From Malthusian War to Solovian Peace

Nils-Petter Lagerlöf*
Department of Economics, York University,
4700 Keele St., Toronto ON Canada M3J 1P3
e-mail: lagerlof@econ.yorku.ca

October 27, 2009

*I am grateful for comments from two referees and an Associate Editor. This paper shares some components with (and replaces) an earlier paper, carrying a similar title, that I wrote while visiting at the PSTC at Brown University: I thank everyone there for their hospitality. Mauricio Drelichman, Omar Licandro, and participants at a workshop at EUI in Florence provided useful input on the current version. I am also grateful for financial support from the Social Sciences and Humanities Research Council of Canada. All errors are mine.

Abstract: We present a two-country version of Hansen and Prescott's two-sector long-run growth model, introducing war by letting the countries take land from each other, at the cost of destroying capital and killing people. Because land is an input only in the Malthus sector the transition to a Solow economy brings a decline in warfare, broadly consistent with an observed 19th-century decrease in Great Power wars. We also find, inter alia, that if governments are Malthus-biased (care less about Solow output), the transition can lead temporarily to more war.

JEL Classification codes: D90, J11, N0, N43, N44.

Keywords: Growth, War, Fertility, Land.

1 Introduction

War seems to be linked to economic development. For example, poor countries today tend to be more war prone than rich. A look at long-run time trends reveals that Europe, before escaping poverty, was plagued by frequent inter-state wars. Industrialization came together with a decline in warfare, in particular between Great Power nations. The 19th century was more peaceful than the preceding ones; this was followed by a smaller spike around the 20th century, associated with e.g. the two world wars, after which peace has prevailed.

The task undertaken here is to examine if a standard long-run growth model, extended to allow for war, can replicate such trends. The idea is that a decline in warfare can result from the rise of a non-agrarian sector, in which land – which is here the resource that is contested in war – is not an input. Our framework is a two-country version of the two-sector overlapping generations model in Hansen and Prescott (2002) (HP hereafter), which we analyze quantitatively. A Malthus sector uses capital, labor, and land, and a Solow sector only capital and labor. War amounts to taking land at the cost of destroying capital and killing people.[1] Governments go to war aiming to maximize tax revenues, which are used only for consumption. Productivity grows at an exogenous rate that is faster in the Solow than the Malthus sector, generating a transition to a Solow economy at some point.

This transition leads to the elimination of war over roughly two model periods (about 70 years), broadly consistent with the observed decline in Great Power warfare around the 19th century, when most Great Powers industrialized.

In our baseline setting, we get a war frequency rate around 40% in the Malthusian phase of development, which we argue is not unreasonable. Some sensitivity analysis shows that war is less frequent in the Malthusian phase if the costs of war (the rates of killing and capital destruction in war) are high, or when the amount of land that can be conquered is small. This is quite intuitive.

We also find that there is more war if governments are more risk loving. This is also intuitive because war is risky, although the quantitative effects

[1] Many wars have been about e.g. religion, royal succession, or political influence, rather than land explicitly, but it is also possible that the *ultimate* contested resource of many wars has been territory, although the *proximate* subject of the struggle has been something else.

are relatively minor.

One interesting finding is that if governments care less about Solow output than Malthus output – which may capture disproportional political influence of a landowning class – then war probabilities may actually *increase* initially before declining. A Malthus-biased government favors war partly *because* it destroys capital, thus leading to a reallocation of resources away from the Solow sector.

The model also has the property that one country rarely comes to completely dominate the other, which may fit with a relative stability of the set of countries that constitute the Great Power system. During the Malthusian phase of development, all else equal, war is more attractive to a country where land is scarce and labor abundant. If country 1 takes land from country 2 today, country 2 wants to take it back in the next period. Over several periods, conquests thus tend to be reversed, generating convergence in landholdings. However, there is also a countervailing force: land conquests raise wages, and thus (in a Malthusian environment) population growth and capital accumulation. Since people and capital are inputs in the war conquest function, initial conquests tend to be followed by more, and landholdings tend to diverge. Still, the converging force tends to be strong enough so that war is eliminated not due to one country completely dominating the other, but rather due to the Malthus-to-Solow transition.

The rest of this paper is organized as follows. This introduction continues with an overview of earlier related literature (Section 1.1), and then some descriptions of the Great Power war trends that we want to capture (Section 1.2). Section 2 sets up the model, and describes the events taking place in each period and their timing. We specify the dynamics of all state variables in Section 3. In Section 4 we pursue the quantitative analysis for a baseline case, and Section 5 then presents some sensitivity analysis. Section 6 ends with a concluding discussion.

1.1 Overview of earlier literature

A large microeconomic literature on conflict examines how rational agents weigh appropriation (stealing) against production. (See e.g. Grossman 1991; Grossman and Kim 1995; Hirshleifer 1988, 2001.) However, these usually apply static and partial-equilibrium models.

Grossman and Mendoza (2003) present a model where competition for resources is induced by a desire for survival. They show that if the elasticity of the survival function is decreasing in consumption more scarcity leads to more violence.[2] During the Malthusian phase of development in our model a similar result arises in a more standard macro framework, since governments are more willing to trade living people for land when land is scarce and people are plentiful.

There is also work on how social conflicts within societies can hinder development, both theoretical (e.g. Benhabib and Rustichini 1996; Bridgman 2008) and empirical (e.g. Collier and Hoeffler 1998, 2004; Easterly and Levine 1997). Recent work on changes over time in warfare include Martin et al. (2008), who examine the relationship between trade, war, and the geographical distances between belligerent countries; and Iyigun (2008), who documents the role played by the Ottoman wars in reducing fighting between Christian nations.

However, these papers do not explain the particular century-long time trends in Great Power war that we focus on here, or how these relate to Malthus-to-Solow transitions. In that sense, this paper is closer to a literature on long-run growth. Other than HP, mentioned above, this literature includes Cervellati and Sunde (2005), Galor and Moav (2002), Galor and Weil (2000), Jones (2001), Lagerlöf (2003a,b; 2006), Lucas (2002), and Tamura (1996, 2002).[3] These papers do not discuss war.

Several other papers have a more indirect connection. Alesina and Spolaore (2003, Ch. 7) model defense spending and the optimal size of nations. Johnson et al. (2006) explain why death tolls in many insurgencies (e.g., Iraq and Colombia) tend to follow a power-law distribution. Easterly, Gatti, and Kurlat (2006) examine empirically the link between mass killings (including genocides), per-capita income, and democracy. Gordon (2008) discusses the link between Nazi Germany's expansion in the east and land scarcity. Yared (2009) examines the role of war in extracting concessions in a dynamic two-player war game with asymmetric information. These studies are not directly relevant for understanding century-long war trends, or takeoffs from stagnation to growth.

[2] Other papers where "harder times" lead to more conflict include Dal Bó and Powell (2006), who allow for asymmetric information about the size of the contested pie.

[3] Papers that apply and extend the HP framework specifically include Doepke (2004), Ngai (2004) and Santaeulalia-Llopis (2008).

1.2 Some empirical patterns

Figure 1(a) shows the number of ongoing Great Power wars per year, averaged over decades. This is based on a list compiled by Levy (1983) over 119 wars fought from 1495 to 1973 by Great Power nations. The trend is clearly towards less war.

We focus here on Great Power wars, which are defined as wars involving two or more Great Powers on different sides of the conflict. The decline in Great Power warfare has been more emphasized than wars in general.[4]

A Great Power is a strong military power, in a broad sense. Levy's (1983, p. 16) definition contains the component that Great Powers are "basically invulnerable to military threats by non-Powers and need only fear other Great Powers."

Which nations or empires are defined as Great Powers has changed over time, but also shows a great deal of continuity. Other than Russia/USSR and the Ottoman Empire, all Great Powers were Western European up until the mid-19th century. Then the US and Japan (and later China) became Great Powers.[5] In that sense, the decline in Great Power warfare was probably not due to one single power coming to dominate.

Although the trends are volatile and non-monotonic, a closer look at Figure 1(a) may reveal something about the causes of the decline. Warfare first bottomed out in the 19th century, which saw long periods with no Great Power wars at all. Then followed the 20th century with a resurgence of Great Power conflicts, associated with the two world wars, and subsequently a complete elimination of warfare. By the 20-century agriculture was already in relative decline in most Great Power nations, so we may not expect a model where war is about land to explain 20th-century warfare.[6] However,

---

[4] Not all wars fought by Great Powers are Great Power wars. Even though Great Powers continue to exist to this day, the last Great Power war was the Korean war 1950-1953. Recent wars, such as the wars in Iraq and Afghanistan, are not Great Power wars, since they have no two opposing contestants that are both Great Powers.

[5] Levy (1983, Table 2.1) defines the following nations/empires as Great Powers in the time periods indicated in parentheses: France (1495-1975); England/Great Britain (1495-1975); Austrian Habsburgs/Austria (1495-1519, 1556-1918); Spain (1495-1519, 1556-1808); Ottoman Empire (1495-1699); United Habsburgs (1519-1556); The Netherlands (1609-1713); Sweden (1609-1713); Russia/Soviet Union (1721-1975); Prussia/Germany/FRG (1740-1975); Italy (1861-1975); United States (1898-1975); Japan (1905-1945); China (1949-1975).

[6] However, it is worth noting that many believe that the German expansion in the east during World War II was driven by land scarcity (see, in particular, Gordon 2008).

our model is able to explain the first decline. (Section 6 discusses how to possibly model 20th-century wars.)

We want to ensure that the trends in Figure 1(a) are not driven by changes in the composition of the Great Powers, or their numbers (which, however, do not show any particular trend). One way to do this is to look separately at England and France, which are the only two nations that are classified by Levy (1983) as Great Powers for the entire period 1495-1973. Figure 1(b) shows the number of years in any given 35-year period (corresponding to a period in our model later) in which the country was involved in a Great Power war. The changes are even less monotone than those in Figure 1(a) but the overall pattern is similar: a broad long-run trend towards less war, with a marked decline at the onset of the 19th century, which was relatively peaceful; thereafter a temporary resurgence in war in the 20th century, and eventually permanent peace.[7]

In Figure 1(a) we also see that the decline in Great Power warfare came together with a rise in per-capita incomes in Western Europe (data is from Maddison 2003). This may suggest a connection between warfare and economic development among Great Power nations. Our model is aimed at exploring such a link.

2 The model

The setting we use is a modified version of HP. Agents live for two periods in overlapping generations. There are two countries and in each country there are two sectors, producing the same good: a Malthus sector uses land, capital, and labor; a Solow sector uses only capital and labor.

2.1 The timing of war and production

When modelling war we need to decide how production factors are allocated between military and civilian production. In many real-world examples, capital and labor have shifted back and forth between these uses, such as when

[7]The rise toward the end of the period is due to the last Great Power wars involving Great Britain and France: in particular World War II (1939-1945) and the Korean War (1950-1953). Since they have not been involved in any war against another Great Power since then, the pattern in Figure 1(b) would indicate zero war years for the 35-year intervals starting 1954 and onwards, if we extended the data beyond 1975.

civilians have been conscripted for the war effort, as happened with the *levée en masse* in post-revolutionary France. A corresponding phenomenon for physical capital could be when civilian merchant ships were requisitioned, as happened in the Spanish-English wars (Hansen 2003, pp. 34-35).[8]

For simplicity, we here let war and production take place in different subperiods. All available capital and labor (or constant fractions thereof) are used in the war phase (if there is a war), before production takes place. More precisely, in each period $t \geq 0$ the timing of events (in both countries) is as follows:

1. The two countries start off with given endowments of capital, young population (i.e. labor force), and shares of a unit-sized amount of land.

2. With probability 1/2 one of the countries (but never both, or none) is given the opportunity to start a war. The cost of war is destroyed capital and young agents (labor) being killed (in both countries); the gain (for the attacker) is a random conquest of land. In each country, the government's decision whether or not to start a war (if given the opportunity) is made with the aim to maximize the expected utility of tax revenues.

3. After the war (if any), the post-war capital, labor, and land endowments of the two countries are used in production. The government receives its taxes. Those young agents who survived the war work, save and rear children, thus updating population and capital to the next period.

Up until Section 2.5 the analysis concerns the production phase (stage 3 above).

2.2 Firms

Let K_t, N_t, and L_t be country 1's total amounts of capital, labor, and land, after a potential war (i.e., at stage 3 above). Up until Section 2.5 the analysis refers to one country (country 1), and we do not yet impose any notation to

[8] When a whole economy is engaged in war it is sometimes called "total war," a term associated with von Clausewitz (see e.g. Browning 2002).

distinguish between the two countries. Thus, let Y_{Mt} (Y_{St}) be period-t output in the Malthus (Solow) sector, given by the following production functions:

$$Y_{Mt} = A_{Mt} K_{Mt}^{\phi} N_{Mt}^{\mu} L_t^{1-\mu-\phi}, \tag{1}$$

and

$$Y_{St} = A_{St} K_{St}^{1-\mu} N_{St}^{\mu}, \tag{2}$$

where A_{it}, K_{it}, and N_{it} denote total factor productivity, capital, and labor, respectively, in each sector i ($i = M, S$). L_t denotes the total amount of land available (in country 1) in period t. The total factor productivities are the same for both countries and evolve according to

$$\begin{aligned} A_{Mt+1} &= \gamma_M A_{Mt}, \\ A_{St+1} &= \gamma_S A_{St}, \end{aligned} \tag{3}$$

where $\gamma_S > \gamma_M > 1$.

