendix shows that, if $m_{j,t}A_{j,t}/P_{j,t} > \overline{c}$, the equilibrium time spent in resource competition, $R_{j,t}$, becomes:[12]

$$R_{j,t} = \frac{\overline{c}}{c_{j,t}} = \frac{\overline{c}P_{j,t}}{m_{j,t}A_{j,t}}. \tag{4}$$

Using (1), (2), (3), and (4) the equilibrium survival rate from starvation (if $m_{j,t}A_{j,t}/P_{j,t} > \overline{c}$) can be derived as:

$$s_{j,t} = q\left(\frac{m_{j,t}A_{j,t}}{P_{j,t}}\right)(1 - R_{j,t}) = (1 - R_{j,t})^2 = \left(1 - \frac{\overline{c}P_{j,t}}{m_{j,t}A_{j,t}}\right)^2. \tag{5}$$

4.2 Technology

The level of technology in country j is updated as follows:

$$A_{j,t+1} = A_{j,t}^{\alpha}P_{j,t}^{\beta}. \tag{6}$$

Letting population enter (6) may capture the type of scale effects occurring in models with non-rivalries in the use of technology (Kremer 1993, Jones 2005). That is, the more people there are to make inventions and discoveries, the faster is the rate of technological progress.

To ensure that a peaceful balanced growth path exists we make the following assumption:

Assumption 1 *$\alpha \in (0, 1)$, $\beta > 0$, $\alpha + \beta > 1$.*

4.3 Four-dimensional dynamics

In peace, $\overline{n}s_{j,t}$ children per adult agent survive to adulthood; in war, a fraction $1 - v_{j,t}$ of these children are killed, and never enter the adult phase. We let the war survival rate be given by $v_{j,t} = \lambda/(\lambda + A_{i,t})$, where $\lambda > 0$. Note that $v_{j,t}$ is decreasing in the *enemy's* technology, which makes intuitive sense. However, we shall soon impose assumptions which imply that the two

[12]If $m_{j,t}A_{j,t}/P_{j,t} \leq \overline{c}$ the population would die out. Simulations will later verify that we can choose parameters and initial conditions such that the economy follows a path where $m_{j,t}A_{j,t}/P_{j,t} > \overline{c}$ in all periods, and in both countries (j =I,II).

countries are identical in all periods, so letting $v_{j,t}$ depend on $A_{j,t}$ instead of $A_{i,t}$ would not change the results.

This gives the following dynamic equation for population:

$$P_{j,t+1} = w_{j,t}\overline{n}s_{j,t}P_{j,t} = w_{j,t}\overline{n}\left(1 - \frac{\overline{c}P_{j,t}}{m_{j,t}A_{j,t}}\right)^2 P_{j,t}, \tag{7}$$

where

$$w_{j,t} = \begin{cases} \frac{\lambda}{\lambda + A_{i,t}} = v_{j,t} & \text{in war,} \\ 1 & \text{in peace,} \end{cases} \tag{8}$$

for $(i, j) \in \{(\text{I,II}), (\text{II,I})\}$.

At any given $m_{j,t}$ (j =I,II), (6), (7), (8) constitute a state-dependent four-dimensional system of difference equations, with two dynamic equations and two state variables for each country ($A_{j,t}$ and $P_{j,t}$, for j =I,II). What path the economy follows depends on whether it is in a state of war or peace. Moreover, as we shall see below, it switches between these states with a probability which in turn evolves endogenously over time.

4.4 War and peace

The state of the world (war or peace) depends on the decisions made by the governments (or leaders) of the two countries. We could assume any sort of government objective function, obviously without fully capturing the belligerents' motives in every single war ever fought. The ambition here is to provide some plausible microfoundation for war. To that end, we assume that the governments maximize the survival rates of their citizens.

The potential benefit of war is the conquest of land. Each country j (j =I,II) starts off period t with $m_{j,t-1}$ units of land. If one country behaves aggressively, and the other is non-aggressive (i.e., fails to defend itself), the aggressor wins a fraction χ_t of the non-aggressor's territory, where (as explained below) χ_t is stochastic. If both countries are aggressive, or if both are non-aggressive, no territory changes hands ($m_{j,t} = m_{j,t-1}$). That is, let the variable $S_{j,t}$ be such that $S_{j,t}$ =**A** if country j's government is aggressive, and $S_{j,t}=$**N** if it is non-aggressive ($j=$I,II). Then $m_{j,t}$ is given as follows:

$$m_{j,t}=\begin{cases} m_{j,t-1}+\chi_t m_{i,t-1} & \text{if } S_{j,t}=\mathbf{A} \text{ and } S_{i,t}=\mathbf{N}, \\ m_{j,t-1}(1-\chi_t) & \text{if } S_{j,t}=\mathbf{N} \text{ and } S_{i,t}=\mathbf{A}, \\ m_{j,t-1} & \text{if } S_{j,t}=S_{i,t}, \end{cases} \tag{9}$$

for $(i,j)\in\{(\text{I,II}),(\text{II,I})\}$. Note again that no territory is redistributed if both countries behave identically, either aggressively, or non-aggressively.

The land-conquest variable, χ_t, is an i.i.d. random variable, uniformly distributed on $[0,1]$ (and, for simplicity, the same for both governments, and known to both of them[13]). These shocks will make it possible to talk about the *probability* of war breaking out. They also have some intuitive and interesting interpretations. They may be thought of as changes in military circumstances that impact how much land can be seized. Alternatively, we may think of the world as having some true constant χ (perhaps zero), but leaders being irrational, so that the random realizations of χ_t capture how much land they "believe" they can take. That is, a high χ_t could be interpreted as a relatively war-prone leadership, such as that of Hitler or the Japanese military command in the 1930's.

The governments' payoffs are given by the survival rates of their citizens from war and starvation. Let the payoff to country j's government be denoted $\pi_{j,t}=w_{j,t}s_{j,t}$. Using (5), (8), and (9) we can write this as follows:

$$\pi_{j,t}=\begin{cases} v_{j,t}\left[1-\frac{\bar{c}P_{j,t}}{(m_{j,t-1}+\chi_t m_{i,t-1})A_{j,t}}\right]^2 \equiv \pi_{j,t}^{\text{A,N}} & \text{if } S_{j,t}=\mathbf{A} \text{ and } S_{i,t}=\mathbf{N}, \\ v_{j,t}\left[1-\frac{\bar{c}P_{j,t}}{m_{j,t-1}(1-\chi_t)A_{j,t}}\right]^2 \equiv \pi_{j,t}^{\text{N,A}} & \text{if } S_{j,t}=\mathbf{N} \text{ and } S_{i,t}=\mathbf{A}, \\ v_{j,t}\left[1-\frac{\bar{c}P_{j,t}}{m_{j,t-1}A_{j,t}}\right]^2 \equiv \pi_{j,t}^{\text{A,A}} & \text{if } S_{j,t}=S_{i,t}=\mathbf{A}, \\ \left[1-\frac{\bar{c}P_{j,t}}{m_{j,t-1}A_{j,t}}\right]^2 \equiv \pi_{j,t}^{\text{N,N}} & \text{if } S_{j,t}=S_{i,t}=\mathbf{N}, \end{cases} \tag{10}$$

for $(i,j)\in\{(\text{I,II}),(\text{II,I})\}$.

We next assume that the two countries have identical levels of technology and population in period 0, and that both start off with territorial holdings equal to unity.

[13] If the conquest shocks differ between the two countries our results below still hold, but with χ_t replaced by $\max\{\chi_{\text{I},t},\chi_{\text{II},t}\}$.

Assumption 2 *$P_{I,0} = P_{II,0}$, $A_{I,0} = A_{II,0}$, and $m_{I,-1} = m_{II,-1} = 1$.*

We can now state the following, proven in Section A.4 of the Appendix.

Proposition 1 *Under Assumption 2 and considering only pure-strategy equilibria, in all periods $t \geq 0$ it holds that $P_{I,t} = P_{II,t} = P_t$, $A_{I,t} = A_{II,t} = A_t$, and $m_{I,t-1} = m_{II,t-1} = 1$.*

We can thus do away with all subindices j. The state variables, A_t and P_t, and all variables which are functions of these, now refer to both countries. Note in particular that the survival rate in war, $v_t = \lambda/(\lambda + A_t)$, is the same for both countries.

When there is no risk of confusion we shall from now on refer to this twin pair of economies as one single economy.

Next we determine when the two countries will be at war or peace.

Proposition 2 *(a) If*

$$R_t = \frac{P_t \bar{c}}{A_t} \leq \frac{(1 + \chi_t)\left(1 - \sqrt{\frac{\lambda}{\lambda + A_t}}\right)}{1 + \chi_t - \sqrt{\frac{\lambda}{\lambda + A_t}}}, \tag{11}$$

*then two (pure-strategy) equilibria exist: $S_{II,t} = S_{I,t} =$**A** (war) and $S_{II,t} = S_{I,t} =$**N** (peace).*
*(b) If (11) is reversed, then $S_{II,t} = S_{I,t} =$**A** (war) is the unique (pure-strategy) equilibrium.*

The proof is in Section A.4 of the Appendix.

Intuitively, the inequality (11) holds if resources are abundant (because P_t is low, and/or A_t is high). Then a peaceful equilibrium is more likely to exist, because the temptation for the government to try to conquer new territory is low. Likewise, a high mortality rate in war (a low survival rate, $v_t = \lambda/[\lambda + A_t]$) makes war less likely, as it deters governments from aggression. Also, a low territorial conquest shock, χ_t, implies that the governments are less likely to end up in war.

Note that no country gains from war in equilibrium, since no land is conquered. The reason the two governments still end up fighting these wars is that they cannot commit to non-aggression, even though it is socially optimal. Only if technology is advanced enough, and/or population low enough, does a peaceful equilibrium exist, by making a one-sided deviation from a peaceful equilibrium not worthwhile.

We shall next assume that the two countries are able to coordinate on a peaceful equilibrium whenever such exists, i.e., whenever (11) holds; when (11) is reversed, war breaks out. Using (8) we can thus write w_t as

$$w_t = w(A_t, P_t, \chi_t) = \begin{cases} \frac{\lambda}{\lambda+A_t} = v_t & \text{if} \quad R_t = \frac{P_t\bar{c}}{A_t} > \frac{(1+\chi_t)\left(1-\sqrt{\frac{\lambda}{\lambda+A_t}}\right)}{1+\chi_t-\sqrt{\frac{\lambda}{\lambda+A_t}}}, \\ \\ 1 & \text{if} \quad R_t = \frac{P_t\bar{c}}{A_t} \leq \frac{(1+\chi_t)\left(1-\sqrt{\frac{\lambda}{\lambda+A_t}}\right)}{1+\chi_t-\sqrt{\frac{\lambda}{\lambda+A_t}}}. \end{cases} \tag{12}$$

Given that χ_t is uniformly distributed on $[0,1]$ it can also be shown that the probability of war, denoted z_t, is given by:

$$z_t = \begin{cases} 2 - \frac{R_t\sqrt{\frac{\lambda}{\lambda+A_t}}}{R_t-1+\sqrt{\frac{\lambda}{\lambda+A_t}}} & \text{if} \quad R_t = \frac{P_t\bar{c}}{A_t} > \frac{2\left(1-\sqrt{\frac{\lambda}{\lambda+A_t}}\right)}{2-\sqrt{\frac{\lambda}{\lambda+A_t}}}, \\ \\ 0 & \text{if} \quad R_t = \frac{P_t\bar{c}}{A_t} \leq \frac{2\left(1-\sqrt{\frac{\lambda}{\lambda+A_t}}\right)}{2-\sqrt{\frac{\lambda}{\lambda+A_t}}}. \end{cases} \tag{13}$$

4.5 Two-dimensional dynamics

4.5.1 Always war or always peace

Since the two countries' state variables are identical in all periods (cf. Proposition 1), we can write (6) and (7) as:

$$\begin{aligned} A_{t+1} &= A_t^{\alpha} P_t^{\beta}, \\ P_{t+1} &= \bar{n} w(A_t, P_t, \chi_t) \left[1 - \tfrac{\bar{c}P_t}{A_t}\right]^2 P_t, \end{aligned} \tag{14}$$

where $w(\cdot)$ is given by (12).

Given a sequence of values for χ_t, (14) and (12) define a two-dimensional system of difference equations. We can gain a lot of insight about its qualitative properties by considering the behavior of an economy which are constantly at war, or constantly at peace.

Proposition 3 *Consider an economy which evolves according to (14).*
(a) If the economy is always at peace, so that $w_t = w(A_t, P_t, \chi_t) = 1$ for all $t \geq 0$, then there exists a balanced growth path, where A_t, P_t, and $c_t = A_t/P_t$ exhibit sustained growth.
(b) If the economy is always at war, so that $w_t = w(A_t, P_t, \chi_t) = \lambda/(\lambda + A_t)$ for all $t \geq 0$, then there can be no sustained growth in either A_t, P_t, or $c_t = A_t/P_t$.

The proof is in Section A.4 of the Appendix.

Intuitively, technological progress requires growth in population, because of the scale effect in the production of new technologies. Wars become increasingly lethal as technology progresses, so permanent war rules out sustained growth in population, making technological progress come to a halt. Sustained growth in technology, population, and their ratio, thus requires peace.