The labor share, μ, is assumed to be the same in both sectors. This enables us to derive closed-form solutions for the equilibrium shares of capital and labor allocated to each sector. (HP impose a common labor share when they calibrate their model.) We also assume that $\mu + \phi < 1$ (to ensure a positive marginal product of land in the Malthus sector).

There is a single competitive firm in each sector, whose output is taxed by a government at some exogenous rate $\tau \in (0, 1)$. The tax is sector neutral, and so does not affect factor allocations. As explain later, the role of this government is to decide whether or not to go to war.

Let w_t be the wage rate, and $r_{K,t}$ and $r_{L,t}$ the rental rates of capital and land, respectively. Profits in the Malthus sector then equal:

$$\pi_{Mt} = (1-\tau) A_{Mt} K_{Mt}^{\phi} N_{Mt}^{\mu} L_t^{1-\mu-\phi} - w_t N_{Mt} - r_{K,t} K_{Mt} - r_{L,t} L_t, \tag{4}$$

and in the Solow sector:

$$\pi_{St} = (1-\tau) A_{St} K_{St}^{1-\mu} N_{St}^{\mu} - w_t N_{St} - r_{K,t} K_{St}. \tag{5}$$

In a competitive equilibrium where both sectors are operative (conditions for which are derived soon), the wage and rental rates can be written:

$$w_t = \mu(1-\tau) A_{Mt} K_{Mt}^{\phi} N_{Mt}^{\mu-1} L_t^{1-\mu-\phi} = \mu(1-\tau) A_{St} K_{St}^{1-\mu} N_{St}^{\mu-1}, \tag{6}$$

$$r_{K,t} = \phi(1-\tau) A_{Mt} K_{Mt}^{\phi-1} N_{Mt}^{\mu} L_t^{1-\mu-\phi} = (1-\mu)(1-\tau) A_{St} K_{St}^{-\mu} N_{St}^{\mu}, \tag{7}$$

and

$$r_{L,t} = (1-\mu-\phi)(1-\tau)A_{Mt}K_{Mt}^{\phi}N_{Mt}^{\mu}L_t^{-(\mu+\phi)}. \tag{8}$$

Since K_t and N_t are the economy's total amounts of capital and labor it follows that $K_{Mt} + K_{St} = K_t$ and $N_{Mt} + N_{St} = N_t$. Let $z_{Kt} = K_{St}/K_t$ be the fraction of the capital stock allocated to the Solow sector in equilibrium, and $z_{Nt} = N_{St}/N_t$ the fraction labor in the Solow sector. Section A in the Appendix shows that we can then use (6) and (7) to derive the following closed-form solutions for the equilibrium z_{Kt} and z_{Nt}:

$$z_{Kt} = 1 - V_t, \tag{9}$$

and

$$z_{Nt} = \frac{z_{Kt}}{z_{Kt} + \left(\frac{1-\mu}{\phi}\right)V_t} = \frac{\phi(1-V_t)}{\phi + (1-\mu-\phi)\,V_t}, \tag{10}$$

where the second equality uses (9), and where

$$V_t = \min\left\{1, \left[\left(\frac{\phi}{1-\mu}\right)^{1-\mu}\frac{A_{Mt}}{A_{St}}\right]^{\frac{1}{1-\mu-\phi}}\frac{L_t}{K_t}\right\}. \tag{11}$$

This defines the factor shares as functions of (a subset of all) the state variables (of country 1), A_{Mt}, A_{St}, K_t, and L_t.

Note that when $V_t = 1$, no capital or labor is allocated to the Solow sector, so $z_{Kt} = z_{Nt} = 0$.

The shares of capital and labor allocated to the Solow sector are increasing in A_{St} and K_t, and decreasing in A_{Mt} and L_t. Therefore, an increase in land and/or a destruction of capital, which both can be caused by war, lead to a reduction in resources being allocated to the Solow sector (if the Solow sector is operative).

2.3 Households

Agents live for a maximum of two periods. When old they live off savings, net of what is destroyed in war, plus interest. When young they first fight in a potential war, and thereafter (if they survive) earn a labor income and rents from land.

We deviate from the HP model by letting land be owned (or controlled) by the young, rather than the old. We also abstract from markets for buying and selling land and assume that agents pass it on through a sort of forced inheritance to their children. Similarly, land conquered in war is distributed among the (surviving) young. This serves to simplify the dynamic analysis later. It can also be motivated from the observation that many preindustrial societies have had poor or imperfect property rights to land (see e.g. Easterly 2006, p. 90-96).

Let s_t denote the saving of an agent who is young (and alive) in period t, and w_t his labor income. Consumption when young, $c_{1,t}$, is then given by

$$c_{1,t} = w_t + r_{L,t} l_t - s_t, \tag{12}$$

where l_t denotes land per young agent in the post-war phase of period t, i.e. $l_t = L_t/N_t$, and (recall) $r_{L,t}$ is the rental price of land. The same agent's consumption when old, $c_{2,t+1}$, is given by:[9]

$$c_{2,t+1} = \begin{cases} (1 + r_{K,t+1})(1 - \delta_{Kt})s_t & \text{if war,} \\ (1 + r_{K,t+1})s_t & \text{if no war,} \end{cases} \tag{13}$$

where δ_{Kt} is the fraction of the capital that is destroyed in the war phase in period t (explained later). The assumption that underlies this formulation is that the government can temporarily confiscate the citizens' (labor and) capital during the war phase (of which some is destroyed), but not keep it during the production phase. The government thus hands back the (old) agents' capital net of war destruction.

Agents maximize

$$(1 - \beta) \ln(c_{1,t}) + \beta \ln(c_{2,t+1}), \tag{14}$$

subject to the budget constraints in (12) and (13). Due to the log utility, regardless of whether or not there will be war in the next period, agents save a constant fraction of their first-period income:

$$s_t = \beta \left[w_t + r_{L,t} l_t \right]. \tag{15}$$

[9]Different from HP, the formulation in (13) interprets $r_{K,t+1}$ as the net return on savings, implicitly assuming zero capital depreciation (other than the destruction taking place in war). However, with log utility and no income in the second period of life, this plays no role for saving or capital accumulation. The capital stock (net of war destruction) is simply eaten by old people, rather than depreciated in production.

2.4 Fertility

Fertility, n_t, evolves according to an exogenously given function similar to HP. The major difference is that we let fertility depend on the wage rate, w_t, rather than consumption:[10]

$$n_t = n(w_t) = \begin{cases} b + \frac{(n^*-b)w_t}{w^*} & \text{if } w_t < \eta w^*, \\ d - \left(\frac{\overline{n}-1}{\nu-\eta}\right)\frac{w_t}{w^*} & \text{if } w_t \in [\eta w^*, \nu w^*], \\ 1 & \text{if } w_t > \nu w^*, \end{cases} \tag{16}$$

where

$$d = \frac{\nu\overline{n} - \eta}{\nu - \eta}, \tag{17}$$

and

$$b = \frac{\eta n^* - \overline{n}}{\eta - 1}. \tag{18}$$

The parameters defining fertility behavior (w^*, $\overline{n}$, n^*, ν and η) are such that $w^* > 0$, $\overline{n} > n^*$, and $\nu > \eta > 1$. It is easily seen that $n'(w_t) > 0$ for $w_t < \eta w^*$ and that w^* and n^* are the wage and fertility rates on a Malthusian balanced growth path (i.e., when the Solow sector is not active); (16) shows that $n(w^*) = n^*$. When the wage rate reaches ηw^* fertility peaks (at $\overline{n}$). Fertility then declines until the wage rate equals νw^* after which fertility equals one, and population remains constant (absent war deaths). [Lagerlöf 2009 graphs (16) for the parameter values used in the calibration later, together with data from England.]

As shown in Section C of the Appendix, by using the Malthusian production function in (1), the technological progress function in (3), and that population (in peace) evolves according to $N_{t+1} = N_t n_t$, we can derive an expression for the Malthusian fertility rate:

$$n^* = \gamma_M^{\frac{1}{1-\phi-\mu}}. \tag{19}$$

[10] In HP's setting, where young earn only labor income, consumption is just a constant times the wage. Here consumption depends also on land income (since young own land). Letting fertility depend on the wage rate thus keeps the model closer to HP. It can also be motivated by the idea that fertility should depend on the time cost of children, which is proportional to labor income.

2.5 War

Recall that L_t, K_t, and N_t denote post-war levels of land, capital, and young population; now let L_t^p, K_t^p, and N_t^p denote the corresponding pre-war levels. Recall also that the total size of the contested territory equals one, so that (preceding a potential war in period t) L_t^p belongs to country 1 and $1 - L_t^p$ to country 2.

To model war, we let each of the two countries with probability 1/2 be given the opportunity of conquering a fraction of the other country's territory. This opportunity shock could represent random shifts in the relative strength or competence in the two countries' political leadership or the degree of internal fractionalization. This is a stylized but convenient way to generate a mechanism through which only one country at the time can take territory from the other, despite them being symmetric (at least initially).

If country 1 is given the opportunity to start a war, and acts on it, then L_t^p, K_t^p, and N_t^p are updated as follows between the war phase and the production phase:

$$\begin{aligned} L_t &= L_t^p + g_t(1 - L_t^p), \\ K_t &= (1 - \delta_{Kt})K_t^p, \\ N_t &= (1 - \delta_{Nt})N_t^p, \end{aligned} \tag{20}$$

where δ_{Kt} and δ_{Nt} are the (possibly endogenous) fractions of the capital stock destroyed, and young population killed, as a result of war; g_t is the (endogenous) fraction of country 2's territory that country 1 conquers in war. Next we shall describe how we model these variables.

2.5.1 Gains from war

We let the fraction of the opponent's territory that is conquered, g_t, be given by

$$g_t = \min\{0.99, x_t^k x_t^u H(K_t^p, \widetilde{K}_t^p, N_t^p, \widetilde{N}_t^p)\}, \tag{21}$$

where x_t^k and x_t^u are random variables, both distributed with identical probabilities 1/11 on $\{0, 0.1, ..., 1\}$, with mean 1/2. The government knows x_t^k before going to war, whereas x_t^u is unknown. We cap the potential conquest at 99% of the loser's land, to ensure that no country becomes extinct.

We let $H(\cdot)$ take a type of Tullock/Cobb-Douglas form:

$$H(K_t^p, \widetilde{K}_t^p, N_t^p, \widetilde{N}_t^p) = 2\left(\frac{K_t^p}{K_t^p + \widetilde{K}_t^p}\right)^{1-\mu}\left(\frac{N_t^p}{N_t^p + \widetilde{N}_t^p}\right)^{\mu}, \tag{22}$$

where μ is the same parameter as in the Solow-sector production function. From now on, we let tildes denote variables referring to country 2, so that if X_t refers to country 1, then $\widetilde{X}_t$ is the same variable for country 2.

We have normalized the conquest function so that if the countries are symmetric ($K_t^p = \widetilde{K}_t^p$ and $N_t^p = \widetilde{N}_t^p$), then $H(\cdot) = 1$; this explains the constant 2 in (22). In other words, $x_t^k x_t^u$ measures the fraction territory that is conquered by the attacker if the contestants are symmetric, and $2x_t^k x_t^u$ measures the conquest by country 1 if it is much larger than country 2 (so that $\widetilde{K}_t^p / K_t^p$ and $\widetilde{N}_t^p / N_t^p$ are close to zero).[11]

The conquest function in (21) and (22) may seem arbitrary, and we will look at some alternative formulations later in the sensitivity analysis in Section 5. Note, however, that it also has some plausible properties.

First, letting warfare be conducted with a Solow-type of military technology makes sense if we think that the production process in the military at any stage of development is similar to that of contemporary industrial production (even before industry becomes commercially active).

Second, we borrow the property from a standard Tullock contest function that the attacking country's conquest is greater the larger is its war inputs, relative to that of the other country.

Third, we noted earlier that our formulation assumed "total war" by letting each country's total (pre-war) endowments of capital and labor enter the conquest function. However, that is not as restrictive as it might seem: since $H(\cdot)$ is homogenous of degree zero, nothing would change if only some exogenous fraction of each country's capital and labor were used.[12] That exogenous fraction could also be time dependent.

Fourth, the conquest function in (21) and (22) allows for some interesting and empirically plausible forces to drive the dynamics during the Malthusian phase of development. The increased war-proneness of relative land scarcity implies that if country 1 takes land from country 2 today, country 2 is more likely to try to take it back in the next period, if given the opportunity. As these opportunities shift back and forth over several periods conquests

[11] However, recall that if $x_t^k x_t^u H(\cdot)$ is large enough, the conquest is capped at 0.99.