The dynamics are illustrated in the phase diagrams in Figure 3. In an always-war economy (the upper panel) the locus along which population is constant ($P_{t+1} = P_t$) bends downward for high levels of technology, due to technology's effect on death rates in war; this rules out sustained growth in either technology or population. The same locus in an always-peace economy (the lower panel) is a straight line, enabling sustained growth.[14]

A detail worth noting is that the always-war dynamics are oscillatory, making technology and population move in cycles. A high level of technology implies lethal warfare, making population decline over time, thus reducing technology through the scale effect; this in turn makes warfare less lethal, allowing population levels to rise again.

Consider now an economy where the war probability is endogenous. While in a state of war the economy may move along a trajectory towards the war

[14] As long as $\alpha + \beta > 1$, the $(A_{t+1} = A_t)$-locus has infinite slope around the origin, implying that there also exists an unstable steady state, with an associated saddle path. In Figure 3 this steady state can be thought of as located very close to the origin, and not visible. In principle, an economy starting off below the saddle path contracts to the origin; if it starts off above it converges either to the balanced growth path (in the always-peace economy), or to the non-growing steady state (in an always-war economy). For the parameters chosen in the simulations later this steady state plays no role.

steady state, along which it experiences a decline in the probability of war. Two factors drive this: as A_t increases in absolute terms war becomes more costly; and as A_t increases relative to P_t (which happens if the economy approaches its steady state from the "west" in the upper panel of Figure 3) resource competition (R_t) declines. This, roughly, is how peace breaks out endogenously in this model.

4.5.2 Endogenous war: a simulation

Next we let the state of war or peace be determined endogenously by (12). To do this, we simulate an economy where χ_t is i.i.d. and uniformly distributed on $[0, 1]$. Given a sequence of values for χ_t, exogenous parameters, and initial conditions, we then update the state variables (P_t and A_t) period by period, using (12) and (14).[15] Note that, aside from the shocks, all variables which evolve over time do so endogenously.

Figure 4 shows the result from a single simulation over 200 periods. (That is, the paths are generated from one particular sequence of 200 values for χ_t.) The transition from stagnation to growth is seen in Panel A, where technology and population begin to grow at sustained rates and diverge. This leads to reduced mortality from starvation, and starvation going to zero in the limit (Panel B), as per-capita consumption begins to exhibit sustained growth (Panel C).

Panel B also shows how peace becomes more frequent over time; note that peace prevails when the war death rate [as given by $1 - w_t$; see (12)] equals zero. Over time, the war death rate first rises as technology improves, then drops to zero as wars are no longer fought. This gives the path an inverse U-shape.

Panel D provides an illustration of what drives this inverse U-shape. The probability of war, as given by (13), declines, and the death rate in war if

[15] The parameter values are set as follows: $\alpha = 0.55$, $\beta = 0.65$, $\lambda = 250$, $\bar{c} = 0.1$, and $\overline{n} = 1.2$. Initial conditions are set to $P_0 = 1.5$ and $A_0 = P_0^{\beta/(1-\alpha)} = 1.8$. However, the simulation here is only for illustration and these numbers are not important. As discussed below, since fertility is exogenous, the present model will by definition fail to generate realistic population growth rates (a demographic transition). Section 5.8 provides a slightly more ambitious calibration exercise in a framework where fertility is endogenous.

war breaks out, as given by $1 - v_t = A_t/(\lambda + A_t)$, rises. Wars thus become less frequent and more deadly over time, and are most deadly just before the probability of war drops to zero.

Due to the random component of this model, all simulations are different from one another (although they are qualitatively very similar). Figure 5 shows the time paths for consumption, time spent in resource competition, and the war death rate, for three different economies. Note the differences in timing: the first country to experience a growth takeoff is the first to experience a decline in resource competition, and a peak in its war death rate.

Figure 6 displays the result of a Monte Carlo simulation, where the paths show averages across 500 runs. The features are qualitatively similar to those of a single run, but the paths are smoother. The smoother war death rate path reflects the different timing across countries of the peaks, and subsequent drops, in war death rates.

Here we can also see an inversely U-shaped time path for the standard deviation in war death rates across the 500 countries (Panel C). This fits with the observed increase over time in the variance in war death rates shown in Figure 2, and discussed in Section 3.3.

4.6 Discussion

The model presented thus far can generate several joint trends observed in Western Europe over the last several centuries: a rise in the growth rate of technology and living standards, together with a parallel decline in the frequency, and a rise in the lethality, of war, as well as a rise in the variance of war death rates.

Some details could be modelled differently, without necessarily altering the main results. For example, an alternative approach may be to let some elite in each country own all land: rising population density and falling marginal product of labor might increase the risk of a revolution, and to avoid this the landed class might join with the landless in an effort to conquer land from the other country.

The model also makes many brave abstractions. For example, it generates technological progress by simply allowing population to enter the dynamic function for technology. Such scale effects from population to technological progress may have mattered at pre-industrial stages of development. However, in the world today skills, or human capital, are probably more important inputs in the production of new technologies.

The model is also inconsistent with some widely documented facts about how population tends to evolve in the course of economic development. Rather than increasing monotonically, population growth has been inversely U-shaped over time (Figure 7), the decline being driven by fertility reductions as parents have substituted away from quantity and into quality (education) of children. This is not captured in the setting presented thus far. Moreover, many have argued that the increased levels of education that came with this quality-quantity substitution generated further acceleration in technological progress (Galor and Weil 1999, 2000). The model set up in the next section addresses these shortcomings.

5 Model B: endogenous fertility

5.1 Structure

The model presented here shares many of the mechanisms at work in Model A. The major difference is that agents now choose the number of children to rear and how much human capital to invest in each child. To model this we specify two new functions: a utility function defined over the number of children a parent has and the children's human capital levels; and a human capital production function.

Moreover, in the spirit of Galor and Weil (2000), the human capital production function allows technology to influence the return to investing in children's human capital.

We also assume a slightly different timing of mortality events. Death from starvation or war occurs in adulthood, before agents have children. Moreover, the starvation survival probability now equals $q(c_{j,t})$, where the form for $q(c)$ is the same as in (2), i.e., $q(c) = \max\{0, (c - \overline{c})/c\}$. Like in Model A, the subindex j refers to the country (j =I,II). Consumption is given by (3), i.e.,

$c_{j,t} = (r_{j,t}/R_{j,t})(m_{j,t}A_{j,t}/P_{j,t})$. The rest of the notation is largely identical to Model A: $r_{j,t}$ is the individual agent's own time in resource competition; $R_{j,t}$ is the average level of $r_{j,t}$ across all agents in country j; $P_{j,t}$ is the (pre-war adult) population; $m_{j,t}$ is the size of country j's (post-war) territory; and $A_{j,t}$ is country j's level of technology.

Conditional on surviving war and starvation, adult agents rear $n_{j,t}$ children, all of whom survive to become adults in the next period.

Agents' preferences are defined over their number of children, $n_{j,t}$, and the human capital of each child, $h_{j,t+1}$, as captured by this utility function:

$$U_{j,t} = h_{j,t+1} n_{j,t}^{\gamma}, \tag{15}$$

where $\gamma \in (0, 1)$.

The probability that the agent lives to have children equals the product of the probability of surviving starvation, $q(c_{j,t})$, and the probability of surviving war, $v_{j,t}$ (if there is a war). An agent who dies in war or starvation is assigned utility zero.[16] Thus, expected utility can be written

$$E_t(U_{j,t}) = \begin{cases} v_{j,t} q(c_{j,t}) h_{j,t+1} n_{j,t}^{\gamma} + [1 - v_{j,t} q(c_{j,t})] \times 0 & \text{if war,} \\ q(c_{j,t}) h_{j,t+1} n_{j,t}^{\gamma} + [1 - q(c_{j,t})] \times 0 & \text{if peace.} \end{cases} \tag{16}$$

5.2 Human capital

Human capital in country j ($j =$I,II) of an agent who is adult in period t is denoted by $h_{j,t}$. Human capital transmitted to each child, $h_{j,t+1}$, depends on three factors: education time per child, $e_{j,t}$; the parent's own human capital, $h_{j,t}$; and an overall productivity factor, $\Gamma_{j,t}$. We use the following specification:

$$h_{j,t+1} = \Gamma_{j,t} e_{j,t} h_{j,t}^{\theta}, \tag{17}$$

where $\theta \in (0, 1)$.

The productivity factor, $\Gamma_{j,t}$, is assumed to have the following properties. First, at a given amount of time spent educating each child, $e_{j,t}$, educational productivity is increasing in the number of children being educated. One

[16] Alternatively, we could let (15) define the difference in utility from being alive and dead.

interpretation is that students and pupils may be able to help each other; another that having more students allows for specialization in teaching. Second, $\Gamma_{j,t}$ is assumed to be increasing in the level of technology, $A_{j,t}$. The idea is that books, computers, laboratory technologies, the internet, and other new technologies enhance the productivity of the human capital accumulation process.

To get tractable analytical solutions we use this functional form:

$$\Gamma_{j,t} = \frac{Bn_{j,t}}{F(A_{j,t}) + n_{j,t}}, \tag{18}$$

where $B > 0$, and $F(A)$ is non-increasing in its argument. It will be seen later that optimal fertility, $n_{j,t}$, simply equals a constant times $F(A_{j,t})$. Intuitively, technological progress lowers $F(A_{j,t})$, which generates higher returns to education. Agents respond by reducing fertility and instead invest in their children's human capital. This mechanism relates to Galor and Weil (2000), and Lagerlöf (2006), with the difference that here improvements in the level of technology, rather than its rate of change, generate the increase in the return to education.[17]

Total time spent educating children equals the time the parent spends outside of resource competition. The time endowment is set to unity so total education time equals

$$e_{j,t} = \frac{1 - r_{j,t}}{n_{j,t}} \tag{19}$$

per child. We use the convention that capital letters denote average levels of variables, so that $H_{j,t+1}$ denotes the average $h_{j,t+1}$ across agents in country j (and, recall, $R_{j,t}$ is average $r_{j,t}$). Using (17), (18), and (19), we can then write:

$$H_{j,t+1} = B\left[\frac{1 - R_{j,t}}{F(A_{j,t}) + n_{j,t}}\right] H_{j,t}^{\theta}, \tag{20}$$

for j =I,II. Note that intense resource competition (a high $R_{j,t}$) is detrimental to human capital accumulation.

[17]There is yet another interpretation of the productivity variable, $\Gamma_{j,t}$. One characteristic of models in which children's human capital is proportional to education time per child, $e_{j,t}$, is that parents can make human capital per child arbitrarily large by setting fertility, $n_{j,t}$, sufficiently small. Here this is impossible because the productivity of education goes to zero as the number of children being educated goes to zero.

5.3 Technology

Technological progress in country j depends on its levels of human capital and population. We use the following specification:

$$A_{j,t+1} = A_{j,t}[1 + D(P_{j,t})H_{j,t}], \tag{21}$$

for $(i,j) \in \{(\text{I,II}), (\text{II,I})\}$. The factor $D(P_{j,t})$ captures a scale effect from population size to efficiency in human capital accumulation, similar to Model A, and is given by:

$$D(P) = \min\{\phi P, D^*\}, \tag{22}$$

where $D^* > 0$, and $\phi > 0$. The idea is that increases in population levels (or density) raise technological progress at early stages of development, and only up to some maximum level. Thereafter increases in human capital alone drive improvements in technological progress.

5.4 Utility maximization

In Section A.3 in the Appendix it is shown that optimal fertility, $n_{j,t}$, is given by:

$$n_{j,t} = \left(\frac{\gamma}{1-\gamma}\right) F(A_{j,t}) \equiv n(A_{j,t}). \tag{23}$$

We can thus choose $F(A)$ to completely characterize the function $n(A)$. The following functional form is easy to work with:

$$n(A_{j,t}) = \overline{n} + (\underline{n} - \overline{n}) \max\left\{0, \frac{A_{j,t} - \widehat{A}}{A_{j,t}}\right\}, \tag{24}$$

where $\underline{n} < \overline{n}$, and $\widehat{A}$ is a threshold level of technology; while technology falls below this threshold fertility is constant at $\overline{n}$. That is, $n(A) = \overline{n}$ for $A \leq \widehat{A}$; $n'(A) < 0$ for $A > \widehat{A}$; and $\lim_{A\to\infty} n(A) = \underline{n}$.[18]

Section A.3 of the Appendix also shows that the optimal choice of $r_{j,t}$ is such that in equilibrium (where $r_{j,t} = R_{j,t}$) time spent in resource competition is given by (4).

[18]More precisely, we let $F(A) = \overline{F} + (\underline{F} - \overline{F})\max\{0, (A - \widehat{A})/A\}$, for some exogenous $\underline{F}$ and $\overline{F}$ and then define $\underline{n} = \underline{F}\gamma/(1-\gamma)$ and $\overline{n} = \overline{F}\gamma/(1-\gamma)$. Together with (23) this gives (24).