[12] That is, let the fractions $\alpha \in (0, 1)$ and $\beta \in (0, 1)$ of both countries' capital and labor, respectively, be used in warfare. Then, from (22), we see that nothing changes, since

$$H(\alpha K_t^p, \alpha \widetilde{K}_t^p, \beta N_t^p, \beta \widetilde{N}_t^p) = H(K_t^p, \widetilde{K}_t^p, N_t^p, \widetilde{N}_t^p).$$

thus tend to be reversed, making landholdings converge between the two countries. The conquest function allows for a countervailing force. In a Malthusian environment conquering land today also raises incomes today, and thus population and capital tomorrow. This makes the conqueror today more likely to conquer more land tomorrow, if given the opportunity; and the loser less likely to act on an opportunity to take lost territory back. This makes landholdings diverge, pushing one country towards domination.[13]

A fifth property is that the random elements in (21) allow us to think of both the outbreak and outcome of war as random, and to study how the governments' risk preferences affect the frequency of war. More precisely, the unknown random component, x_t^u, makes war outcomes uncertain; the attacking country may end up not gaining any territory at all, namely if $x_t^u = 0$. Moreover, the known random component, x_t^k, must be large enough for war to break out, thus generating an endogenous and dynamically evolving frequency, or probability, of war. Also, because x_t^k and x_t^u enter multiplicatively and are distributed uniformly on $\{0, 0.1, ..., 1\}$, it can be seen that the conquest carries more probability weight around zero, generating a sort of status-quo bias in war outcomes. (See Lagerlöf 2009 for illustrations of the distribution of $x_t^k x_t^u$.)

2.5.2 Costs of war

Recall from (20) that the costs of war come from capital being destroyed and labor killed (in both countries, but not necessarily in equal proportions). We let these destruction/killing rates, δ_{Kt} and δ_{Nt}, be given by

$$\delta_{Kt} = \frac{\widehat{\delta}_K}{2}\left[\zeta + (1-\zeta)H(\widetilde{K}_t^p, K_t^p, \widetilde{N}_t^p, N_t^p)\right], \tag{23}$$

and

$$\delta_{Nt} = \frac{\widehat{\delta}_N}{2}\left[\zeta + (1-\zeta)H(\widetilde{K}_t^p, K_t^p, \widetilde{N}_t^p, N_t^p)\right], \tag{24}$$

where $H(\cdot)$ is the function applied in (22), but with the first and third arguments here being the opponent's capital or labor (i.e., that of country 2).

[13] Rather than taking more land in war, a capital and/or labor abundant country could have a higher probability of getting the opportunity to start a war (instead of that probability being 1/2 for both countries). Qualitatively, this would generate similar divergent forces in landholdings. See the discussion in Section 6.

The parameter $\zeta \in [0,1]$ measures how much the destruction/killing rates depend on the relative sizes of the contestants' capital and labor inputs in war. When $\zeta = 1$ these rates are constant at $\widehat{\delta}_K/2$ and $\widehat{\delta}_N/2$, respectively. Setting $\zeta < 1$ allows for the possibility that the weaker country, in terms of capital and people, loses more in war. If the countries are symmetric ($K_t^p = \widetilde{K}_t^p$ and $N_t^p = \widetilde{N}_t^p$), then $H(\cdot) = 1$, so the rates become $\widehat{\delta}_K/2$ and $\widehat{\delta}_N/2$. If country 1 completely dominates country 2 (so that $\widetilde{K}_t^p/K_t^p$ and $\widetilde{N}_t^p/N_t^p$ are close to zero) then $H(\cdot)$ is close to zero, and the rates become $\zeta\widehat{\delta}_K/2$ and $\zeta\widehat{\delta}_N/2$ for country 1: thus, if ζ is also close to zero, then almost none of country 1's capital or population is destroyed or killed; and for country 2 the rates become $\widehat{\delta}_K$ and $\widehat{\delta}_N$.

2.5.3 The government's objective function

The decision about starting a war (if given the opportunity) is made by each country's government. For simplicity, we let these governments maximize the expected utility of total tax revenues in each period. That is, all revenues are consumed immediately, so that the governments effectively live for one period only, and do not care about what happens in the future. We thus abstract from e.g. investments in armaments or intertemporal considerations when making war decisions. This structure could be motivated from the assumption that agents themselves a finitely lived. We may think of the government as being made up of one or more old agents, who care only about their own consumption in the current period (the last period of life), although not necessarily having the same preferences as other agents.

A constant (exogenous) fraction τ of total output in each sector is taxed. To make things a little more interesting, we now also allow for the possibility that taxation of Solow output is associated with some form of waste, not captured explicitly in the model. Let the fraction $1-\theta \in [0,1]$ of the taxes received from the Solow sector go to waste, so that the government extracts τY_{Mt} from the Malthus sector and $\tau\theta Y_{St}$ from the Solow sector. Then the government (in country 1) consumes

$$\begin{aligned} G_t &= \tau Y_{Mt} + \theta\tau Y_{St} \\ &= \tau A_{Mt}\left[(1-z_{Kt})K_t\right]^{\phi}\left[(1-z_{Nt})N_t\right]^{\mu}\left[L_t\right]^{1-\mu-\phi} \\ &\quad +\tau\theta A_{St}[z_{Kt}K_t]^{1-\mu}[z_{Nt}N_t]^{\mu}. \end{aligned} \tag{25}$$

The parameter θ allows for the possibility that the government cares more about output in the Malthus sector (which is the case when $\theta < 1$). This type of Malthus-bias may capture a disproportionate influence of landowners in the political or social process leading up to (the decision to start a) war. In the baseline case below we set $\theta = 1$, but some interesting results arise when setting $\theta < 1$ (see Section 5.2).

The government's utility function (assumed to be the same for both countries) is CRRA:

$$U(G_t) = \frac{G_t^{1-\omega}}{1-\omega}. \tag{26}$$

If given the opportunity, the government starts a war if the known shock, x_t^k, is large enough to make the expected utility of starting a war, conditional on x_t^k, greater than that of not starting a war.

3 Dynamics

Given the assumptions about the timing of events, the dynamics can be analyzed in three steps. (1) First we specify how the pre-war state variables are updated in the war phase of each period, in three different scenarios: when country 1 attacks country 2; when country 2 attacks country 1; and when no one attacks. (2) We then specify each country's decision about whether or not to attack (if given the opportunity), thus determining which scenario plays out under step 1 as a function of the pre-war state variables, the realization of the opportunity shock, and the known shock, x_t^k. (3) Finally, we specify how the post-war state variables are transformed to become the pre-war state variables in the next period.

3.1 Step # 1: How pre-war state variables are updated in the war phase

We first specify the following eight state vectors:

$$\begin{aligned}
\boldsymbol{\lambda}_t &= (A_{Mt}, A_{St}, K_t, \widetilde{K}_t, N_t, \widetilde{N}_t, L_t) \\
\widetilde{\boldsymbol{\lambda}}_t &= (A_{Mt}, A_{St}, \widetilde{K}_t, K_t, \widetilde{N}_t, N_t, 1-L_t) \\
\boldsymbol{\lambda}_t^p &= (A_{Mt}, A_{St}, K_t^p, \widetilde{K}_t^p, N_t^p, \widetilde{N}_t^p, L_t^p) \\
\widetilde{\boldsymbol{\lambda}}_t^p &= (A_{Mt}, A_{St}, \widetilde{K}_t^p, K_t^p, \widetilde{N}_t^p, N_t^p, 1-L_t^p) \\
\boldsymbol{\lambda}_t^w &= (A_{Mt}, A_{St}, (1-\delta_{Kt})K_t^p, (1-\widetilde{\delta}_{Kt})\widetilde{K}_t^p, (1-\delta_{Nt})N_t^p, (1-\widetilde{\delta}_{Nt})\widetilde{N}_t^p, \Psi(\boldsymbol{\lambda}_t^p, x_t^k, x_t^u)) \\
\widetilde{\boldsymbol{\lambda}}_t^w &= (A_{Mt}, A_{St}, (1-\widetilde{\delta}_{Kt})\widetilde{K}_t^p, (1-\delta_{Kt})K_t^p, (1-\widetilde{\delta}_{Nt})\widetilde{N}_t^p, (1-\delta_{Nt})N_t^p, \Psi(\widetilde{\boldsymbol{\lambda}}_t^p, x_t^k, x_t^u)) \\
\boldsymbol{\lambda}_t^l &= (A_{Mt}, A_{St}, (1-\delta_{Kt})K_t^p, (1-\widetilde{\delta}_{Kt})\widetilde{K}_t^p, (1-\delta_{Nt})N_t^p, (1-\widetilde{\delta}_{Nt})\widetilde{N}_t^p, 1-\Psi(\widetilde{\boldsymbol{\lambda}}_t^p, x_t^k, x_t^u)) \\
\widetilde{\boldsymbol{\lambda}}_t^l &= (A_{Mt}, A_{St}, (1-\widetilde{\delta}_{Kt})\widetilde{K}_t^p, (1-\delta_{Kt})K_t^p, (1-\widetilde{\delta}_{Nt})\widetilde{N}_t^p, (1-\delta_{Nt})N_t^p, 1-\Psi(\boldsymbol{\lambda}_t^p, x_t^k, x_t^u))
\end{aligned} \tag{27}$$

The vector $\boldsymbol{\lambda}_t$ is the post-war state vector for country 1, and $\widetilde{\boldsymbol{\lambda}}_t$ that of country 2. To see this, recall the following: A_{Mt} and A_{St} are common for both countries; K_t and N_t ($\widetilde{K}_t$ and $\widetilde{N}_t$) refer to country 1 (2); and country 1's (2's) territory is L_t $(1-L_t)$.

$\boldsymbol{\lambda}_t^p$ and $\widetilde{\boldsymbol{\lambda}}_t^p$ are the corresponding pre-war state vectors; if there is no war in period t, then $\boldsymbol{\lambda}_t$ and $\widetilde{\boldsymbol{\lambda}}_t$ equal $\boldsymbol{\lambda}_t^p$ and $\widetilde{\boldsymbol{\lambda}}_t^p$, respectively.

If there is war in period t, then $\boldsymbol{\lambda}_t$ and $\widetilde{\boldsymbol{\lambda}}_t$ equal either $\boldsymbol{\lambda}_t^w$ and $\widetilde{\boldsymbol{\lambda}}_t^l$, or $\boldsymbol{\lambda}_t^l$ and $\widetilde{\boldsymbol{\lambda}}_t^w$. More precisely: $\boldsymbol{\lambda}_t^w$ ($\widetilde{\boldsymbol{\lambda}}_t^w$) is the post-war state vector for country 1 (2) if it attacks country 2 (1); $\boldsymbol{\lambda}_t^l$ ($\widetilde{\boldsymbol{\lambda}}_t^l$) is the post-war state vector of country 1 (2) if it is attacked by (i.e., loses to) country 2 (1). The function

$$\Psi(\boldsymbol{\lambda}_t^p, x_t^k, x_t^u) = L_t^p + \max\{0.99, x_t^k x_t^u H(K_t^p, \widetilde{K}_t^p, N_t^p, \widetilde{N}_t^p)\}\,[1-L_t^p] \tag{28}$$

gives the post-war landholdings of country 1 if it attacks country 2, where $H(\cdot)$ is defined in (22). Similarly, $\Psi(\widetilde{\boldsymbol{\lambda}}_t^p, x_t^k, x_t^u)$ is the post-war landholdings of country 2 if it attacks country 1. Since their landholdings add up to unity, it follows that $1-\Psi(\boldsymbol{\lambda}_t^p, x_t^k, x_t^u)$ [or $1-\Psi(\widetilde{\boldsymbol{\lambda}}_t^p, x_t^k, x_t^u)$] is the landholdings of country 2 (or 1) if it is attacked by country 1 (or 2); this explains the last elements in the vectors $\boldsymbol{\lambda}_t^l$ and $\widetilde{\boldsymbol{\lambda}}_t^l$.

3.2 Step # 2: Determining the decision to go to war

Next let $O_t \in \{1, 2\}$ be the random variable that determines which country has the opportunity to go to war against the other: $O_t = 1$ if country 1 has the opportunity; $O_t = 2$ if country 2 has.

Eq. (A15) in Section B of the Appendix defines output in the Malthus and Solow sectors as functions of the post-war state vector: $Y_M(\boldsymbol{\lambda}_t)$ and $Y_S(\boldsymbol{\lambda}_t)$, the argument being $\widetilde{\boldsymbol{\lambda}}_t$ for country 2. The sum gives total output, $Y(\boldsymbol{\lambda}_t) = Y_M(\boldsymbol{\lambda}_t) + Y_S(\boldsymbol{\lambda}_t)$. Using (25) we can thus define government consumption, G_t, as a function of $\boldsymbol{\lambda}_t$:

$$G_t = \tau Y_M(\boldsymbol{\lambda}_t) + \tau\theta Y_S(\boldsymbol{\lambda}_t) = G(\boldsymbol{\lambda}_t). \tag{29}$$

Recall that country 1 attacks country 2 if it has the opportunity ($O_t = 1$) and if doing so leads to an increase in expected utility, conditional on the known shock, x_t^k, and analogously for country 2. We can now specify how the post-war state vectors for the two countries are determined:

$$(\boldsymbol{\lambda}_t, \widetilde{\boldsymbol{\lambda}}_t) = \begin{cases} (\boldsymbol{\lambda}_t^w, \widetilde{\boldsymbol{\lambda}}_t^l) & \text{if } O_t = 1 \text{ and } E[U(G(\boldsymbol{\lambda}_t^w)) \mid x_t^k] > U(G(\boldsymbol{\lambda}_t^p)), \\ (\boldsymbol{\lambda}_t^l, \widetilde{\boldsymbol{\lambda}}_t^w) & \text{if } O_t = 2 \text{ and } E[U(G(\widetilde{\boldsymbol{\lambda}}_t^w)) \mid x_t^k] > U(G(\widetilde{\boldsymbol{\lambda}}_t^p)), \\ (\boldsymbol{\lambda}_t^p, \widetilde{\boldsymbol{\lambda}}_t^p) & \text{otherwise,} \end{cases} \tag{30}$$

where $E(X_t \mid x_t^k)$ is the mean of X_t conditional on x_t^k, $U(G)$ is given by (26), and $G(\boldsymbol{\lambda}_t)$ by (29). Note that (30) also determines whether there is war or peace in period t, since peace prevails in the "otherwise" case.