5.5 Six-dimensional dynamics

To find a dynamic equation for population, we first impose equilibrium ($r_{j,t} = R_{j,t}$) in (3). This gives consumption per agent, $c_{j,t} = m_{j,t}A_{j,t}/P_{j,t}$, which gives the survival rate from starvation (as long as $c_{j,t} > \bar{c}$) as

$$q(c_{j,t}) = \frac{c_{j,t} - \bar{c}}{c_{j,t}} = 1 - \frac{\bar{c}P_{j,t}}{m_{j,t}A_{j,t}}. \tag{25}$$

The survival rate from war can be written as $w_{j,t}$ in (8). Recall that a fraction $w_{j,t}q(c_{i,t})$ of the $P_{j,t}$ adults survive war and starvation, and these survivors have $n(A_{i,t})$ children each, so the population evolves according to $P_{j,t+1} = w_{j,t}q(c_{j,t})n(A_{j,t})P_{j,t}$. Using (25) this gives

$$P_{j,t+1} = w_{j,t}n(A_{j,t})\left(1 - \frac{\bar{c}P_{j,t}}{m_{i,j}A_{j,t}}\right)P_{j,t}. \tag{26}$$

Human capital evolves according to (20). Recalling from (4) that $1 - R_{j,t} = \bar{c}P_{j,t}/(m_{j,t}A_{j,t})$, and setting $n_{j,t} = n(A_{j,t})$, this gives

$$H_{j,t+1} = \frac{\gamma B}{n(A_{j,t})}\left[1 - \frac{\bar{c}P_{j,t}}{m_{j,t}A_{j,t}}\right]H^{\theta}_{j,t}. \tag{27}$$

Holding constant $m_{j,t}$ (j =I,II), (21), (26), and (27) constitute a state-dependent six-dimensional dynamical system. Next, like in Model A, we shall assume that the two countries are identical in the initial period. Then they can be seen to be identical also in all subsequent periods, making the dynamics three-dimensional.

5.6 War and peace

Like in Model A, the governments can be aggressive in an attempt to increase their territories, $m_{j,t}$. The next step is thus to find the payoffs to each government from being aggressive, and non-aggressive. The governments' objective functions are here their citizens' expected utility. Recall the notation from Model A: $S_{j,t}$ =**A** if country j's government is aggressive, and $S_{j,t}$ =**N** if it is non-aggressive (j =I,II). Peace prevails if $S_{\mathrm{I},t} = S_{\mathrm{II},t}$ =**N**; else war war breaks out and a fraction $1 - v_{j,t}$ of the agents in country j are killed, where $v_{j,t}$ is given in (8).

The payoffs to the government in one country of choosing **A** or **N** can now be seen to be identical to those of Model A, up to a constant.

Proposition 4 *Expected utility in equilibrium of an agent in country j can be written*

$$E_t(U_{j,t}) = \pi_{j,t} \left[\frac{\gamma B H_{j,t}^{\theta}}{n(A_{j,t})} \right], \tag{28}$$

where $\pi_{j,t}$ is given by (10).

The proof is in Section A.4 of the Appendix.

The factor in square brackets in (28) is taken as given by the government maximizing $E_t(U_{j,t})$ in period t, so the governments choose $S_{j,t}$ to simply maximize $\pi_{j,t}$, making the outcomes identical to those in Model A. To see this, first we add this assumption.

Assumption 3 $H_{I,0} = H_{II,0}$.

We can now state the following, also proven in Section A.4 of the Appendix.

Proposition 5 *Under Assumptions 2 and 3, and considering only pure-strategy equilibria, in all periods $t \geq 0$ it holds that $P_{I,t} = P_{II,t} = P_t$, $A_{I,t} = A_{II,t} = A_t$, $H_{I,t} = H_{II,t} = H_t$, and $m_{I,t-1} = m_{II,t-1} = 1$.*

We can thus do away with all subindices j. Moreover, since the payoffs are identical to those in (10) (up to a factor which the players take as constant) Proposition 2 applies: a peaceful equilibrium exists if (11) holds; else, only a war equilibrium exists (assuming pure strategies). Repeating the assumption of Model A, that the two countries coordinate on the peaceful equilibrium whenever that exists, the survival rate from war, w_t, can be written as in (12).

5.7 Three-dimensional dynamics

Suppressing the country index j in (21), (26), and (27), and setting $m_{j,t} = 1$, we can write the resulting three-dimensional dynamical system as:

$$\begin{aligned} P_{t+1} &= w(A_t, P_t, \chi_t) n(A_t) \left[1 - \tfrac{\bar{c}P_t}{A_t}\right] P_t, \\ A_{t+1} &= A_t[1 + D(P_t)H_t], \\ H_{t+1} &= \tfrac{\gamma B}{n(A_t)} \left[1 - \tfrac{\bar{c}P_t}{A_t}\right] H_t^{\theta}, \end{aligned} \tag{29}$$

where $w(\cdot)$ is given by (12), $n(A)$ by (24), and $D(P)$ by (22).

Given a sequence of values for χ_t and initial conditions, (29) fully determines the evolution of the economy. Since the system is three-dimensional it is difficult to illustrate in a two-dimensional phase diagram. However, if we impose the following assumption to ensure that a peaceful economy can exhibit sustained growth, the system can be seen to share some qualitative features with Model A.

Assumption 4 $D^* > (\underline{n} - 1) \left(\frac{n}{B\gamma}\right)^{1/(1-\theta)}$.

We can now state the following, which is proven in Section A.4 of the Appendix.

Proposition 6 *Consider an economy which evolves according to (29), where $D(P)$ is given by (22), and $n(A)$ is given by (24), where $\underline{n} > 1$. Under Assumption 4, the following holds.*
(a) If the economy is always at peace, so that $w_t = w(A_t, P_t, \chi_t) = 1$ for all $t \geq 0$, then there exists a balanced growth path, where A_t, P_t, and $c_t = A_t/P_t$ exhibit sustained growth.
(b) If the economy is always at war, so that $w_t = w(A_t, P_t, \chi_t) = \lambda/(\lambda + A_t)$ for all $t \geq 0$, then there can be no sustained growth in A_t or P_t.

The intuition is similar to that behind Proposition 3 in Model A. Permanent war rules out sustained growth because wars become increasingly lethal as technology progresses, in turn ruling out sustained population growth.[19]

Sustained growth in both technology, population, and per-capita consumption, thus requires peace. Moreover, recall that Proposition 2 applies, so peace breaks out endogenously as the result technological progress. As in Model A, governments are less inclined to fight more lethal wars. A novel ingredient in this model compared to Model A is that technology, after reaching the threshold $\widehat{A}$, drives a fertility decline, and a rise in human capital levels. This generates a spurt in technological progress, and thus reduces population pressure and further mitigates the Malthusian war motive.

5.7.1 Dynamics in the Malthusian regime

To see the intuition behind the dynamics we may think of an underdeveloped economy in which the level of technology is low and changes slowly over time [as will be the case while human capital and/or population levels are low; see (29)]. The behavior of this economy may be approximated by a holding technology fixed at some level $\overline{A} < \widehat{A}$. Thus, fertility is constant at $\overline{n}$; see (24). For the moment, hold the war survival rate fixed at some level $\overline{w}$ (which may be a function of $\overline{A}$ or equal to one). The three-dimensional system in (29) then boils down to the following two-dimensional system:

$$\begin{aligned} P_{t+1} &= \overline{wn}\left[1 - \tfrac{\bar{c}P_t}{\overline{A}}\right] P_t, \\ H_{t+1} &= \tfrac{\gamma B}{\overline{n}}\left[1 - \tfrac{\bar{c}P_t}{\overline{A}}\right] H_t^{\theta}. \end{aligned} \tag{30}$$

Some properties of the system in (30) can be summarized as follows.

Proposition 7 *Consider an economy which evolves according to (30).*
(a) If $\overline{wn} > 1$, then there exists a steady-state equilibrium, $P_t = P^*$ and $H_t = H^*$, where

$$\begin{aligned} P^* &= \tfrac{\overline{A}}{\bar{c}}\left(1 - \tfrac{1}{\overline{wn}}\right), \\ H^* &= \left(\tfrac{\gamma B}{\overline{wn}^2}\right)^{\frac{1}{1-\theta}}. \end{aligned} \tag{31}$$

[19]Different from Model A, however, here technology cannot regress. Rather, with permanent war, population contracts indefinitely, so that the rate of technology growth goes to zero, and technology converges to a constant level. With constant technology and shrinking population, per-capita consumption, A_t/P_t, actually exhibits sustained growth (although not of the type we observe empirically).

(b) If the economy is always at peace, so that $\overline{w} = 1$ for all $t \geq 0$, then H^* does not depend on $\overline{A}$, and P^* is strictly increasing in $\overline{A}$.
(c) If the economy is always at war, so that $\overline{w} = \lambda/(\lambda + \overline{A})$ for all $t \geq 0$, then both H^* and P^* are strictly increasing in $\overline{A}$.

Intuitively, a rise in technology ($\overline{A}$) leads to higher survival from starvation, but also lower survival from war (lower $\overline{w}$). Thus, with permanent war steady-state population increases less than proportionally to technology. A higher technology-population ratio leads to less resource competition (lower R_t), and thus to more human capital investment.

Consider now what happens if technology evolves endogenously. Then an initial rise in technology raises the level of population and (under permanent war) human capital, which in turn implies further technological progress.

One may also illustrate (30) in a phase diagram, to see that the steady-state equilibrium in (31) appears globally stable. However, discrete systems may nevertheless be oscillatory and/or diverge globally. (Note, in particular, that if population exceeds $\overline{A}/\overline{c}$ it becomes extinct in the next period.) Section 5.8 examines the global behavior of the system in (29) for a reasonable quantitative example, where the outbreak of war is determined endogenously.

5.8 Quantitative analysis

The most straightforward way to illustrate the qualitative behavior of the three-dimensional system in (29) is to simulate it. All functions are given specific forms already, as characterized by 10 parameters: B, γ, θ, $\overline{c}$, $\overline{n}$, $\underline{n}$, $\widehat{A}$, λ, D^*, and ϕ. Because the model is set up in a stylized manner it is hard to pin down an empirically realistic value for every parameter. However, there is some logic to how we have chosen most of them, as described below.

5.8.1 The length of a period

First, we let each period correspond to 5 years. The two-period overlapping-generations structure might suggest that each period be about 20 years. However, 5 years is closer to the duration of e.g. WWI and WWII, allowing longer wars to be interpreted as several periods of subsequent war. Moreover, some of the European 20th-century events that the model is designed to explain lasted for only about half a century, so it is desirable that these phases correspond to more than just a couple of periods in the model.

To translate each period to years A.D. we let the war death rate peak in 1945.

5.8.2 Parameter values

Table 1 lists the parameter values chosen and summarizes what functions they originate from.

We set $\overline{n} = 1.4$, which implies an upper bound for the population growth rate of 40% over 5 years, or about 8% per year. Note that population will never grow at that rate. Rather, this corresponds to a hypothetical growth rate during the Malthusian phase (i.e., while $A_t < \widehat{A}$), if mortality from starvation had been zero [$q(c_t) = 1$], and if no agent died from war.[20]

We set $\underline{n} = 1.015$, which corresponds to a population growth rate on the balanced growth path of 1.5% per period, or about 0.3% per year.

The level of subsistence consumption, $\overline{c}$, is set to 0.025. This value is chosen quite arbitrary, but given how we set other parameters values, it is low enough that the model can generate paths where $c_t = A_t/P_t$ exceeds $\overline{c}$ in every period, thus ensuring that the population never dies out.

The threshold level of technology above which fertility starts to decline, $\widehat{A}$, is set arbitrarily to 10.

Given $\widehat{A} = 10$, we then set λ so that $\lambda/(\lambda+ \widehat{A})$ equals 0.05. This implies a war death rate of about 5% at the onset of the demographic transition.

The value for θ is set to 0.2. This means that a 1% increase in the human capital of the parent (holding constant education) raises the child's human capital by 0.2%. This is consistent with Solon (2002), who reports estimates of the elasticity of a son's earnings with respect to the father's earnings, ranging from 0.11 to 0.42.

This leaves us with four parameters: B, γ, D^*, and ϕ. However, because only the product $B\gamma$ plays any role in the dynamics (holding fixed $\underline{n}$ and $\overline{n}$),

[20] This can also be interpreted as 4 children per agent over a 5-year period, corresponding to 8 children per woman in a two-sex setting, or 1.6 children per woman and year.

we can normalize B to 1, leaving us with only three parameters to pin down: γ, D^*, and ϕ. We choose these to make the model fit the following targets:

First, the per-capita consumption growth rate on the balanced growth path is set to be close to the growth rate in GDP per capita in the developed world today, about 2.5% per year.

Second, the peak of the population growth rate in the model is fit to the maximum annual population growth rate in Western Europe. This was around 0.75% per annum in the period 1870-1913 (see Figure 7), but probably somewhat higher if we were to include those who migrated during this era; we take 1% to be a reasonable target.

Third, the decline in fertility (which, recall, occurs when A_t reaches $\widehat{A}$) is set to begin around 1900, given that the peak in war deaths was set to 1945. Although the timing varies across countries it fits roughly with the actual decline in fertility in most of Western Europe.

Fourth, we set the scale effect from population on technological progress, $D(P_t)$, to reach its maximum in the same period as the quality-quantity shift sets in, i.e., when A_t reaches $\widehat{A}$.

We use the following algorithm to fit these targets. We pick some first guess for γ and ϕ. We then set D^* so that the scale effect from population on technological progress reaches its maximum when A_t reaches $\widehat{A}$.[21] Given this choice of D^* we can then re-adjust ϕ and γ so that the decline in fertility occurs around 1900, population growth peaks around 1%, and the per-capita consumption growth rate is about 2.5% on the balanced growth path.[22] The parameter values listed in Table 1 get us close to these targets.