3.3 Step # 3: Updating post-war state variables to the next period's pre-war state variables

The final step is to specify how the pre-war state vector for period $t+1$ depends on the post-war state vector in period t.

The productivity variables, A_{Mt} and A_{St}, are updated through (3).

The pre-war territory is simply the post-war territory from the previous period, so for country 1 we get:

$$L_{t+1}^p = L_t, \tag{31}$$

and country 2's corresponding landholdings are simply $1 - L_{t+1}^p = 1 - L_t$.

Recall that agents die after the second period of life, so population is updated according to $N_{t+1}^p = n_t N_t$. (Note that N_t denotes the post-war young population.) Fertility is defined as a function of the wage rate in (16).

Using (A16) in Section B of the Appendix, which defines the wage rate as a function of the post-war state variable, $\boldsymbol{\lambda}_t$, we can write

$$\begin{aligned} N_{t+1}^p &= N_t n(w(\boldsymbol{\lambda}_t)), \\ \widetilde{N}_{t+1}^p &= \widetilde{N}_t n(w(\widetilde{\boldsymbol{\lambda}}_t)). \end{aligned} \tag{32}$$

Each period's pre-war capital stock is made up of saving from the previous period, $K_{t+1}^p = s_t N_t$. Section B of the Appendix shows that the capital accumulation function becomes:

$$\begin{aligned} K_{t+1}^p &= \beta(1-\tau)\left[1-\mu-\phi+\tfrac{\mu}{1-z_N(\boldsymbol{\lambda}_t)}\right]\tfrac{Y_M(\boldsymbol{\lambda}_t)}{N_t}, \\ \widetilde{K}_{t+1}^p &= \beta(1-\tau)\left[1-\mu-\phi+\tfrac{\mu}{1-z_N(\widetilde{\boldsymbol{\lambda}}_t)}\right]\tfrac{Y_M(\widetilde{\boldsymbol{\lambda}}_t)}{\widetilde{N}_t}, \end{aligned} \tag{33}$$

where $z_N(\boldsymbol{\lambda}_t)$ and $Y_M(\boldsymbol{\lambda}_t)$ are defined in (A14) and (A15).

4 Quantitative analysis

4.1 Calibration: baseline case

We first choose a baseline set of parameter values, as reported in Table 1. In those cases where we are the most uncertain about what values to pick, we consider some sensitivity analysis (see Section 5 below).

Unless otherwise stated, all parameter values and initial conditions below are the same for both countries.

4.1.1 Parameter values

We set all parameters associated with the production functions and total factor productivity growth (μ, ϕ, γ_M, γ_S) as in HP.

Since tax revenues are just consumed by the government the tax rate is neutral; a higher τ has the same effect as lower initial levels of A_{Mt} and A_{St}. We arbitrarily set τ to 1%.

We normalize the Malthusian wage, w^*, to one.

The relative utility weight on old-age consumption is chosen similarly to HP, who set the weights in the utility function to 1 and β, calibrating β to 1; in our formulation in (14) the weights are $(1-\beta)$ and β, so we set β to 1/2. With a period being 35 years (as in HP) this gives an annual real interest rate on the balanced growth path (where the Solow sector dominates) of

3.8%. This is similar to HP (whose corresponding rate is 4.5%), although interest rates are determined quite differently in our setting. (We let young own land, thus not enforcing returns to land and capital to be equal; this also makes saving for old age, and capital accumulation, independent of the interest rate.)

The parameters in the fertility function are set slightly differently than HP, but along similar principles. We set $\overline{n} = 1.5$, implying that, when population growth is at its peak, population expends by 50% over a 35-year period, or a little over 1% per year, which fits with data from England reported in HP's Table 1.[14] We let fertility peak when the wage rate is 50% above ($\eta = 1.5$) its Malthusian level, w^*. Fertility then declines with the wage rate until the wage equals $\nu w^* = 7w^*$. These numbers are lower than in HP, but generate similar time paths: three periods until fertility peaks, and three more until population is constant. (The difference is due to capital accumulation adjusting more slowly in our setting, since agents save a constant fraction of income earned from both land and labor.) If we set the transition period to 1780, this also generates a good fit between model and data for population and wage levels (see Lagerlöf 2009).

We set $\omega = 0$, so that the government is risk neutral in the baseline setting.

We set $\theta = 1$ in the baseline setting, so that the government cares only about total output, and not its sectoral composition.

We set $\zeta = 1$, implying that the destruction and killing rates in war are constant at $\widehat{\delta}_K/2$ and $\widehat{\delta}_N/2$, independently of any asymmetries in endowments of capital and labor.

Choosing numbers for $\widehat{\delta}_K$ and $\widehat{\delta}_N$ is difficult. Below follows a rationale for the choices we have made.

Setting $\widehat{\delta}_N = 0.12$ implies a death rate in war of $\widehat{\delta}_N/2 = 0.06$. Battle deaths in war have varied a lot, and are better documented for later wars than earlier. According to Browning (2002, Table 18) German battle deaths in World War II amounted to 4.9% of the 1938 population; the corresponding number for the USSR was 13%, for France 0.7%, and for Britain 1%. In World War I battle deaths as a fraction of the 1913 population were 3.8% for

[14] HP instead use data from Lucas (2002) to calibrate the fertility function. Since those data refer to several different regions of the world, many in which fertility peaked later and higher, they set maximum fertility a bit higher, at $\overline{n} = 2$. Since we are here interested in predominantly European and early industrializing countries (which constituted most of the Great Power nations) it makes sense to use HP's English numbers as guide.

Germany, 2.4% for Austria-Hungary, 2.1% for Britain, and 3.6% for France (ibid., Table 16). Most countries whose territories were not battle grounds had much lower death rates (see e.g. Cook 2002, Table A.1). In proportion to world population the two world wars of the 20th century were more lethal than previous wars (Ferguson 2006, Introduction, Figure 1.1). On the other hand, the numbers cited here exclude indirect mortality effects of war, such as famines and epidemics; some wounded returning soldiers may be unable to work, which should perhaps be added to the "depreciation" of the labor force in our model.

For the purpose of arriving at some baseline number, we can think of a typical 5- to 6-year war as having a 2% death rate. We may define war in a model period (35 years) as there being war in about half of those years, or 17.5 years. That makes about 3 consecutive wars, with an accumulated death rate of approximately 6%, i.e., $\widehat{\delta}_N/2 = 0.06$. We thus set $\widehat{\delta}_N = 0.12$.

Numbers for capital destruction are even harder to come by. In World War II about 25% of Japan's real assets were destroyed (Flath, 2005, p. 91). According to Broadberry and Howlett (1998), Britain's losses in World War II, including ships, physical capital on land, and the decline in net asset holdings overseas, have been estimated to 19% of pre-war wealth. Excluding overseas disinvestment the loss is about 9%. Given that offensive weapons systems have become more sophisticated over time (e.g., through the development of bombers) we may guess lower numbers for earlier wars.

With the same reasoning as used for death rates, we may think of a 5- to 6-year war as destroying 10% of the physical capital, and then define a 35-year period as being in war if war is fought in about half those years (17.5 years). This makes for about three wars, and an accumulated capital destruction of about 30%, i.e., $\widehat{\delta}_K/2 = 0.3$. We thus set $\widehat{\delta}_K = 0.6$.

The parameter T, which we set to 5, measures how many periods it takes before a transition sets in (see explanation below). This facilitates comparison to HP, who show simulation results for 5 periods prior to the transition. Also, if the transition year corresponds to 1780, then the initial period corresponds to 1605 (five 35-year periods earlier). This matches broadly with the emergence of the Great Power system in Europe; Levy's (1983) list over Great Power wars starts in 1495, and the frequency was the highest in the early 17th century [see Figure 1(a)].

4.1.2 Initial conditions

We normalize initial total factor productivity in the Malthus sector to unity: $A_{M,0} = 1$.

The unit-sized land is initially distributed equally between the two countries, so that $L_0 = 1/2$.

We set initial labor and capital, N_0 and K_0, so that the economy starts off on a Malthusian balanced growth path. We derive expressions for N_0 and K_0 in Section D in the Appendix; see (A23) to (A25).

We set initial total factor productivity in the Solow sector below the level at which the Solow sector becomes active. As shown in Section E of the Appendix the Solow sector becomes active when A_{St} exceeds A_S^{crit}, given by

$$A_S^{\text{crit}} = \left(\frac{\phi}{1-\mu}\right)^{1-\mu} \frac{(Rn^*)^{1-\mu}\,(w^*)^{\mu}}{(1-\tau)\mu}, \tag{34}$$

where R is defined by (A24). Setting $A_{S,0} = \gamma_M^{-T} A_S^{\text{crit}}$ implies that the Solow sector becomes active after T periods (absent war, although the timing changes very little when introducing war). As explained above we set $T = 5$.

4.2 Simulation results: baseline case

4.2.1 One single run

Figure 2 shows the results from one single run, where the time paths are specific to the realized shocks. As HP, we set the transition period to 0. Panel A shows two population growth paths for each country: (1) the actual (gross) population growth rate, which equals the fertility rate, n_t, in peace, and $n_t(1-\delta_{Nt})$ in war; and (2) the population growth rate absent war, which equals just n_t.

We see that periods of war are associated with a drop in population growth. Note also, when comparing Panels A and B, how the country that has recently gained land exhibits faster population growth

Panel C shows wages. As Malthus sector productivity grows, population initially keeps even pace, thus holding wages stagnant. When the wage rate exceeds ηw^*, the wage-fertility relationship reverses and fertility starts to fall.

Panel D shows a simultaneous rise in the Solow sector's shares of capital and labor. Note that the country that ends up with less territory as peace sets in (here country 1) leads the industrialization process. This happens because productivity is lower in the Malthus sector when land is scarce, inducing an earlier reallocation of people and capital into the Solow sector. In some runs (although not in Figure 1) the land scarce country transits before period 0 (cf. the Monte Carlo simulations in Figure 3). However, the difference in timing is not great. Note also that this result need not hold in a two-good setting, where the Malthus-sector good (say food) is subject to Engel's Law (as in Matsuyama 1992 and Voigtländer and Voth 2006), or consumed only up to some exogenous satiation level (as in Gollin, Parente and Rogerson 2002). In such settings, an increase in land (or in land productivity) would make less labor required for food production, thus freeing up labor for the Solow sector.

4.2.2 Monte Carlo simulations

Figure 3 shows the time paths for the same variables as in Figure 2 in a Monte Carlo simulation where we take the average across 500 runs. We define a country as a winner if it has more than half of the land when war ends. (Note that territory is not redistributed any more when peace breaks out; here we use the last period of the simulation, 7 periods after the transition, to decide which country is the winner.)

As seen in Panels A and B, winners have faster population growth and larger landholdings throughout. Note how the landholdings of the eventual winner grow gradually. This is driven by the way in which initial conquests create growth in population and capital, enabling further conquests.

Panel D shows how the war frequency rate (i.e., the fraction of the runs in which war is fought in any given period) drops from about 0.4 to zero over two generations, as the Malthus-to-Solow transition sets in. This happens because incentives for land conquests vanish. Simultaneously differences in wages and population growth rates go away, as land abundance ceases to matter for those variables.

The Malthusian war frequency rate of 0.4 can be compared to the numbers shown in Table 2. To derive these we first pick what might be considered a Malthusian period, such as 1495-1700; this can be divided into 206 different rolling 35-year periods (the last one starting 1700). (Recall that we let 35 years correspond to a model period.) Based on the Levy (1983) data, we can then define each such a 35-year period as being "in war" if the country in question (England or France) was involved in a Great Power war during more than a half, or a third, of those 35 years; cf. Figure 1(b). We also do the same exercise for the period 1495-1800. The fraction 35-year periods with war, reported in Table 2, can thus be interpreted as the probability of war in a model period in the Malthusian phase. As seen, the numbers vary depending on what period, which country, and which criterion we consider. Most numbers in the table are higher than the model's 0.4, but at least when using the half-of-the-years criterion for England the model is not far off. Moreover, if the economic mechanisms behind war that we model here are only one factor of many that cause war, we would want the simulated numbers generated by the model to be on the low side.