[21] Since the path for A_t is random, we run the model 500 times, calculating for each run what value ϕP_t takes when A_t first exceeds $\widehat{A}$. We then set D^* to the average of these values across all 500 runs.

[22] Per-capita consumption equals $c_t = A_t/P_t$ so we can write the gross growth rate of c_t as $c_{t+1}/c_t = (A_{t+1}/A_t)/(P_{t+1}/P_t)$. On the balanced growth path mortality from war and starvation approaches zero, so $P_{t+1}/P_t = \underline{n}$. Setting $n(A_t) = \underline{n}$ and $R_t = \bar{c}P_t/A_t = 0$ in (29), the (non-growing) level of human capital on the balanced growth path equals $(B\gamma/\underline{n})^{1/(1-\theta)}$. Setting the scale effect from population to its maximum, D^*, gives $A_{t+1}/A_t = 1 + D^*(B\gamma/\underline{n})^{1/(1-\theta)}$. Per-capita consumption growth on the balanced growth path can thus be written $c_{t+1}/c_t = [1 + D^*(B\gamma/\underline{n})^{1/(1-\theta)}]/\underline{n}$.

5.8.3 Initial conditions

We set initial technology, A_0, equal to 10% of $\widehat{A}$. Initial levels of population and human capital, P_0 and H_0, are set to the hypothetical steady state levels they would converge to if technology stayed constant at A_0, and the economy remained in a state of permanent war [i.e., setting $w_0 = \lambda/(\lambda+A_0)$]. Analogous to (31) this gives $P_0 = (A_0/\overline{c})\left[1 - (\lambda + A_0)/(\overline{n}\lambda)\right]$, and $H_0 = \left[B\gamma(\lambda + A_0)/(\overline{n}^2\lambda)\right]^{1/(1-\theta)}$.

5.8.4 Simulation results

Having chosen parameter values and initial conditions the simulation algorithm is the same as for Model A. First, we generate a series of shocks, χ_t, from a uniform distribution on $[0,1]$. We then update the state variables (H_t, P_t, and A_t) period by period, using the difference equations in (29).

Figures 8 and 9 show the results from a Monte Carlo simulation where we run the model 500 times. Each run may represent one country (or, to be precise, one pair of identical countries). The paths show the averages of the different variables across the 500 runs. We display the results from 1600 to 2050, where (as described above) the average war death rate is set to peak in 1945.

Panel A in Figure 8 shows the paths for the levels of technology and population. These grow throughout but around 1900 they diverge as population levels out, due to the arrival of the demographic transition (as A_t reaches $\widehat{A}$). Simultaneously, per-capita consumption starts to grow at a faster rate, and human capital starts to rise, converging to a new, higher level.

Panel B in Figure 8 shows the endogenously evolving paths for the death rates from war, $1-w_t$, and starvation, $1-q(c_t)$. Whereas war mortality shows an inversely U-shaped pattern, non-war mortality is declining monotonically, but faster when growth in per-capita consumption sets in. Both these trends seem consistent with the data. (Recall that the paths show the means across 500 runs; war deaths in each individual run drop to zero when the probability of war does.)

Panel C in Figure 8 shows the two sides of what we usually refer to as the demographic transition: a decline in mortality, followed by a lagged decline in fertility. The decline in fertility begins when A_t reaches $\widehat{A}$.

Panel D shows the probability of being at war for the mean country, as calculated by applying (13) to the mean levels of A_t and P_t, and the fraction of all 500 countries actually being at war in any period.[23] Both are declining, which fits with the centuries-long decline in the frequency of war in Figure 1. Panel D also shows the incline in the deadliness of war, as measured by $1 - v_t = A_t/(\lambda + A_t)$. This also fits with the stylized facts described earlier.

Figure 9 shows the time paths for the annual percentage growth rates of population and per-capita consumption, and the hypothetical no-war population growth path (derived by setting $w_t = 1$).[24] The latter is inversely U-shaped, whereas actual population growth follows a path which can be described as inversely U-shaped with a "dent" around the peak, similar to that in Figure 7. This pattern is driven by accelerating technological progress, which raises living standards, thus making non-war deaths decline, and at the same time makes wars more deadly.

The path for per-capita consumption growth in Figure 9 follows largely the same pattern as that of per-capita GDP growth in Western Europe shown in Figure 7. A slow and steady rise in the per-capita consumption growth rate is followed by a spurt, due to faster technological progress at the onset of the demographic transition. One detail is inconsistent with the facts: since consumption here equals the technology-population ratio, per-capita consumption growth rises in the most war-intense phase during which population growth drops. This need not be true in a model where wars destroy physical capital.

[23]The two lines are not identical, because all countries have different levels of A_t and P_t in any period. When the probability of war reaches zero for the mean country some countries which are slower to develop still fight wars.

[24]The annual population growth rate is calculated as $100 \times \{[w_t n_t q(c_t)]^{1/5} - 1\}$. The hypothetical population growth rate if wars were eliminated is calculated as $100 \times \{[n_t q(c_t)]^{1/5} - 1\}$. (Recall that each period is 5 years.)

6 Conclusions

We have presented two version of a unified growth model in which economies take off endogenously from stagnation to sustained growth in living standards, and from war to peace. At the micro level agents compete for food for their survival. When resources are scarce agents allocate more of their time to competing over resources. Land is in fixed supply, so higher population leads to scarcer resources and more resource competition. However, improvements in technology mitigates resource scarcity.

Governments start wars aiming to increase their country's territories, and thus their citizens' starvation survival probabilities, but weigh this against agents being killed in war. Governments in countries where resources are scarce, and thus resource competition intense, are more inclined to start wars.

Technological progress makes war less frequent by reducing resource scarcity, and thus the desire for land conquest. It also makes wars more deadly, and thus governments less prone to fight them. In the transition to a peaceful balanced growth path where per-capita consumption grows at a sustained rate, an economy can pass a phase of very high war death rates. This may capture what happened in Western Europe in the first half of the 20th century.

We also allow for a random element in the decision to engage in war. Among countries which are initially identical, those hit by worse war shocks tend to take off later from stagnation to growth. Across countries the variance in war death rates displays an initial rise as country after country passes through the most lethal war phase of development, before becoming completely peaceful. This pattern also seems consistent with the data.

An extended setting, where fertility is endogenous, can also replicate a demographic transition, i.e., a decline in fertility following a decline in mortality. The framework reproduces a population growth path which is inversely U-shaped, but with a "dent" around the peak, consistent with the Western European experience around the time of the two world wars; cf. Figures 7 and 9.

We have of course abstracted from many factors which we know matter for the likelihood of war: institutions is one example, in particular democracy.[25] Some wars being fought today (e.g., in Iraq and Afghanistan, not included in the data shown here) have lower death rates than earlier wars (at least when measured by battle deaths as a fraction of the total populations of the United States and/or Europe). However, these wars may be exceptions proving the rule. History's deadliest wars (like WWI and WWII) were fought between the technological leaders at the time, and it was new technologies that made those wars so deadly. According to our theory, the technological leaders today do not fight wars against one another, because the death rates would be too high, and technologies have advanced enough to make land and resource scarcity irrelevant. (Of course, one may think of many other, often related, explanations too.) In fact, we only model wars between technologically symmetric countries; strictly speaking, we are silent about e.g. guerilla wars. However, consistent with our assumptions, it seems that *if* wars between technologically advanced nations were fought today death rates could become very high indeed.[26]

A Appendix

A.1 Data

A.1.1 The frequency of war

Figure 1 shows two indices over the number of ongoing wars. These are constructed using two different lists of wars, with both a start year and an end year for each war. For each list, we simply calculate the total number of wars that went on in each year, and then average over decades. Next we describe these two lists of wars.

[25] See e.g. Easterly et al. (2006) for a discussion. However, some have also argued that particularly young and emerging democracies need not always be more peaceful than dictatorships; see Mansfield and Snyder (2005) and Baliga et al. (2006).

[26] One could take this argument one step further. In our model, wars are started because the conquest of land can potentially reduce death from starvation, making limited war deaths tolerable to the governments who start them. Today, troop deaths in Iraq and Afghanistan are tolerated because (some believe) these can reduce expected death risks from terrorist attacks.

Levy (1983) The Levy (1983) data set consists of 119 wars, which start and end between 1495 and 1975. These are listed in Table A.1. The list is restricted to wars which involve at least one Great Power (as defined below), and excludes "civil wars, unless they become internationalized through the intervention of an external state; and [...] imperial or colonial wars, unless they expand through the intervention of another state" (Levy 1983, p. 51).

The Great Power dummy in Table A.1 indicates which wars involved two Great Powers, or more.

Levy (1983, Table 2.1) defines the following nations/empires as Great Powers in the time periods indicated in parentheses: France (1495-1975); England/Great Britain (1495-1975); Austrian Habsburgs/Austria (1495-1519, 1556-1918); Spain (1495-1519, 1556-1808); Ottoman Empire (1495-1699); United Habsburgs (1519-1556); The Netherlands (1609-1713); Sweden (1609-1713); Russia/Soviet Union (1721-1975); Prussia/Germany/FRG (1740-1975); Italy (1861-1975); United States (1898-1975); Japan (1905-1945); China (1949-1905).

For each war, Levy (1983, Table 4.1) provides data on e.g. start year, end year, duration, severity (number of battle deaths), and intensity (battle deaths in proportion to population).

Intensity is computed as the number of battle deaths per million European population. The motivation for using European population is that "[t]he Great Power system has historically been European-based, and for most of the temporal span of the system national population growth rates have not deviated significantly from that of Europe as a whole" (Levy 1983, p. 87). From 1500 to 1913 the population of Western Europe stayed within a bound of 13.1% and 14.7% of world population; the corresponding numbers for Eastern Europe are 3.1% and 4.4% (Maddison 2003, Table 8a). Thus, the time trends discussed here should not be sensitive to using world population instead.

The death rates shown in Figure 2 are calculated as follows. Let D be Levy's (1983) measure of intensity (battle deaths per million European population), and L the duration of the war (in years). Then we use this measure of the annual death rate:

$$\ln\left\{\left(1+\frac{D}{1,000,000}\right)^{\frac{1}{L}}\right\}.$$

That is, $D/1,000,000$ is the percentage death rate per capita over the whole war. For example, WWI lasted for 4.3 years and battle deaths were 57,616 per million European population ($L = 4.3$, $D = 57,616$). The above formula then gives 0.013027. Note that WWI and WWII would be outliers in Figure 2 if we were to show them.

Figure 2 refers to wars involving two, or more, Great Powers, as indicated by the dummy variable in Table A.1.

Wikipedia The Wikipedia list of wars is copied from the following Web site:

http://en.wikipedia.org/wiki/List_of_conflicts_in_Europe

This list starts earlier than the Levy (1983) data. Disregarding all wars from the Greek and Roman empires 500-71 B.C., and the Norman conquest in 1066, leaves us with a list of 64 wars, starting in 1337 with the Hundred Years' War.

We then subdivide some of these wars into shorter wars. For example, the Hundred Years' War (1337-1453) is divided into the Edwardian War (1337-1360), the Caroline War (1369-1389), and the Lancastrian War (1415-1429). The subdivisions for the Eighty Years' War and the Italian Independence Wars are detailed in Table A.2.

Finally, by following the links provided on the Wikipedia Web site (and using some common sense), we selected those wars which played out in Western Europe. How this selection was done is shown by the Western Europe dummy in Table A.2.

A.2 Optimality conditions in Model A

The agent chooses $r_{j,t}$ to maximize $q(c_{j,t})(1-r_{j,t})$, subject to (3). The solution is given by

$$g(c_{j,t})\frac{\partial c_{j,t}}{\partial r_{j,t}}(1-r_{j,t}) = g(c_{j,t})\frac{1-r_{j,t}}{r_{j,t}} = 1, \tag{A1}$$

where $g(c)$ is the elasticity of $q(c)$ with respect to c, which from (2) becomes

$$g(c)=\frac{q'(c)c}{q(c)}=\frac{\overline{c}}{c-\overline{c}}. \tag{A2}$$

In equilibrium, where $R_{j,t}=r_{j,t}$ and $c_{j,t}=(m_{j,t}A_{j,t})/P_{j,t}$, (A1) can be written

$$g\left(c_{j,t}\right)\left(\frac{1-R_{j,t}}{R_{j,t}}\right)=g\left(\frac{m_{j,t}A_{j,t}}{P_{j,t}}\right)\left(\frac{1-R_{j,t}}{R_{j,t}}\right)=1. \tag{A3}$$

Using (A2) and (A3) gives (4). More generally, (A3) demonstrates that $R_{j,t}$ is decreasing (increasing) in $c_{j,t}$ if $g(c_{j,t})$ is decreasing (increasing) in $c_{j,t}$, which is the result of Grossman and Mendoza (2003).

A.3 Optimality conditions in Model B

Agents maximize utility in (16) subject to (18), (17), and (19). Note that the survival rate of war (if there is a war) is simply given by a constant, $v_{j,t}$, in front of the objective function in peace. Since $v_{j,t}$ and the probability of war are taken as given by each agent, the utility maximization problem is the same in both war or peace, and can be written:

$$\max_{(r_{j,t},n_{j,t})\in[0,1]\times\Re_+} q\left(\frac{r_{j,t}m_{j,t}A_{j,t}}{R_{j,t}P_{j,t}}\right)\frac{B(1-r_{j,t})h_{j,t}^{\theta}n_{j,t}^{\gamma}}{F(A_{j,t})+n_{j,t}}, \tag{A4}$$

where $q(\cdot)$ is given in (2). The first-order condition for fertility can be written

$$q(c_{j,t})B(1-r_{j,t})\left\{\frac{\gamma n_{j,t}^{\gamma-1}\left[F(A_{j,t})+n_{j,t}\right]-n_{j,t}^{\gamma}}{\left[F(A_{j,t})+n_{j,t}\right]^2}\right\}h_{j,t}^{\theta}=0, \tag{A5}$$

which gives (24).

Disregarding factors in (A4) which do not involve $r_{j,t}$, the optimal choice of $r_{j,t}$ is given by maximizing $q(c_{j,t})(1-r_{j,t})$, subject to $c_{j,t}=(r_{j,t}m_{j,t}A_{j,t})/(R_{j,t}P_{j,t})$. As shown in Section A.2 above, this gives (4).

A.4 Proofs

Proof of Proposition 1: From (10) it is seen that $\pi_{j,t}^{\mathrm{A,A}}>\pi_{j,t}^{\mathrm{N,A}}$ always holds, since $v_{j,t}<1$. Thus, non-aggression is never an optimal response if the other country is aggressive, and $S_{\mathrm{II},t} \neq S_{\mathrm{I},t}$ cannot hold in any pure-strategy equilibrium. It follows from (9) that no territory will be redistributed in equilibrium, so $m_{\mathrm{I},t} = m_{\mathrm{II},t} = 1$ for all $t \geq 0$. The dynamic equations in (7) and (6) then give the same $P_{j,t+1}$ and $A_{j,t+1}$ in both economies, $j =$I,II. $\|$

Proof of Proposition 2: First set $P_{\mathrm{I},t} = P_{\mathrm{II},t} = P_t$, $A_{\mathrm{I},t} = A_{\mathrm{II},t} = A_t$, and $m_{\mathrm{I},t-1} = m_{\mathrm{II},t-1} = 1$ in (10). Consider now part (a). For $S_{\mathrm{II},t} = S_{\mathrm{I},t} =$**A** to be an equilibrium, it must hold that no country wants to be non-aggressive if the other is aggressive: $\pi_{\mathrm{I},t}^{\mathrm{A,A}} \geq \pi_{\mathrm{I},t}^{\mathrm{N,A}}$ and $\pi_{\mathrm{II},t}^{\mathrm{A,A}} \geq \pi_{\mathrm{II},t}^{\mathrm{N,A}}$. Using (10), these inequalities become identical and can be written $v_t \left[1 - \overline{c}P_t/\{(1-\chi_t)A_t\}\right]^2 \leq \left[1 - \overline{c}P_t/A_t\right]^2$. This always holds, since $v_t \leq 1$, and $\chi_t \in [0,1]$. Thus, $S_{\mathrm{II},t} = S_{\mathrm{I},t} =$**A** is always an equilibrium. Next consider part (b). For $S_{\mathrm{II},t} = S_{\mathrm{I},t} =$**N** to be an equilibrium it must hold that no country wants to make a one-sided deviation from mutual non-aggression: $\pi_{\mathrm{I},t}^{\mathrm{N,N}} \geq \pi_{\mathrm{I},t}^{\mathrm{A,N}}$ and $\pi_{\mathrm{II},t}^{\mathrm{N,N}} \geq \pi_{\mathrm{II},t}^{\mathrm{A,N}}$. Using (10), these inequalities become identical and can be written $v_t \left[1 - \overline{c}P_t/A_t\right]^2 \geq \left[1 - \overline{c}P_t/\{(1+\chi_t)A_t\}\right]^2$, which gives (11); equivalently, peace cannot be an equilibrium if (11) is reversed. Part (a) showed that a war equilibrium always exists. Thus, if (11) holds both war and peace equilibria exist. $\|$

Proof of Proposition 3: To prove (a), first conjecture that a balanced growth path (BGP) exists where A_t/P_t exhibits sustained growth. On that BGP, P_t/A_t must be zero. It then follows from (14), $\overline{n} > 1$, and $w_t = 1$ that $P_{t+1}/P_t = \overline{n} > 1$. Let g^* denote the (gross) growth rate of A_t on the BGP, i.e., $A_{t+1}/A_t = A_t^{\alpha-1}P_t^{\beta} = g^*$. On the BGP it must hold that $A_{t+1}^{\alpha-1}P_{t+1}^{\beta} = g^*$, so $(A_{t+1}/A_t)^{\alpha-1}(P_{t+1}/P_t)^{\beta} = 1$, or $g^{*\alpha-1}\overline{n}^{\beta} = 1$. It follows that $g^* = \overline{n}^{\beta/(1-\alpha)} > \overline{n}$, where the inequality follows from $\overline{n} > 1$, and Assumption 1. Thus, $A_{t+1}/A_t = g^* > P_{t+1}/P_t = \overline{n}$, verifying that A_t/P_t indeed exhibits sustained growth on this BGP. The proof of (b) is done through contradiction: if A_t were to exhibit sustained growth, $w_t = \lambda/(\lambda + A_t)$ would approach zero, ruling out sustained growth in P_t, which from (14) also rules out sustained growth in A_t, and A_t/P_t. $\|$

Proof of Proposition 4: Expected utility in equilibrium is given by setting $r_{j,t} = R_{j,t}$, $h_{j,t} = H_{j,t}$, and $h_{j,t+1} = H_{j,t+1}$ in (16). Using (20), (24), (25), and recalling from (4) that $1 - R_{j,t} = 1 - \overline{c}P_{j,t}/(m_{j,t}A_{j,t})$, we can write (16) as in

(28), where

$$\pi_{j,t} = \begin{cases} v_{j,t}\left[1 - \frac{\bar{c}P_{j,t}}{m_{j,t}A_{j,t}}\right]^2 & \text{if war,} \\ \left[1 - \frac{\bar{c}P_{j,t}}{m_{j,t}A_{j,t}}\right]^2 & \text{if peace.} \end{cases} \tag{A6}$$

Recall that war prevails unless $S_{\mathrm{I},t} = S_{\mathrm{II},t} = \mathbf{N}$. Using (9) to substitute for updated territory, $m_{j,t}$, in (A6), gives (10). ‖

Proof of Proposition 5: From (10) it is seen that $\pi_{j,t}^{\mathrm{A,A}} > \pi_{j,t}^{\mathrm{N,A}}$ always holds, since $v_{j,t} < 1$. Thus, non-aggression is never an optimal response if the other country is aggressive, and $S_{\mathrm{II},t} \neq S_{\mathrm{I},t}$ cannot hold in any pure-strategy equilibrium. It follows from (9) that no territory will be redistributed in equilibrium, so $m_{\mathrm{I},t} = m_{\mathrm{II},t} = 1$ for all $t \geq 0$. The dynamic equations in (21), (26), and (27) then give the same $P_{j,t+1}$, $H_{j,t+1}$, and $A_{j,t+1}$ in both economies, j =I,II. ‖

Proof of Proposition 6: To prove (a), first conjecture that a balanced growth path (BGP) exists where A_t and A_t/P_t exhibit sustained growth. Thus, $n(A_t)$ goes to $\underline{n} > 1$, and H_t to $(B\gamma/\underline{n})^{1-\theta}$, and P_t exhibits sustained growth at gross rate $\underline{n} > 1$. This means that $D(P_t)$ equals D^*, and A_t grows at gross rate $1 + D^*(B\gamma/\underline{n})^{1-\theta}$, which under Assumption 4 exceeds the gross population growth rate, $\underline{n}$. The proof of (b) is done through contradiction: if A_t were to exhibit sustained growth, $w_t = \lambda/(\lambda + A_t)$ would approach zero, implying that P_t goes to zero. Thus, using (22), $D(P_t) = \phi P_t$ would go to zero, ruling out sustained growth in A_t. ‖

Proof of Proposition 7: Part (a) follows from setting $P_{t+1} = P_t = P^*$ and $H_{t+1} = H_t = H^*$; then (30) gives (31). Part (b) follows from simply differentiating H^* and P^* with respect to $\overline{A}$, holding $\overline{w} = 1$. To prove Part (c), we first note that H^* depends on $\overline{A}$ only through $\overline{w}$, which is decreasing in $\overline{A}$. To see that P^* are strictly increasing in $\overline{A}$, use $\overline{w} = \lambda/(\lambda + \overline{A})$ and (31) to derive that:

$$\begin{gathered} \frac{\partial P^*}{\partial \overline{A}} = \frac{1}{\bar{c}}\left(1 - \frac{1}{\overline{w}\overline{n}}\right) + \frac{\overline{A}}{\bar{c}\overline{n}}\frac{\partial \overline{w}}{\partial \overline{A}}\frac{1}{\overline{w}^2} \\ = \frac{\overline{A}}{\bar{c}}\left(1 - \frac{\lambda + \overline{A}}{\lambda \overline{n}}\right) - \frac{\overline{A}}{\bar{c}}\left(\frac{1}{\lambda \overline{n}}\right) = \frac{\overline{n} - 1}{\bar{c}\overline{n}} > 0, \end{gathered} \tag{A7}$$

where we have used $\partial \overline{w}/\partial \overline{A} = -\lambda/(\lambda + \overline{A})^2$. ‖

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Parameter	**Value**	**From function**
$\overline{c}$	0.025	Starvation survival function in (2)
B	1	Human capital production function in (17) and (18)
θ	0.2	
$\overline{n}$	1.4	Fertility function in (24)
$\underline{n}$	1.015	
$\widehat{A}$	10	
ϕ	0.010	Technological updating in (21) or (22)
D^*	0.813	
γ	0.250	Utility function in (15)
λ	190	War survival function in (12)

Table 1: Parameter values in Model B.

Table A.1: The Levy war data

ID	Name of war	Start	End	Duration	Intensity	Great Powers dummy
1	War of the League of Venice	1495	1497	2	119	1
2	Polish-Turkish War	1497	1498	1	45	0
3	Venetian-Turkish War	1499	1503	4	60	0
4	First Milanese War	1499	1500	1	29	0
5	Neapolitan War	1504	1501	3	269	1
6	War of the Cambrian League	1508	1509	1	145	0
7	War of the Holy League	1511	1514	3	261	1
8	Austro-Turkish War	1512	1519	7	343	1
9	Scottish War	1513	1515	2	57	0
10	Second Milanese War	1515	1515	0.5	43	1
11	First War of Charles V	1521	1526	5	420	1
12	Ottoman War	1521	1531	10	958	1
13	Scottish War	1522	1523	1	41	0
14	Second War of Charles V	1526	1529	3	249	1
15	Ottoman War	1532	1535	3	384	1
16	Scottish War	1532	1534	2	55	0
17	Third War of Charles V	1536	1538	2	438	1
18	Ottoman War	1537	1547	10	1329	1
19	Scottish War	1542	1550	8	176	0
20	Fourth War of Charles V	1542	1544	2	629	1
21	Siege of Boulogne	1544	1546	2	107	1
22	Arundel’s Rebellion	1549	1550	1	79	1
23	Ottoman War	1551	1556	5	578	1
24	Fifth War of Charles V	1552	1556	4	668	1
25	Austro-Turkish War	1556	1562	6	676	1
26	Franco-Spanish War	1556	1559	3	316	1
27	Scottish War	1559	1560	1	78	1
28	Spanish-Turkish War	1559	1564	5	310	1
29	First Huguenot War	1562	1564	2	77	1
30	Austro-Turkish War	1565	1568	3	306	0
31	Spanish Turkish War	1569	1580	11	608	1
32	Austro-Turkish War	1576	1583	7	600	0
33	Spanish-Portuguese War	1579	1581	2	50	0
34	Polish-Turkish War	1583	1590	7	210	0
35	War of the Armada	1585	1604	19	588	1
36	Austro-Polish War	1587	1588	1	49	0
37	War of Three Henries	1589	1598	9	195	1
38	Austro-Turkish War	1593	1606	13	1086	1
39	Franco-Savoian War	1600	1601	1	24	0
40	Spanish-Turkish War	1610	1614	4	175	1
41	Austro-Venetian War	1615	1618	3	70	0
42	Spanish-Savoian War	1615	1617	2	23	0
43	Spanish-Venetian War	1617	1621	4	58	0
44	Spanish-Turkish War	1618	1619	1	69	1
45	Polish-Turkish War	1618	1621	3	173	0
46	Thirty Years’ War - Bohemian	1618	1625	7	3535	1
47	Thirty Years’ War - Danish	1625	1630	5	3432	1
48	Thirty Years’ War - Swedish	1630	1635	5	3568	1
49	Thirty Years’ War - Swedish-French	1635	1648	13	12933	1
50	Spanish-Protuguese War	1642	1668	26	882	0