Figure 4 illustrates changes over time in the distribution of land and population and per-capita income gaps. The left-hand panel shows the outcomes across different Monte Carlo runs in country 1's population share. Initially, country 1 has exactly half of the population; the variance grows over time, but stays relatively constant after the transition, when land is no longer changing hands. In other words, whereas population *growth rates* converge, population *levels* mirror landholdings long after the transition.

The right-hand panel of Figure 4 shows that, in those runs where it takes a greater land share, country 1's per-capita income is also higher, but only during the Malthusian phase. After the transition, per-capita income differences vanish, although land inequality stays constant.

5 Sensitivity analysis

5.1 A risk averse or risk loving government

The parameter ω measures the government's degree of risk aversion when evaluating the gains from war. In the baseline setting we let the government be risk neutral, setting ω in (26) to zero. (Note, however, that since output in the Malthus sector is concave in land there is effectively a form of risk aversion imposed on the government's behavior, even when $\omega = 0$. Setting $\omega > 0$ introduces some further risk aversion.)

The left-hand panel of Figure 5 shows that with risk averse governments ($\omega = 10$) the war frequency is lower, and with risk loving governments ($\omega = -10$) it is (slightly) higher.[15] This is intuitive, since war is risky. Moreover, in many non-democratic Great Power nations it may have been the case

[15] Due to random variation, in some periods the war frequency associated with $\omega = -10$ in Figure 5 is lower than in the baseline case.

that risk-loving agents sought, or were selected into, military and political leadership roles, so negative values for ω may not be unreasonable. However, considering that these values for ω are extreme when compared to standard risk-aversion assumptions for households, the differences in war frequencies are not very big.

5.2 A Malthus-biased government

The parameter θ measures how important Solow sector output is in the governments' decisions to go to war. The left-hand panel of Figure 5 shows the war frequencies with different θ's. (Recall that $\theta = 1$ in the baseline case.) When $\theta = 0.75$ the decline in war frequency is delayed by one period. For θ even lower the war frequency *increases* as the Malthus-to-Solow transition sets in.

Intuitively, when the Solow sector is active, recall from (9) to (11) that the destruction of capital leads to a reallocation of labor and capital back into the Malthus sector, thus raising Malthus sector output. This comes with a reduction in both total and Solow sector output, but for low enough θ that "trade" is still optimal for the government.

A low θ may represent a relatively influential landowning class in the political process leading up to war. There is some evidence that landowners have been more war prone than capital owners, e.g. in the run-up to World War I (Ferguson 1999, Ch. 1-2; Hewitson 2004, Ch. 2). This may say something about the increase in warfare in the 20th century; see Figures 1(a)-(b) and the discussion in Section 1.2. The timing is a bit off: in the data war frequencies first drop in the 19th century and then rise again in the 20th century, whereas the simulations show a delay by one period, or an initial increase. However, recall that our results here refer to Monte Carlo simulations; in each run, random shocks play a role too. See Section 6 for some further discussion of 20th-century wars.

5.3 Alternative functions for destruction and killing

In the baseline case we let the costs of war (δ_{Kt} or δ_{Nt}) be constant by setting $\zeta = 1$; see (23) and (24). Figure 6 shows how the time paths of war frequencies and the winner's landholdings change when setting $\zeta = 0$. Little happens to war frequencies, but the country that ends up being the winner takes a larger share of the land. This happens because initial land conquests, which generate population growth and capital accumulation, in the long run lead to lower war costs, thus inducing the winning country to keep acting on opportunities to conquer more land.

Figure 5 also shows the effects of changing $\widehat{\delta}_K$ and $\widehat{\delta}_N$ in the baseline case (i.e., with constant war costs, $\zeta = 1$). Not surprisingly, lower (higher) war costs are associated with higher (lower) war frequencies. The effects can be quite big: varying $\widehat{\delta}_N$ between 0.02 and 0.2, and $\widehat{\delta}_K$ between 0.2 and 1, we get war frequencies fluctuating from around 0.2 to 0.6 throughout the Malthusian phase. The decline comes at most a period later when $\widehat{\delta}_K$ and $\widehat{\delta}_N$ are at their lowest, and a period earlier when they are at their highest.

5.4 Alternative conquest functions

We next consider some alternative functional forms for the conquest function in (21) and (22). The Monte Carlo results for war frequencies and the landholdings of the winner are summarized in Figure 7.

In the baseline case we let $H(\cdot) = 1$ for symmetric countries; recall that the overall conquest is scaled by the constant 2 in (22). Now consider lowering that constant by 25% to 1.5, so that (22) is replaced by

$$\text{Alternative I:} \quad H(K_t^p, \widetilde{K}_t^p, N_t^p, \widetilde{N}_t^p) = 1.5 \left(\frac{K_t^p}{K_t^p + \widetilde{K}_t^p} \right)^{1-\mu} \left(\frac{N_t^p}{N_t^p + \widetilde{N}_t^p} \right)^{\mu}. \tag{35}$$

Thus, the conquest in a war between symmetric countries now equals $1.5x_t^k x_t^u/2 = 0.75x_t^k x_t^u$. As seen in Figure 7, this generates Malthusian war frequencies around 0.3 (compared to 0.4 in the baseline case) and a slightly earlier decline. Note also that the winner ends up with less land than in the baseline case. This is not surprising: with lower conquest returns to war, governments start wars less often, and less land is redistributed.

The reduction in the overall conquest by 25% considered in Alternative I does not change the results in any drastic or unrealistic ways; in light of the numbers in Table 2 a war frequency of 0.3 is not necessarily unreasonable if some wars have other causes than competition for land. However, if the reduction is 50% [implying $H(\cdot) = 1/2$ for symmetric countries], or larger, wars are eliminated completely. Intuitively, even if the known shock is at its maximum ($x_t^k = 1$) in the first period, no country wants to start a war; symmetry and peace is thus preserved to the next period, and to the next, and so on. In other words, given the assumption of initial symmetry, the overall conquest must exceed a certain threshold for wars to get started.

Another sensitivity test is to eliminate the known stochastic component, by letting x_t^k in the baseline formulation in (21) be replaced by its mean, i.e. 1/2. However, then no wars are ever fought, for the same reason that peace prevails when the overall conquest is too low, as shown with Alternative I above.[16] Some more interesting results arise if we replace x_t^k by 0.6, i.e.,

$$\text{Alternative II:} \quad g_t = 0.6 x_t^u H(K_t^p, \widetilde{K}_t^p, N_t^p, \widetilde{N}_t^p). \tag{36}$$

This raises the war frequencies in Figure 7, which is not surprising since the expected fraction land conquered is higher than that associated with the mean of x_t^k. (The war probability equals one in the first period since the countries are initially identical: either both want to start a war if given the opportunity, or none does.) The left-hand panel shows that the winning country ends up with less land compared to the baseline case, even though 0.6 is greater than the mean of x_t^k. Note that, in the baseline case, small realizations of x_t^k do not matter, because governments do not go to war in those events. Thus, there is a type of Jensen's inequality built into the model, through which the randomization of x_t^k matters. Part of the divergence effect goes away when x_t^k is non-random, because without large conquests there are no large changes in the gaps in population and capital between the countries.

Another alternative form for the conquest function is to let the relative total size of (hypothetical) Solow output, rather than the relative inputs, determine the conquest, i.e.,

$$\text{Alternative III:} \quad H(K_t^p, \widetilde{K}_t^p, N_t^p, \widetilde{N}_t^p) = 2\left[\frac{\left(K_t^p\right)^{1-\mu}\left(N_t^p\right)^{\mu}}{\left(\widetilde{K}_t^p\right)^{1-\mu}\left(\widetilde{N}_t^p\right)^{\mu}+\left(K_t^p\right)^{1-\mu}\left(N_t^p\right)^{\mu}}\right]. \tag{37}$$

With this alteration, the differences from the baseline case are hard to even detect in Figure 7.

A final alternative is to let the size of the conquest be independent of the belligerent countries' capital and labor endowments, i.e.,

$$\text{Alternative IV:} \quad H(K_t^p, \widetilde{K}_t^p, N_t^p, \widetilde{N}_t^p) = 1. \tag{38}$$

As seen in Figure 7, war frequencies do not change much, but the time path for the winner's landholdings increases more slowly. Here there is no divergent force at play during the Malthusian phase, since initial land conquests do

[16] If the two countries are symmetric, and if parameter values are set as in the baseline case, then war breaks out only when $x_t^k \geq 0.6$.

not feed back into future conquests, as they do in the baseline formulation by generating more people and capital. However, there is still some divergence before peace breaks out, since we are showing the mean of the landholdings among those countries that through luck end up with more land.

6 Conclusions

We have presented a version of the Hansen-Prescott long-run growth model, where we allow for war between two initially identical economies. War amounts to seizing land from the opponent, at the cost of destroying capital and killing people. The model can generate a downward trend in the frequency of warfare that is broadly consistent with an observed decline in Great Power warfare in the 19th century, shown in Figures 1(a)-(b).

Our baseline quantitative analysis reveals some interesting insights. The dynamics that are inherent to the model tend to create an asymmetric situation where one country holds more land than the other country, but not all land. The probability of war in any given period is about 40% during the Malthusian phase of development, dropping to zero over a couple of generations as the transition sets in.

Because conquests are uncertain, war in our model is risky. Sensitivity analysis suggests that there is more war with more risk loving governments, and less with governments that are risk averse, but the differences are quantitatively relatively small. Another finding from the sensitivity analysis refers to a Malthus-biased government. This may be interpreted as a landowning class having a disproportional influence on decisions about going to war. We find that with such a government war probabilities can *increase* initially as the Solow transition sets in, before declining. A Malthus-biased government favors war partly *because* it destroys capital, thus leading to a reallocation of resources away from the Solow sector.

We have obviously made many brave assumptions along the way. For example, there is no trade and no movement of labor or capital between the two countries; their only interaction is through war. This may not be too unrealistic, if e.g. economies were relatively more closed in preindustrial times than they are today. However, it would be interesting to think about (and model) the interaction between trade and war in a framework similar to the one used here.

We could have assumed that abundance of capital and people affects e.g.

the probability of getting the opportunity to start a war, rather than the land conquest, g_t. (We now let those probabilities be 1/2 for both countries in all periods.) The qualitative results, in particular regarding divergence in landholdings, need not change in such a setting. However, one would ideally want to model endogenously how such war opportunities might arise, e.g. as the result of internal power struggles that weaken one country's government, or random shocks to technological superiority.

We have not allowed governments to have longer time horizons when deciding whether, or not, to start a war. For example, one country's conquest today induces the losing country to try to recapture it in the next period if given the opportunity; here governments are finitely lived and do not consider such repercussions. Neither do governments use their tax revenues to invest in armaments, or the hiring of a professional army. Such extensions are left for future work.

In terms of the timing, our model is not able to explain 20th-century warfare after the rise of the Solow sector, in particular the two world wars. One possibility is that those wars were caused by competition for other resources than land, typical Solow-sector inputs such as coal, petroleum, and metals. This raises the question why such resource wars died out, at least in Europe. One possibility could be that technological progress has reduced the need for resource inputs or made war too costly; changes in trade could be another factor. These are also interesting questions left for future research.

When modelling conquests we assumed that the winner gains only land, not people. In that sense, land conquests in our model should be followed by the movement of people, i.e. refugees. This has certainly happened in European history. The alternative approach would be to allow a power to occupy both the land and the people who live there, which has also been common in Europe. Allowing for that may not require too much change to the current framework, although one may then want to also think about insurgencies, and allow the costs of holding on to territory to be a function of how long it has been occupied.

APPENDIX

A Factor allocations across sectors

If both sectors are active, so that $z_{Kt} > 0$ and $z_{Nt} > 0$, then both (6) and (7) hold. Substituting $K_{St} = z_{Kt}K_t$, $K_{Mt} = (1 - z_{Kt})K_t$, $N_{St} = z_{Nt}N_t$, and $N_{Mt} = (1 - z_{Nt})N_t$ into (6) we get

$$A_{Mt}\left[(1 - z_{Kt})K_t\right]^{\phi}\left[(1 - z_{Nt})N_t\right]^{\mu-1} L_t^{1-\mu-\phi} = A_{St}[z_{Kt}K_t]^{1-\mu}[z_{Nt}N_t]^{\mu-1}, \tag{A1}$$

or, rearranging,

$$W_t(1 - z_{Kt})^{\phi} z_{Kt}^{\mu-1} = \left[\frac{1 - z_{Nt}}{z_{Nt}}\right]^{1-\mu}, \tag{A2}$$

where

$$W_t = \frac{A_{Mt}}{A_{St}}\left(\frac{L_t}{K_t}\right)^{1-\mu-\phi}. \tag{A3}$$

Solving (A2) for z_{Nt} gives

$$z_{Nt} = \frac{z_{Kt}}{z_{Kt} + (1 - z_{Kt})^{\frac{\phi}{1-\mu}} W_t^{\frac{1}{1-\mu}}}. \tag{A4}$$

Next substituting $K_{St} = z_{Kt}K_t$, $K_{Mt} = (1 - z_{Kt})K_t$, $N_{St} = z_{Nt}N_t$, and $N_{Mt} = (1 - z_{Nt})N_t$ into (7) we get

$$\phi A_{Mt}\left[(1 - z_{Kt})K_t\right]^{\phi-1}\left[(1 - z_{Nt})N_t\right]^{\mu} L_t^{1-\mu-\phi} = (1 - \mu)A_{St}[z_{Kt}K_t]^{-\mu}[z_{Nt}N_t]^{\mu}, \tag{A5}$$

or, rearranging,

$$\frac{\phi}{1-\mu}\left\{\frac{A_{Mt}}{A_{St}} K_t^{\phi+\mu-1} L_t^{1-\mu-\phi}\right\}(1 - z_{Kt})^{\phi-1} z_{Kt}^{\mu} = \left(\frac{z_{Nt}}{1 - z_{Nt}}\right)^{\mu}, \tag{A6}$$

where we note from (A3) that the expression in curly brackets equals W_t. Solving (A6) for z_{Nt} gives

$$z_{Nt} = \frac{W_t^{\frac{1}{\mu}} z_{Kt}}{\left(\frac{1-\mu}{\phi}\right)^{\frac{1}{\mu}}(1 - z_{Kt})^{\frac{1-\phi}{\mu}} + W_t^{\frac{1}{\mu}} z_{Kt}}. \tag{A7}$$

Equalizing (A4) and (A7) we can write

$$\frac{z_{Kt}}{z_{Kt}+(1-z_{Kt})^{\frac{\phi}{1-\mu}}W_t^{\frac{1}{1-\mu}}}=\frac{W_t^{\frac{1}{\mu}}z_{Kt}}{\left(\frac{1-\mu}{\phi}\right)^{\frac{1}{\mu}}(1-z_{Kt})^{\frac{1-\phi}{\mu}}+W_t^{\frac{1}{\mu}}z_{Kt}}. \tag{A8}$$

which can be rearranged as

$$1-z_{Kt}=\left(\frac{\phi}{1-\mu}\right)^{\frac{1-\mu}{1-\mu-\phi}}W_t^{\frac{1}{1-\mu-\phi}}=\left[\left(\frac{\phi}{1-\mu}\right)^{1-\mu}\frac{A_{Mt}}{A_{St}}\right]^{\frac{1}{1-\mu-\phi}}\frac{L_t}{K_t}, \tag{A9}$$

where the second equality uses (A3). If the right-hand side of (A9) exceeds one, the fraction capital (and labor) in the Solow sector must be zero. Defining V_t as in (11) we can accordingly write z_{Kt} as in (9).

Since $1-z_{Kt}=V_t$ we then note that

$$(1-z_{Kt})^{\frac{\phi}{1-\mu}}W_t^{\frac{1}{1-\mu}}=V_t^{\frac{\phi}{1-\mu}}W_t^{\frac{1}{1-\mu}}. \tag{A10}$$

When the Solow sector is active (i.e., $V_t<1$) we can use (11), (A3), (A10), and some algebra, to find that

$$W_t^{\frac{1}{1-\mu}}=\left(\frac{1-\mu}{\phi}\right)V_t^{\frac{1-\mu-\phi}{1-\mu}}. \tag{A11}$$

Now (A10) and (A11) can be seen to imply that

$$(1-z_{Kt})^{\frac{\phi}{1-\mu}}W_t^{\frac{1}{1-\mu}}=\left(\frac{1-\mu}{\phi}\right)V_t. \tag{A12}$$

The expression for z_{Nt} in (10) then follows directly from (9), (A4), and (A12).

B Dynamics

B.1 Output and wages as functions of the post-war state vector

Recall that (11) defines V_t as function of the post-war state vector, $\boldsymbol{\lambda}_t$:

$$V_t=V(\boldsymbol{\lambda}_t)=\min\left\{1,\left[\left(\frac{\phi}{1-\mu}\right)^{1-\mu}\frac{A_{Mt}}{A_{St}}\right]^{\frac{1}{1-\mu-\phi}}\frac{L_t}{K_t}\right\}. \tag{A13}$$

Since (9) and (10) define z_{Kt} and z_{Nt} as functions of V_t we can write these as functions of $\boldsymbol{\lambda}_t$:

$$\begin{aligned} z_{Kt} &= z_K(\boldsymbol{\lambda}_t) = 1 - V(\boldsymbol{\lambda}_t), \\ z_{Nt} &= z_N(\boldsymbol{\lambda}_t) = \tfrac{1-V(\boldsymbol{\lambda}_t)}{\phi+(1-\phi-\mu)V(\boldsymbol{\lambda}_t)}. \end{aligned} \tag{A14}$$

Using (6) and (2), we can write Y_{Mt}, Y_{St}, and their sum, Y_t, as functions of $\boldsymbol{\lambda}_t$:

$$Y_{Mt} = Y_M(\boldsymbol{\lambda}_t) = [1 - z_K(\boldsymbol{\lambda}_t)]^{\phi} [1 - z_N(\boldsymbol{\lambda}_t)]^{\mu} A_{Mt} K_t^{\phi} N_t^{\mu} L_t^{1-\mu-\phi},$$

$$Y_{St} = Y_S(\boldsymbol{\lambda}_t) = [z_K(\boldsymbol{\lambda}_t)]^{1-\mu} [z_N(\boldsymbol{\lambda}_t)]^{\mu} A_{St} K_t^{1-\mu} N_t^{\mu}, \tag{A15}$$

$$Y_t = Y(\boldsymbol{\lambda}_t) = Y_M(\boldsymbol{\lambda}_t) + Y_S(\boldsymbol{\lambda}_t).$$

Using (1) and (A15) we can write the wage rate as a function of $\boldsymbol{\lambda}_t$:

$$w_t = w(\boldsymbol{\lambda}_t) = \frac{\mu(1-\tau)Y_M(\boldsymbol{\lambda}_t)}{[1 - z_N(\boldsymbol{\lambda}_t)]\, N_t}. \tag{A16}$$

B.2 Capital accumulation

Using (8) and $l_t = L_t/N_t$ we can write:

$$r_{L,t} l_t = (1-\tau)(1-\mu-\phi)\frac{Y_{Mt}}{L_t}\frac{L_t}{N_t} = (1-\tau)(1-\mu-\phi)\frac{Y_{Mt}}{N_t}. \tag{A17}$$

The pre-war capital stock in period $t+1$, K_{t+1}^p, is made up of aggregate savings by the young in the period t, $s_t N_t$. Using $s_t = \beta [w_t + r_{L,t} l_t]$ [see (15)] we can thus write $K_{t+1}^p = s_t N_t = \beta [w_t + r_{L,t} l_t] N_t$; together with (A16) and (A17) this gives (33).

C The Malthusian balanced growth path

We define the Malthusian balanced growth path as the path that the economy converges to if we close down the Solow sector permanently, and assume away war. The wage and interest rates must be constant on the balanced growth path, and population (i.e. labor) and capital must grow at constant rates, here denoted n^* and g_K^*. The first equality in (6) and $A_{Mt+1} = \gamma_M A_{Mt}$ imply that

$$1 = \gamma_M \left(g_K^*\right)^{\phi} \left(n^*\right)^{\mu-1}, \tag{A18}$$

and since the real interest rate is also constant, the first equality in (7) and $A_{Mt+1} = \gamma_M A_{Mt}$ imply that

$$1 = \gamma_M \left(g_K^*\right)^{\phi-1} \left(n^*\right)^{\mu}. \tag{A19}$$

Together, (A18) and (A19) imply that

$$n^* = g_K^* = \gamma_M^{\frac{1}{1-\mu-\phi}}. \tag{A20}$$

D Initial conditions

We choose initial labor and capital, N_0 and K_0, so that the economy starts off on a Malthusian balanced growth path (as defined in Section C above). Since the Solow sector is not operative ($z_N = z_K = 0$), we can use (A15) and (33) to see that the growth rate of K_t in the first period is given by

$$g_{K,1} = \frac{K_1}{K_0} = \beta \left(1-\phi\right) \left(1-\tau\right) A_{M,0} K_0^{\phi} N_0^{\mu-1} L_0^{1-\mu-\phi}. \tag{A21}$$

(Recall that we are considering a no-war balanced growth path, where $K_t^p = K_t$ in all periods.) From (6) we can write the initial wage rate as:

$$w_0 = \mu(1-\tau) A_{M,0} K_0^{\phi} N_0^{\mu-1} L_0^{1-\mu-\phi}. \tag{A22}$$

Denote the wage rate on the Malthusian balanced growth path by w^*. We now want to choose K_0 and N_0 so that $w_0 = w^*$ and $g_{K,1} = g_K^* = n^*$ [recall (A20)]. Using (A21) and (A22) we get

$$K_0 = \left(\frac{w^*}{Rn^*}\right) N_0, \tag{A23}$$

where

$$R = \frac{\mu}{\beta(1-\phi)}. \tag{A24}$$

Substituting (A23) into (A22), and again using $w_0 = w^*$, gives initial population:

$$N_0 = L_0 \left[\left(\frac{\mu(1-\tau)A_{M,0}}{w^*}\right) \left(\frac{w^*}{Rn^*}\right)^{\phi} \right]^{\frac{1}{1-\mu-\phi}}. \tag{A25}$$

Together, (A23) and (A25) then give the initial capital stock, K_0.

E Deriving A_S^{crit}

The Solow sector becomes active when V_t in (11) falls below unity. From (11) we learn that this occurs when

$$A_{S,t} \geq \left(\frac{\phi}{1-\mu}\right)^{1-\mu} \left[A_{M,t}\left(\frac{L_t}{K_t}\right)^{1-\mu-\phi}\right]. \tag{A26}$$

Recall that we have chosen K_0 and N_0 so that the economy starts off on a Malthusian balanced growth path, where K_t and N_t grow at the same rate, $n^* = g_K^*$ [recall (A20)]. From (A23) it follows that

$$\frac{K_t}{N_t} = \frac{K_0}{N_0} = \frac{w^*}{Rn^*} \tag{A27}$$

in all periods while the Solow sector is non-operative. Inserting (A27) into (A26) we get

$$A_{S,t} \geq \left(\frac{\phi}{1-\mu}\right)^{1-\mu} \left(\frac{Rn^*}{w^*}\right)^{1-\mu-\phi} \left[A_{M,t}\left(\frac{N_t}{K_t}\right)^{1-\mu-\phi}\right]. \tag{A28}$$

Next we can use the expression for w^* on the Malthusian balanced growth path, and (A27), to write

$$w^* = \mu(1-\tau)A_{M,t}K_t^{\phi}N_t^{\mu-1}L_t^{1-\mu-\phi} = \mu(1-\tau)A_{M,t}\left[\frac{w^*}{Rn^*}\right]^{\phi} N_t^{\phi+\mu-1}L_t^{1-\mu-\phi}, \tag{A29}$$

or

$$A_{M,t}\left(\frac{L_t}{Nt}\right)^{1-\mu-\phi} = \frac{w^*}{(1-\tau)\mu}\left(\frac{Rn^*}{w^*}\right)^{\phi}. \tag{A30}$$

Inserting (A30) into (A28) we see that the Solow sector becomes active when $A_{S,t} \geq A_S^{\text{crit}}$, where A_S^{crit} is given in (34).

References

[1] Alesina, A., Spolaore, E., 2003, The Size of Nations. MIT Press, Cambridge, Massachusetts.

[2] Benhabib, J., Rustichini, A., 1996, Social conflict and growth, Journal of Economic Growth 1, 125-142.

[3] Bridgman, B., 2008, Why are ethnically divided countries poor?, Journal of Macroeconomics 30, pp. 1-18.

[4] Broadberry, S., Howlett, P., 1998, The United Kingdom: 'Victory at all Costs,' pp. 43-73. In: Harrison, M. (Ed.), The Economics of World War II. Cambridge University Press, Cambridge.

[5] Browning, P., 2002, The Changing Nature of Warfare – The Development of Land Warfare from 1792 to 1945. Cambridge University Press, Cambridge, United Kingdom.

[6] Cervellati, M., Sunde, U., 2005, Human capital formation, life expectancy, and the process of development, American Economic Review 95, 1653-1672.

[7] Collier, P., Hoeffler, A., 1998, On economic causes of civil war, Oxford Economic Papers 50, 563-573.

[8] —–, 2004, Greed and grievance in civil wars, Oxford Economic Papers 56, 563-595.

[9] Cook, D., 2002, World War II and convergence, Review of Economics and Statistics 84, 131-138.

[10] Dal Bó, E., Powell, R., 2006, Conflict and compromise in hard times, Institute of Governmental Studies Working Paper 2006-33.

[11] Doepke, M., 2004, Accounting for fertility decline during the transition to growth, Journal of Economic Growth 9, 347-383.

[12] Easterly, W., 2006, The White Man's Burden – Why the West's Efforts to Aid the Rest Have Done So Much Ill and So Little Good. Penguin Books, New York.

[13] Easterly, W., Levine, R., 1997, Africa's growth tragedy: policies and ethnic divisions, Quarterly Journal of Economics 112, 1203-1250.

[14] Easterly, W., Gatti, R., Kurlat, S. , 2006, Development, democracy, and mass killings, Journal of Economic Growth 11, 129-156.

[15] Ferguson, N., 1999, The Pity of War – Explaining World War I. Basic Books, New York.