Table A.1 continued

51	Turkish-Venetian War	1645	1664	19	791	0
52	Franco-Spanish War	1648	1659	11	1187	1
53	Scottish War	1650	1651	1	22	0
54	Anglo-Dutch Naval War	1652	1655	3	282	0
55	Great Northern War	1654	1660	6	238	1
56	English-Spanish War	1656	1659	3	161	1
57	Dutch-Portuguese War	1657	1661	4	43	0
58	Ottoman War	1657	1664	7	1170	1
59	Sweden-Bremen War	1665	1666	1	11	0
60	Anglo-Dutch Naval War	1665	1667	2	392	1
61	Devolutionary War	1667	1668	1	42	1
62	Dutch War of Louis XIV	1672	1678	6	3580	1
63	Turkish-Polish War	1672	1676	4	52	0
64	Russo-Turkish War	1677	1681	4	125	0
65	Ottoman War	1682	1699	17	3954	1
66	Franco-Spanish War	1683	1684	1	51	1
67	War of the League of Augsburg	1688	1697	9	6939	1
68	Second Northern War	1700	1721	21	640	1
69	War of Spanish Succession	1701	1713	12	12490	1
70	Ottoman War	1716	1718	2	98	0
71	War of the Quadruple Alliance	1718	1720	2	245	1
72	Britsh-Spanish War	1726	1729	3	144	1
73	War of the Polish Succession	1733	1738	5	836	1
74	Ottoman War	1736	1739	3	359	0
75	War of the Austrian Succession	1739	1748	9	3379	1
76	Russo-Swedish War	1741	1743	2	94	0
77	Seven Years War	1755	1763	8	9118	1
78	Russo-Turkish War	1768	1774	6	127	0
79	Confederation of Bar	1768	1772	4	149	0
80	War of the Bavarian Succession	1778	1779	1	3	1
81	War of the American Revolution	1778	1784	6	304	1
82	Ottoman War	1787	1792	5	1685	0
83	Russo-Swedish War	1788	1790	2	26	0
84	French Revolutionary Wars	1792	1802	10	5816	1
85	Napoleonic Wars	1803	1815	12	16112	1
86	Russo-Turkish War	1806	1812	6	388	0
87	Russo-Swedish War	1808	1809	1.5	51	0
88	War of 1812	1812	1814	2.5	34	0
89	Neapolitan War	1815	1815	0.2	17	0
90	Franco-Spanish War	1823	1823	0.9	3	0
91	Navarino Bay	1827	1827	0.1	2	0
92	Russo-Turkish War	1828	1829	1.4	415	0
93	Austro-Sardinian War	1848	1849	1	45	0
94	First Schleswig-Holstein War	1849	1849	1.2	20	0
95	Roman Republic War	1849	1849	0.2	4	0
96	Crimean War	1853	1856	2.4	1743	1
97	Anglo-Persian War	1856	1857	0.4	4	0
98	War of Italian Unification	1859	1859	0.2	159	1
99	Franco-Mexican War	1862	1867	4.8	64	0
100	Second Schleswig-Holstein War	1864	1864	0.5	12	0
101	Austro-Prussian War	1866	1866	0.1	270	1

Table A.1 continued

102	Franco Prussian War	1870	1871	0.6	1415	1
103	Russo-Turkish War	1877	1878	0.7	935	0
104	Sino-French War	1884	1885	1	16	0
105	Russo-Japanese War	1904	1905	1.6	339	0
106	Italo-Turkish War	1911	1912	1.1	45	0
107	World War I	1914	1918	4.3	57616	1
108	Russian Civil War	1918	1921	3	37	1
109	Manchurian War	1931	1933	1.4	73	0
110	Italo-Ethiopian War	1935	1936	0.6	29	0
111	Sino-Japanese War	1939	1939	4.4	1813	0
112	Russo-Japanese War	1937	1941	0.4	116	1
113	World War II	1939	1945	6	93665	1
114	Russo-Finnish War	1939	1940	0.3	362	0
115	Korean War	1950	1953	3.1	6821	1
116	Russo-Hungarian War	1956	1956	0.1	50	0
117	Sinai War	1956	1956	0.1	0	0
118	Sino-Indian War	1962	1962	0.1	1	0
119	Vietnam War	1965	1973	8	90	0

Note: Intensity refers to battle deaths per million European population. The Great Powers dummy equals 1 if two or more Great Powers were involved. Source: Levy (1983); see Appendix for more details.

Table A.2: The Wikipedia war data

Start	End	War	Remark	Duration	Western Europe dummy
1337	1360	Edwardian War	Hundred Years' War	24	1
1369	1389	Caroline War	Hundred Years' War	21	1
1415	1429	Lancastrian War	Hundred Years' War	15	1
1455	1485	Wars of the Roses		31	1
1496	1499	Russo-Swedish War, 1496-1499		4	1
1522	1559	Habsburg-Valois Wars		38	1
1554	1557	Russo-Swedish War, 1554-1557		4	1
1558	1583	Livonian War		26	1
1568	1609	Eighty Years' War I	Ceasefire 1609-1621	26	1
1621	1648	Eighty Years' War II	Ceasefire 1609-1621	26	1
1590	1595	Russo-Swedish War, 1590-1595		6	1
1594	1603	Nine Years War (Ireland)		10	1
1610	1617	Ingrian War		8	1
1618	1648	Thirty Years' War		31	1
1641	1649	Wars of Castro		9	1
1642	1651	English Civil War		10	1
1656	1658	Russo-Swedish War, 1656-1658		3	1
1667	1668	War of Devolution		2	1
1667	1683	Great Turkish War		17	0
1688	1691	Williamite war in Ireland		4	1
1700	1721	Great Northern War		22	1
1701	1713	War of the Spanish Succession		13	1
1733	1738	War of the Polish Succession		6	1
1739	1740	War of Jenkin's Ear		2	1
1740	1748	War of Austrian Succession		9	1
1741	1743	Russo-Swedish War, 1741-1743		3	1
1756	1763	Seven Years' War		8	1
1788	1790	Russo-Swedish War, 1788-1790		3	1
1789	1799	French Revolution		11	1
1798	1798	Irish Rebellion of 1798		1	1
1792	1815	Napoleonic Wars		24	1
1808	1809	Finnish War		2	1
1848	1849	First Italian Independence War	Italian Independence Wars	2	1
1859	1859	Austro-Sardinian War/Second Italian Independence War	Italian Independence Wars	1	1
1866	1866	Third Italian Independence War	Italian Independence Wars	1	1
1854	1856	Crimean War		3	1
1866	1866	Austro-Prussian War		1	1
1870	1871	Franco-Prussian War		2	1
1877	1878	Russo-Turkish War, 1877-78		2	0
1893	1896	Cod War of 1893		4	1
1897	1897	First Greco-Turkish War		1	0
1912	1913	Balkan Wars		2	0
1914	1918	World War I		5	1
1916	1916	Easter Rising		1	1
1917	1920	Estonian Liberation War		4	1
1918	1919	Czechoslovakia-Hungary War		2	0
1918	1918	Finnish Civil War		1	1
1918	1920	Russian Civil War		3	0
1919	1921	Irish War of Independence		3	1
1922	1923	Irish Civil War		2	1
1936	1939	Spanish Civil War		4	1
1939	1945	World War II		7	1
1958	1958	First Cod War		1	1
1972	1973	Second Cod War		2	1
1974	1974	Turkish Invasion of Cyprus		1	0
1975	1976	Third Cod War		2	1
1994	1996	First Chechen War		3	0
1991	1991	War in Slovenia		1	0
1991	1995	Croatian War of Independence		5	0
1992	1995	Bosnian War		4	0
1996	1999	Kosovo War		4	0
1999	2006	present Second Chechen War		8	0
2001	2001	Conflict in Macedonia		1	0
2001	2001	Conflict in Southern Serbia		1	0

See Appendix for source and other details

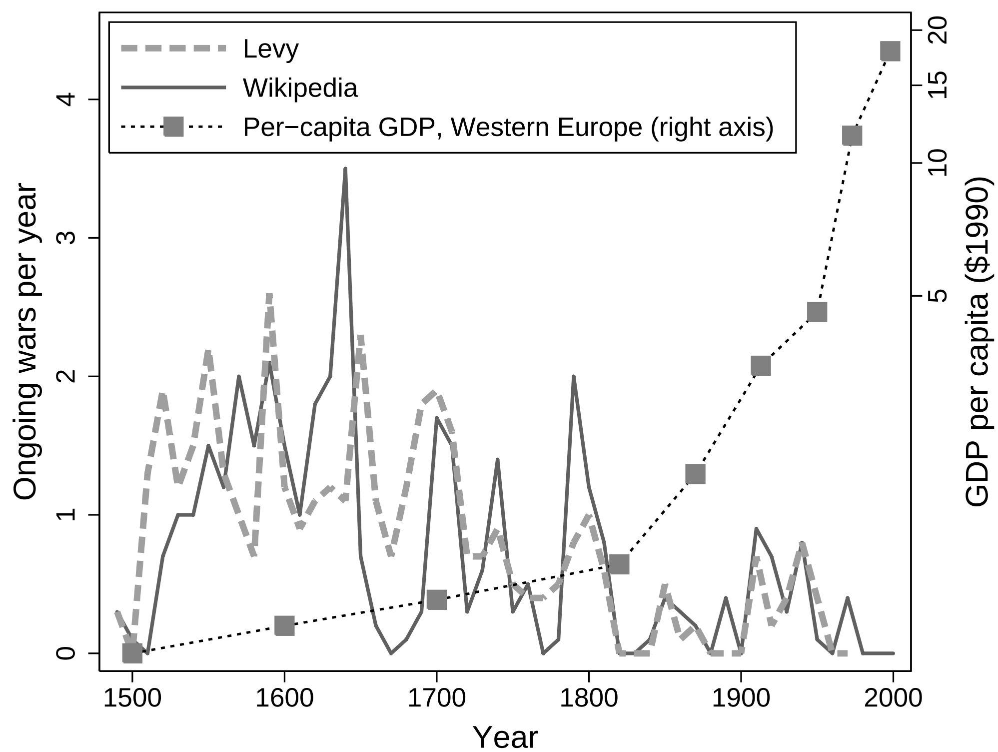


Figure 1: The figure shows a decline in the frequency of war, and a rise in per-capita GDP in Western Europe. Sources: GDP per capita is from Maddison (2003, Tables 8a,b); the war indexes are computed from two data sets, Levy (1983) and a list of European conflicts published by Wikipedia, as explained in detail in Section A.1 of the Appendix.

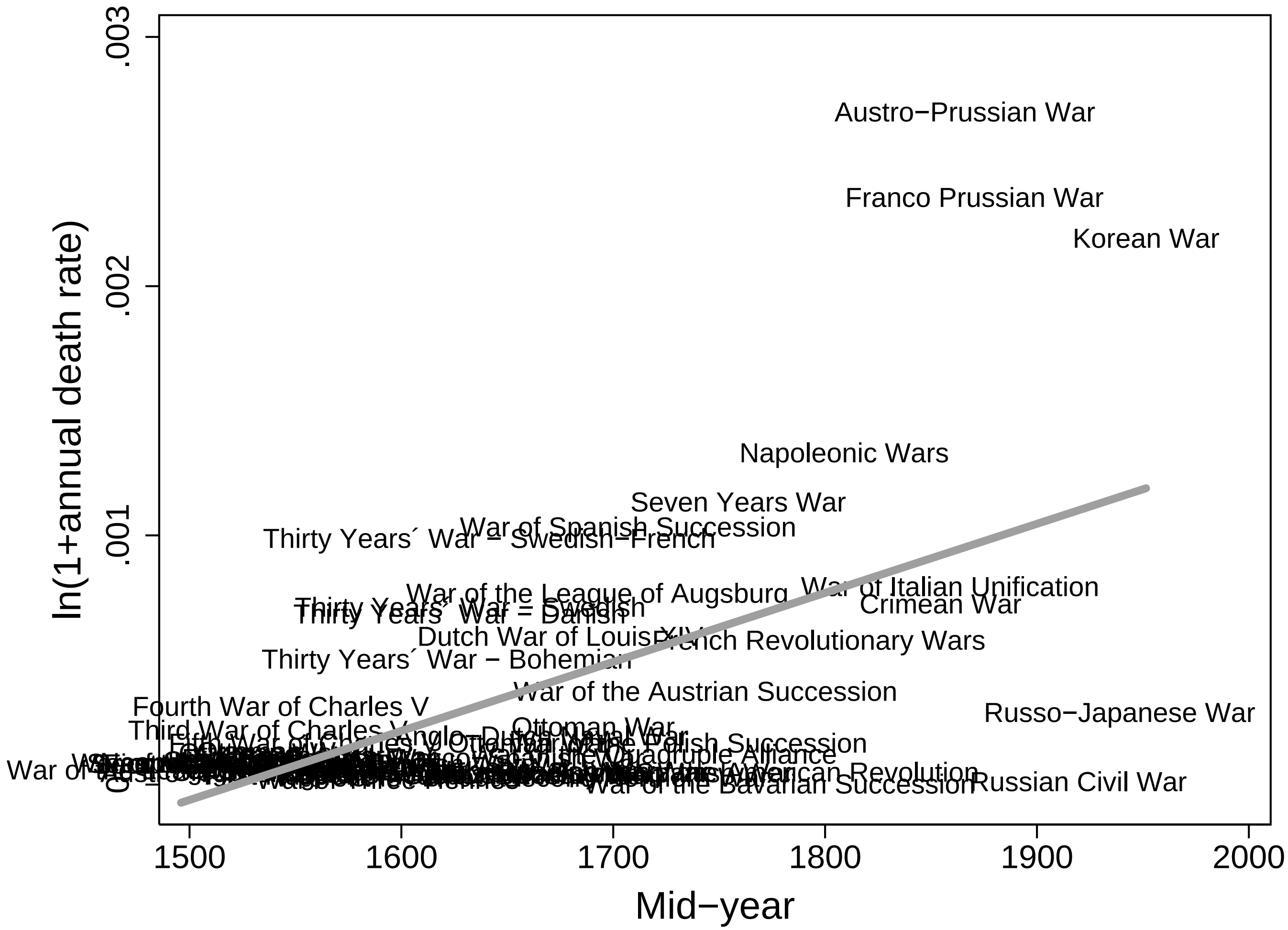
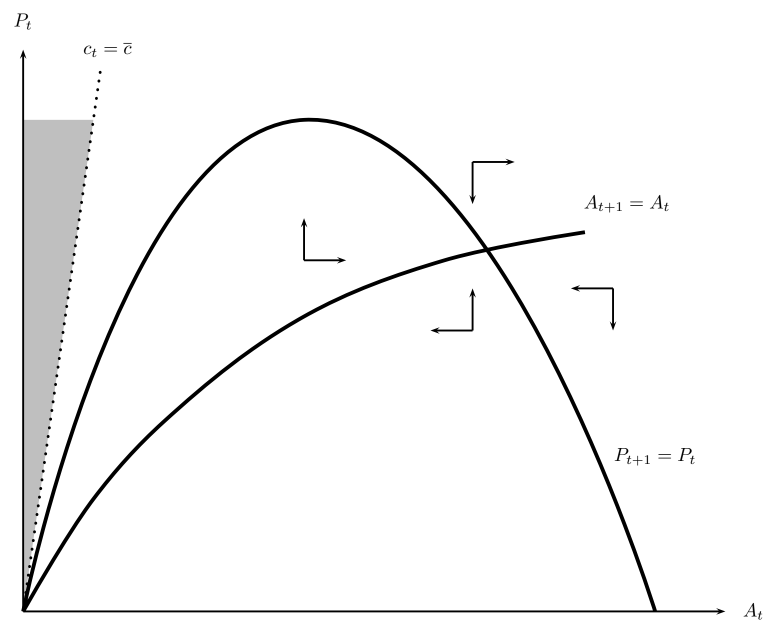