[16] —–, 2006, The War of the World – Twentieth-Century Conflict and the Descent of the West. The Penguin Press, New York.

[17] Flath, D., 2005, The Japanese Economy, Second Edition. Oxford University Press, Oxford, England.

[18] Galor, O., Moav, O. , 2002, Natural selection and the origin of economic growth, Quarterly Journal of Economics 117, 1133-1192.

[19] Galor, O., Weil, D., 2000, Population, technology, and growth: from Malthusian stagnation to the demographic transition and beyond. American Economic Review 90, 806-829.

[20] Gollin, D., Parente, S., Rogerson, R., 2002, The role of agriculture in development, American Economic Review Papers and Proceedings 92, 160-164.

[21] Gordon, R.J., 2008, Did economics cause World War II?, NBER Working Paper 14560.

[22] Grossman, H.I., 1991, A general equilibrium model of insurrections, American Economic Review 81, 913-921.

[23] Grossman, H.I., Kim, M., 1995, Swords and plowshares? A theory of the security of claims to property, Journal of Political Economy 103, 1275-1288.

[24] Grossman, H., Mendoza, J., 2003, Scarcity and appropriative competition, European Journal of Political Economy 19, 747-758.

[25] Hansen, N., 2003, The Confident Hope of a Miracle – The True Story of the Spanish Armada. Alfred A. Knopf, New York.

[26] Hansen, G.D., Prescott, E.C., 2002, Malthus to Solow, American Economic Review 92, 1205-1217.

[27] Hewitson, M., 2004, Germany and the Causes of the First World War. Berg Publishers, New York.

[28] Hirshleifer, J., 1988, The analytics of continuing conflict, Synthese 76, 201-233.

[29] —–, 2001, The Dark Side of the Force: Economic Foundations of Conflict Theory. Cambridge University Press.

[30] Iyigun, M., 2008, Luther and Suleyman, Quarterly Journal of Economics 123, 1465-1494.

[31] Johnson, N., Spagat, M., Restrepo, J., Bohórquez, J., Suárez, N., Restrepo, E., Zarama, R., 2006, Universal patterns underlying ongoing wars and terrorism, manuscript, Royal Holloway.

[32] Jones, C.I., 2001, Was the industrial revolution inevitable? Economic growth over the very long run, Advances in Macroeconomics 1, 1-43.

[33] Lagerlöf, N.P., 2003a, From Malthus to modern growth: can epidemics explain the three regimes?, International Economic Review 44, 755-777.

[34] —–, 2003b, Mortality and early growth in England, Sweden, and France, Scandinavian Journal of Economics 105, 419-439.

[35] —–, 2006, The Galor-Weil model revisited: a quantitative exercise, Review of Economic Dynamics 9, 116-142.

[36] —–, 2009, From Malthusian War to Solovian Peace – Supplementary Notes, mimeo, York University.

[37] Levy, J., 1983, War in the Modern Great Power System, 1495-1975. The University Press of Kentucky, Lexington, Kentucky.

[38] Lucas, R.E., 2002, Lectures on Economic Growth. Harvard University Press, Cambridge.

[39] Maddison, A., 2003, The World Economy – Historical Statistics, CD-ROM. Development Centre of the Organisation for Economic Co-operation and Development, Paris, France.

[40] Martin, P., Mayer, T.M., Thoenig, M., 2008, Make trade not war?, Review of Economic Studies 75, 865-900.

[41] Matsuyama, K., 1992, Agricultural productivity, comparative advantage, and economic growth, Journal of Economic Theory 58, 317-334.

[42] Ngai, L.R., 2004, Barriers and the transition to modern growth, Journal of Monetary Economics 51, 1353-1383.

[43] Santaeulalia-Llopis, R., 2008, Aggregate effects of AIDS on development, mimeo, Washington University.

[44] Tamura, R., 1996, From decay to growth: a demographic transition to economic growth, Journal of Economic Dynamics and Control 20, 1237-1261.

[45] ––—, 2002, Human capital and the switch from agriculture to industry, Journal of Economic Dynamics and Control 27, 207-242.

[46] Voigtländer, N., Voth, H.J., 2006, Why England? Demographic factors, structural change and physical capital accumulation during the Industrial Revolution, Journal of Economic Growth 11, 319-361.

[47] Yared, P., 2009, A dynamic theory of concessions and war, mimeo, Columbia Business School.

Parameter	**Value**	**Comment**
μ	0.6	Labor share in Solow and Malthus; same as HP
ϕ	0.1	Land share in Malthus; same as HP
γ_M	1.032	Growth rate Malthus; same as HP
γ_S	1.518	Growth rate Solow; same as HP
τ	0.01	Tax rate; arbitrary, does not affect results
$\overline{n}$	1.5	Max fertility
w^*	1	Malthusian wage rate; normalized
η	1.5	From fertility function; Dem. Trans. as in HP
ν	7	From fertility function; Dem. Trans. as in HP
β	0.5	Relative weight on fertility; same as HP
θ	1	Weight on Solow income in gov't's obj. function
T	5	Periods before transition; same as HP
ζ	1	Constant destruction/killing rates in war
$\widehat{\delta}_K$	0.6	Capital destruction in war is 0.3 per period
$\widehat{\delta}_N$	0.12	Death rate in war is 0.06 per period

Initial condition	**Value**	**Comment**
$A_{M,0}$	1	Arbitrary
L_0	0.5	Countries have equal initial holdings of unit-sized territory
N_0	0.0773	Start off on Malthusian balanced growth path
K_0	0.0522	Start off on Malthusian balanced growth path
$A_{S,0}$	0.1404	γ_M^{-T} times level of A_S where Solow sector becomes active

Table 1: Baseline parameter values and initial conditions.

Period	Great Power	Half	Third
1495-1700	England	0.35	0.85
1495-1700	France	0.59	0.74
1495-1800	England	0.45	0.90
1495-1800	France	0.62	0.83

Table 2: Fraction 35-year periods with Great Power war during more than half (17.5) and more than a third (11.7) of those 35 years. See text for discussion.

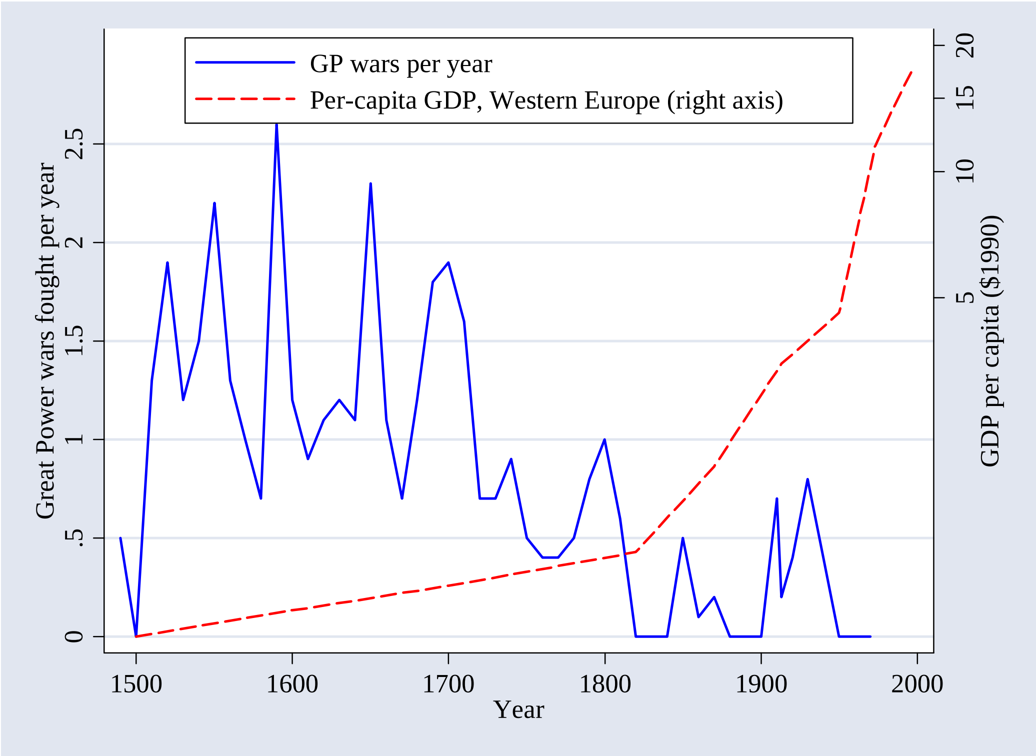

Figure 1(a) Trends in the overall frequency of Great Power wars and per-capita income.

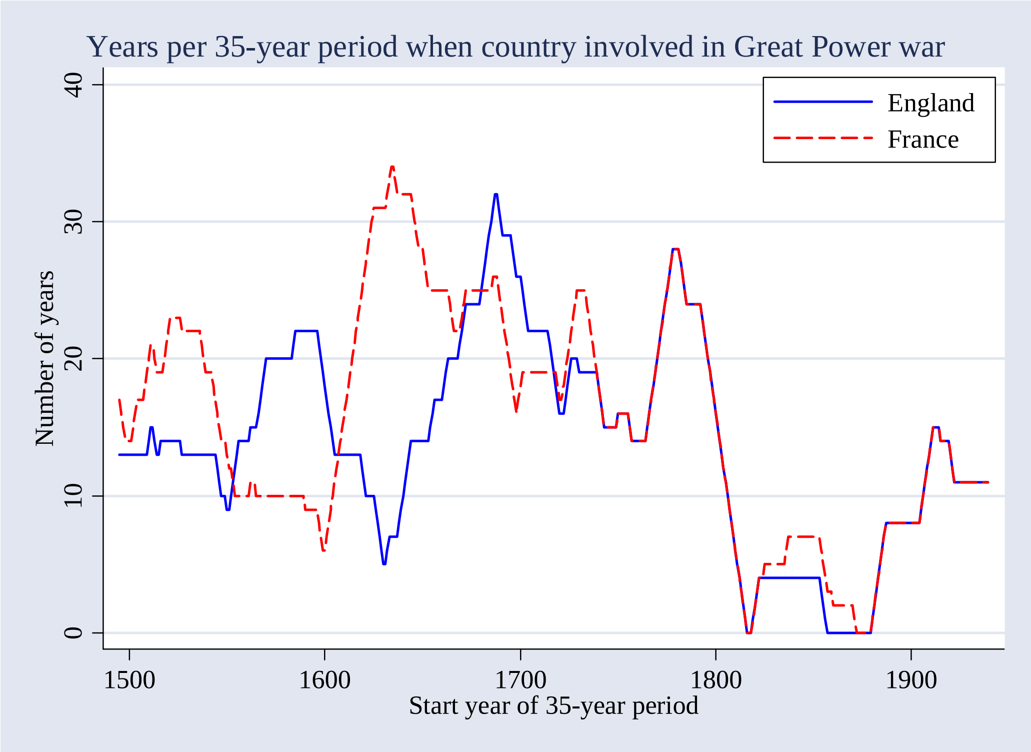

Figure 1(b) Years in Great Power wars for England and France over 35-year periods.

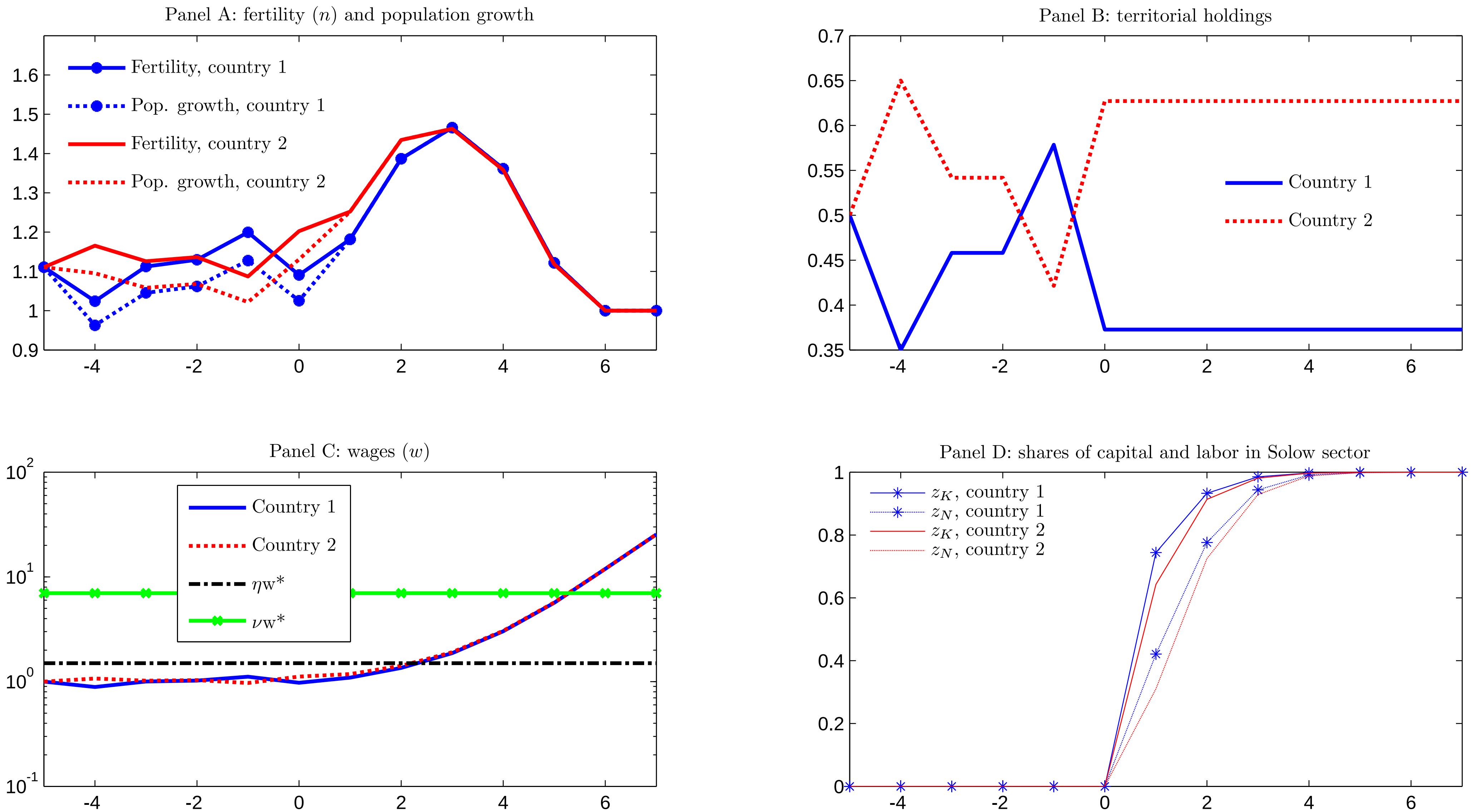

Figure 2: One single run.