Figure 2: The figure plots annual death rates of wars involving at least two Great Powers, against the mid-year of the war. WWI and WWII are excluded. These are outliers with very high death rates, and including them would strengthen the upward trend in war death rates. Source: Levy (1983); see Table A.1 and Section A.1 in the Appendix for details.



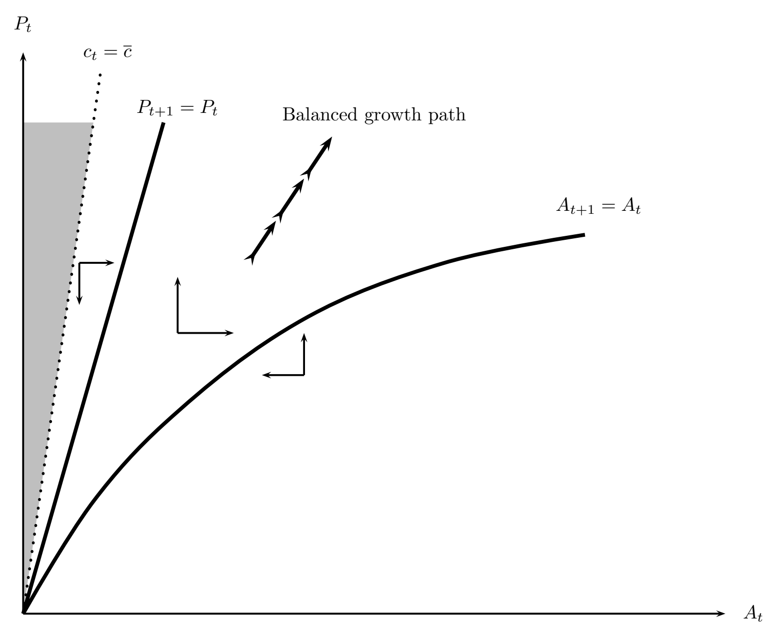


Figure 3: Phase diagrams illustrating the dynamics of Model A. The upper panel refers to an always-war economy, the lower to an always-peace economy. The shaded regions indicate where population would instantly die out.

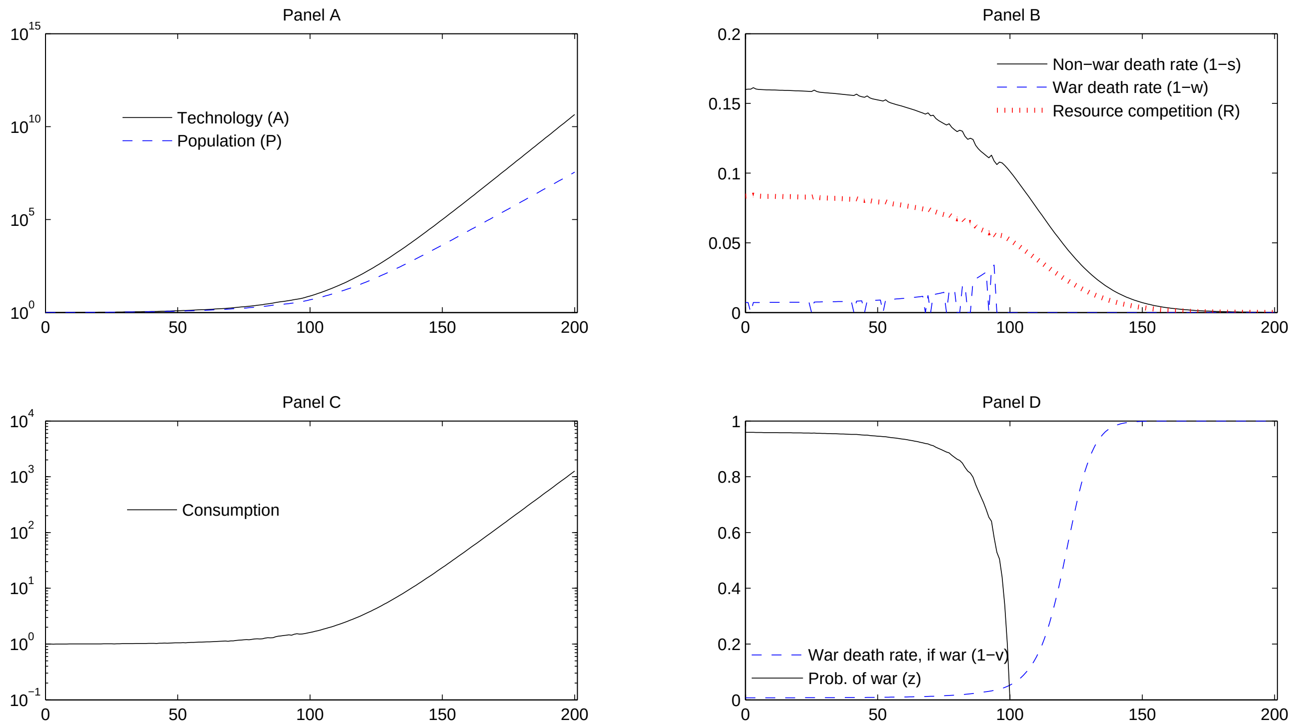


Figure 4: Simulation of a single economy (Model A). Panel D shows how the probability of war (z_t) declines, simultaneously with a rise in the deadliness of war when war if fought $(1 - v_t = A_t/(\lambda + A_t))$. This generates the pattern in Panel B where the actual war death rate $(1 - w_t)$ temporarily falls to zero in times of peace, which happens with increasing frequency; this is interrupted by higher peaks in times of war; eventually peace becomes permanent.

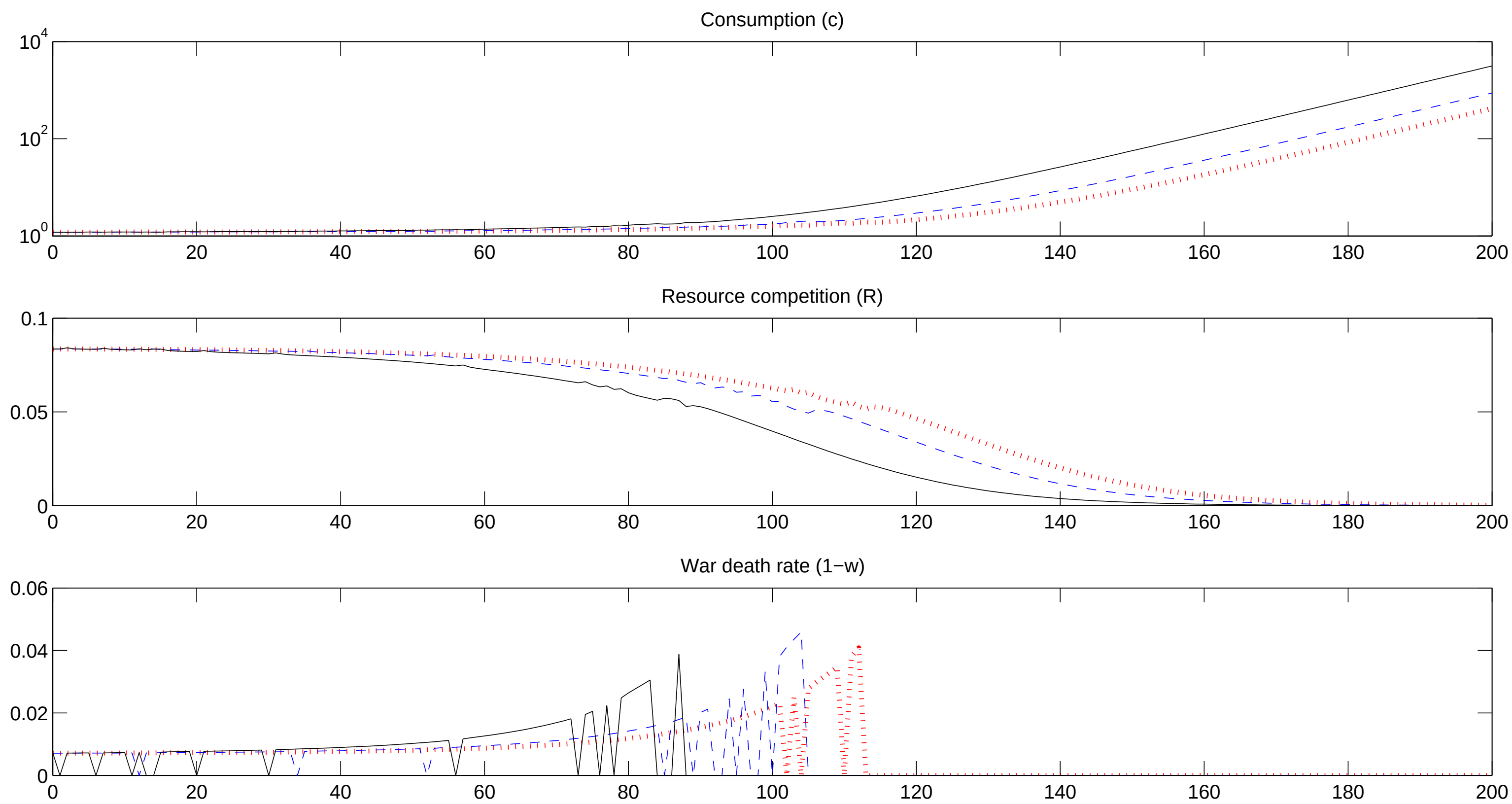


Figure 5: Simulation of three different economies (Model A). Note the timing: a later takeoff from stagnation to growth is associated with a later rise and fall in war death rates.

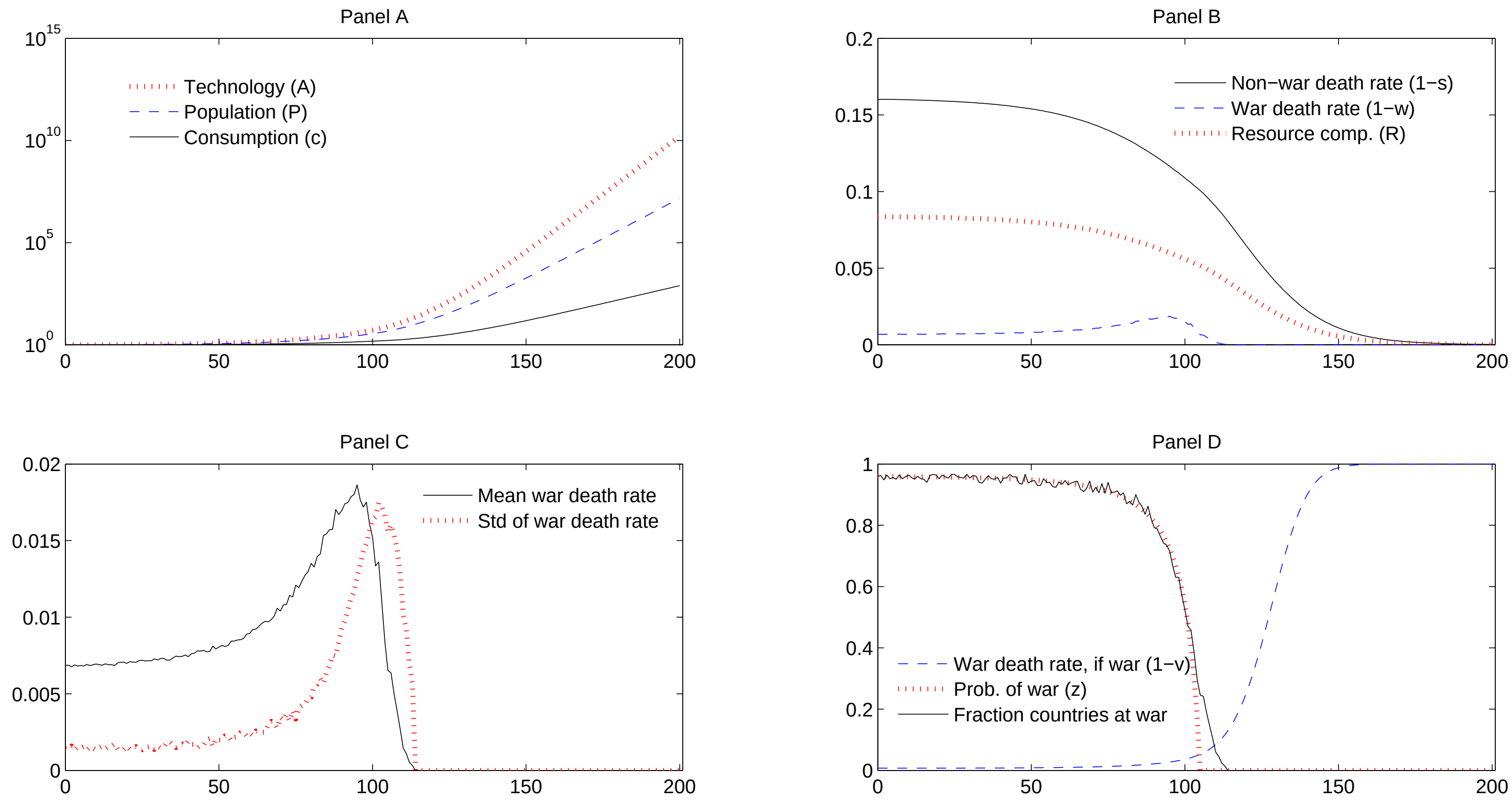


Figure 6: Monte Carlo simulation of Model A. The paths show averages across 500 runs. [The exception is the probability of war (z_t in Panel D), which is calculated for a hypothetical economy where A_t and P_t equal that of the average economy.] Note the inverse U-shape of the the standard deviation of the war death rate in Panel C, as each country's peak in war death rates is differently timed (cf Figure 5).

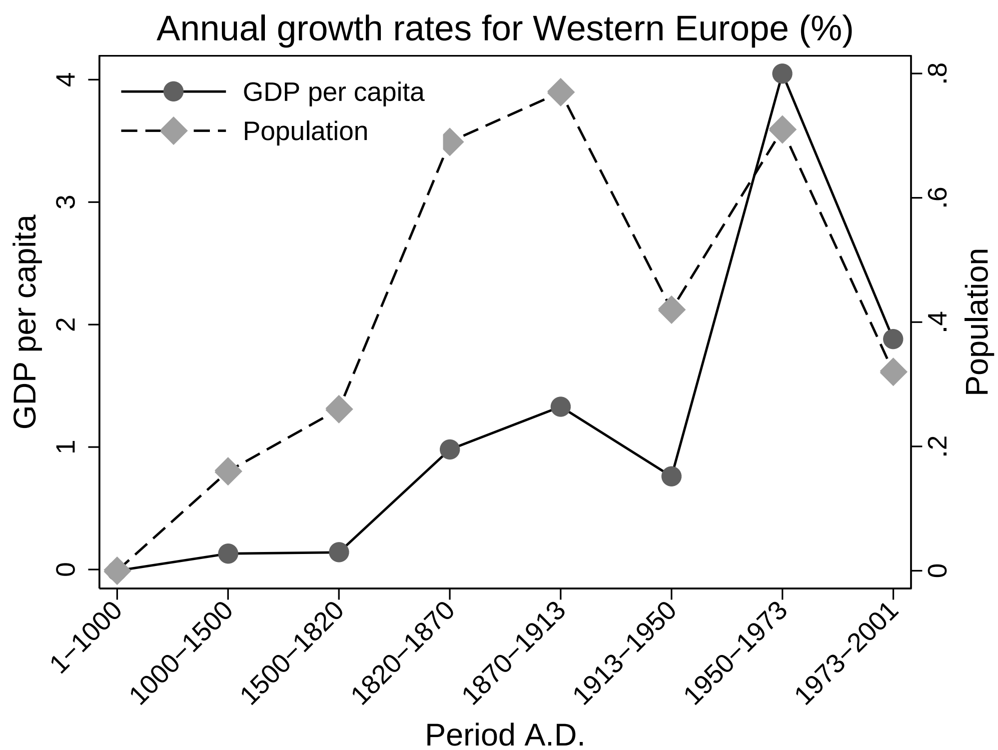


Figure 7: The figure shows the growth rates of population and per-capita income in Western Europe. The population growth rate is inversely U-shaped with a "dent" in the period 1913-1950, due to the two world wars. Source: Maddison (2003, Tables 8a,b).

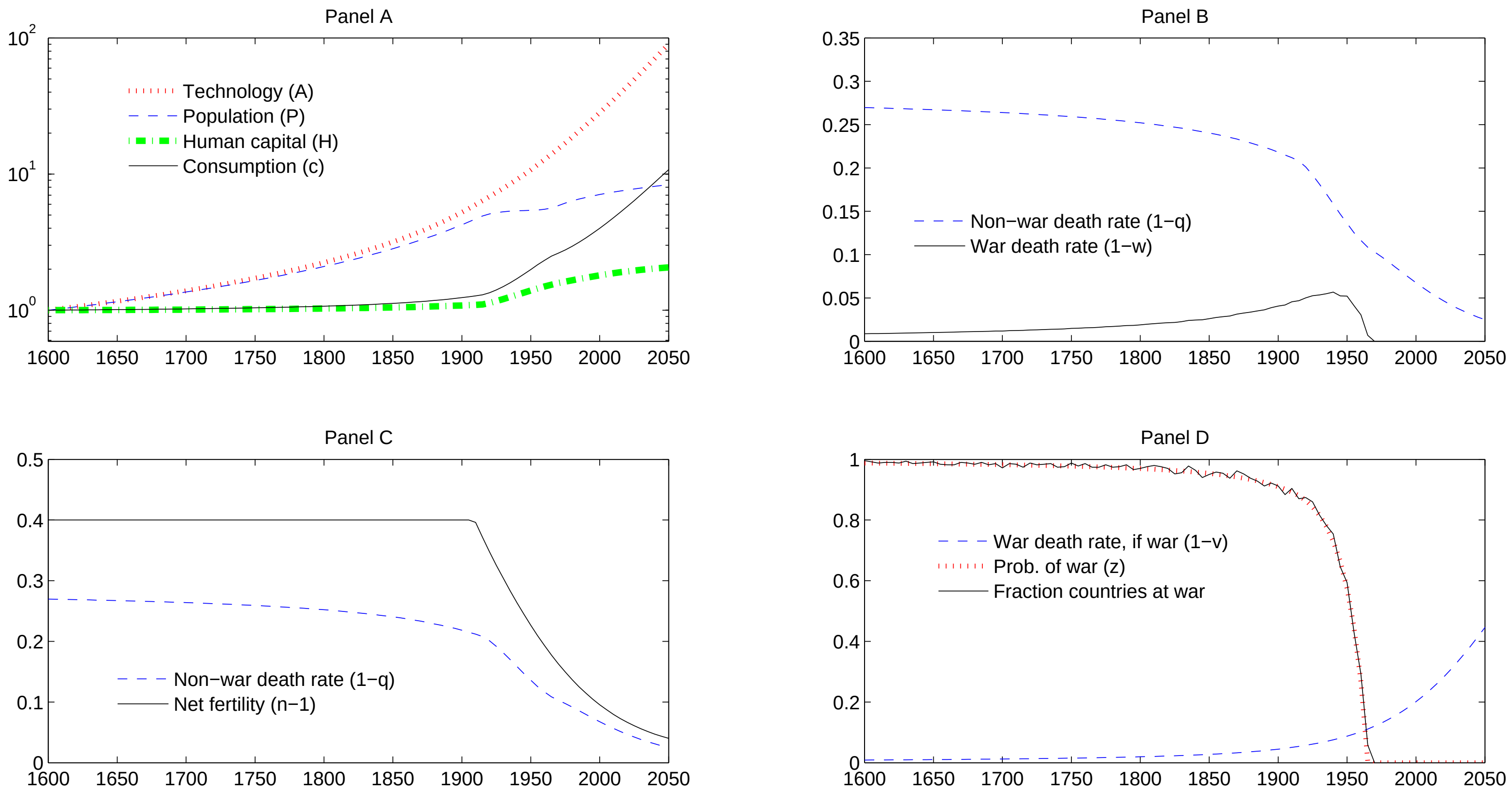


Figure 8: Monte Carlo simulation of Model B. The paths show averages across 500 runs. [Like in Figure 6, the exception is the probability of war (z_t in Panel D), which is calculated for a hypothetical economy where A_t and P_t equal that of the average economy.] From around 1900 fertility starts to decline (Panel C). Simultaneously, human capital starts to rise, making technology and per-capita consumption grow faster (Panel A). Improved technology raises death rates in war, which together with rising consumption levels and declining resource competition eventually leads to permanent peace, thus generating a decline in war death rates (Panel B). The years are chosen so that war death rates peak in 1945.

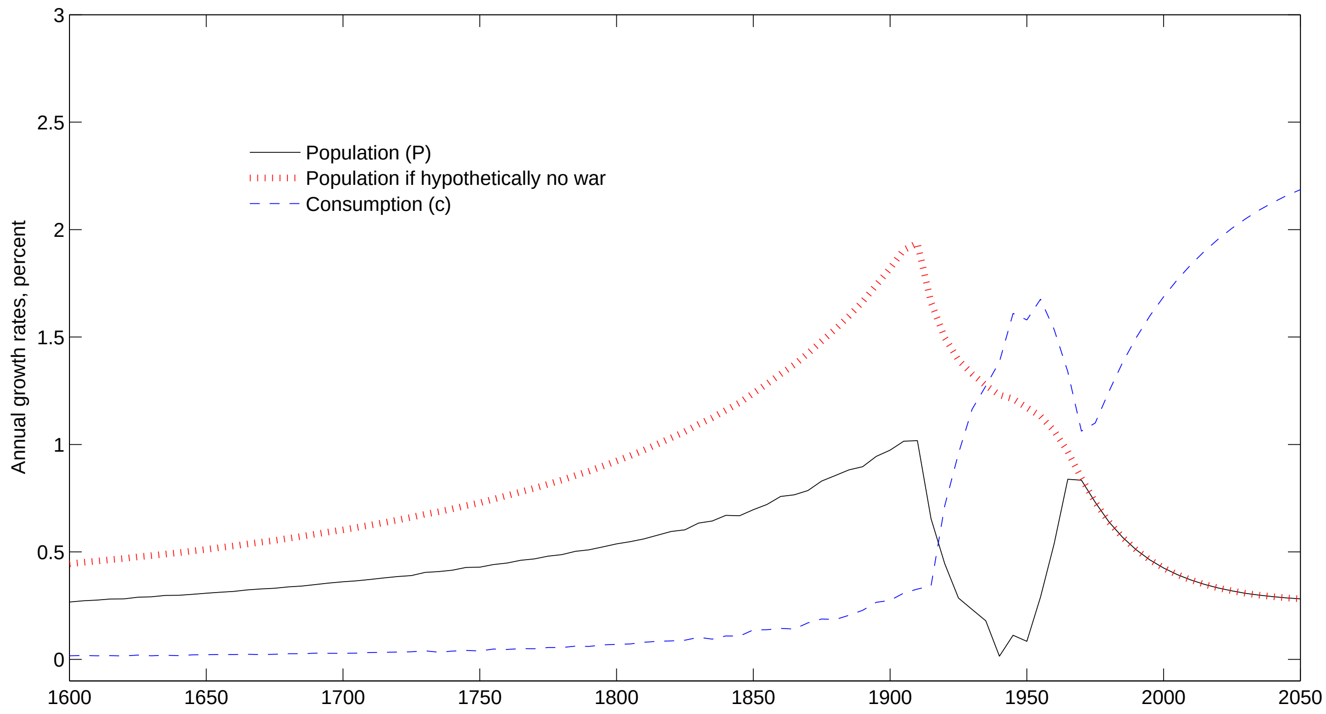


Figure 9: Results from the same Monte Carlo simulation of Model B as that in Figure 8, but here showing the annualized growth rates of population and per-capita consumption, averaged over 500 runs. The figure also shows a hypothetical growth path for population, in which war deaths are eliminated. Note that the actual population growth rate is inversely U-shaped, with a "dent" similar to that experienced by Western Europe in 1913-1950 (see Figure 7).