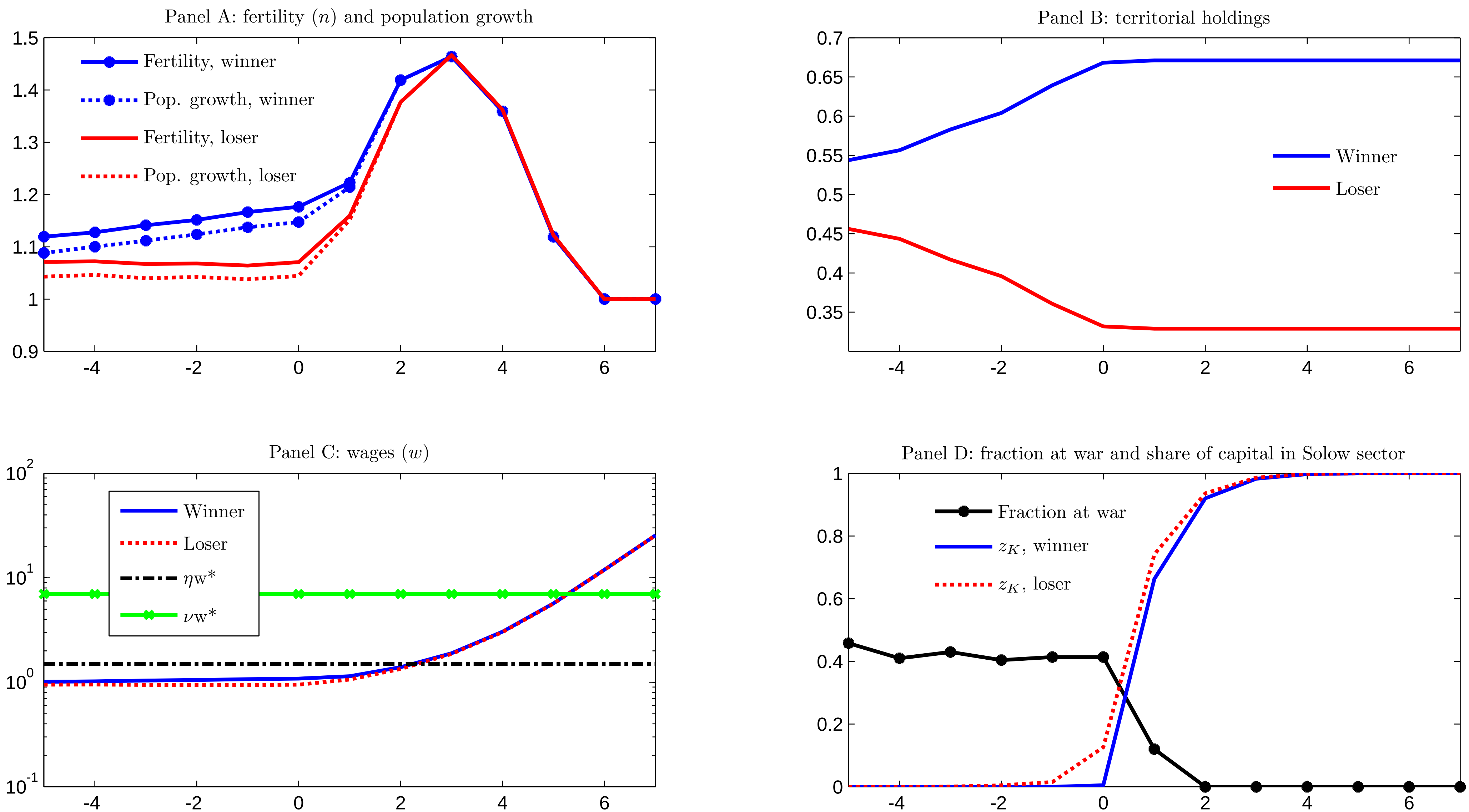

Figure 3: Monte Carlo simulations.

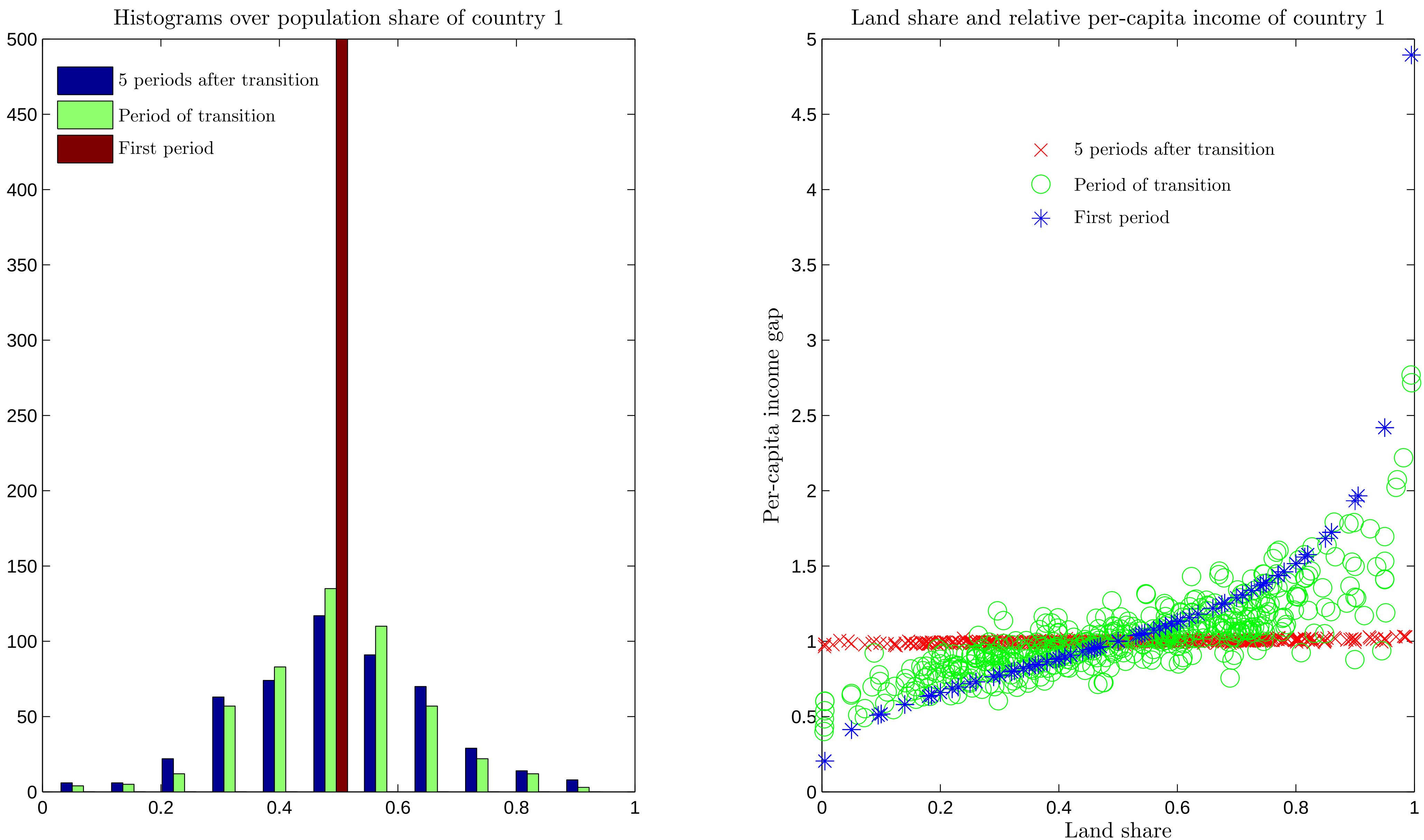

Figure 4: The dynamics of the land and population distribution.

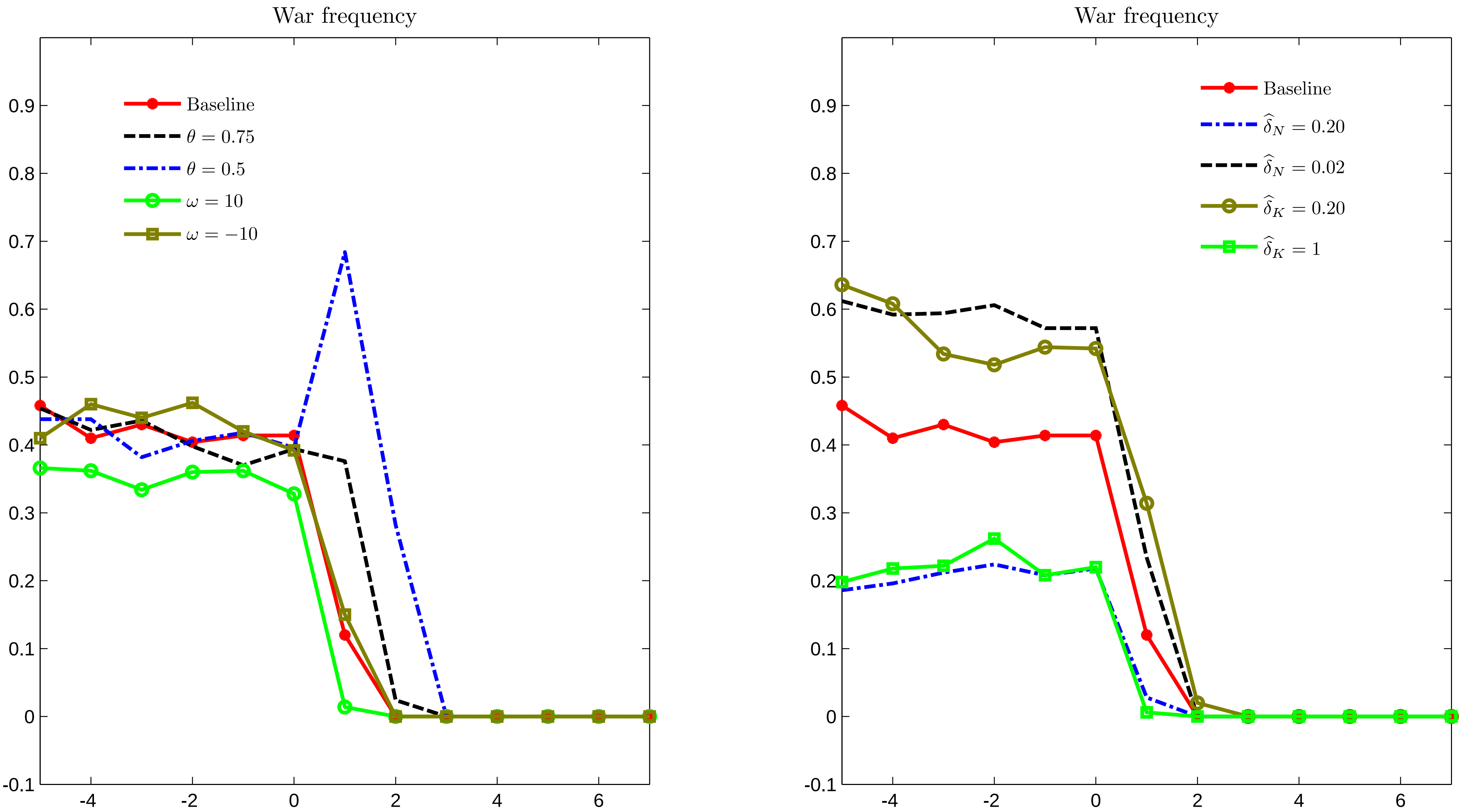

Figure 5: Results when changing ω, θ, $\widehat{\delta}_N$, and $\widehat{\delta}_K$.

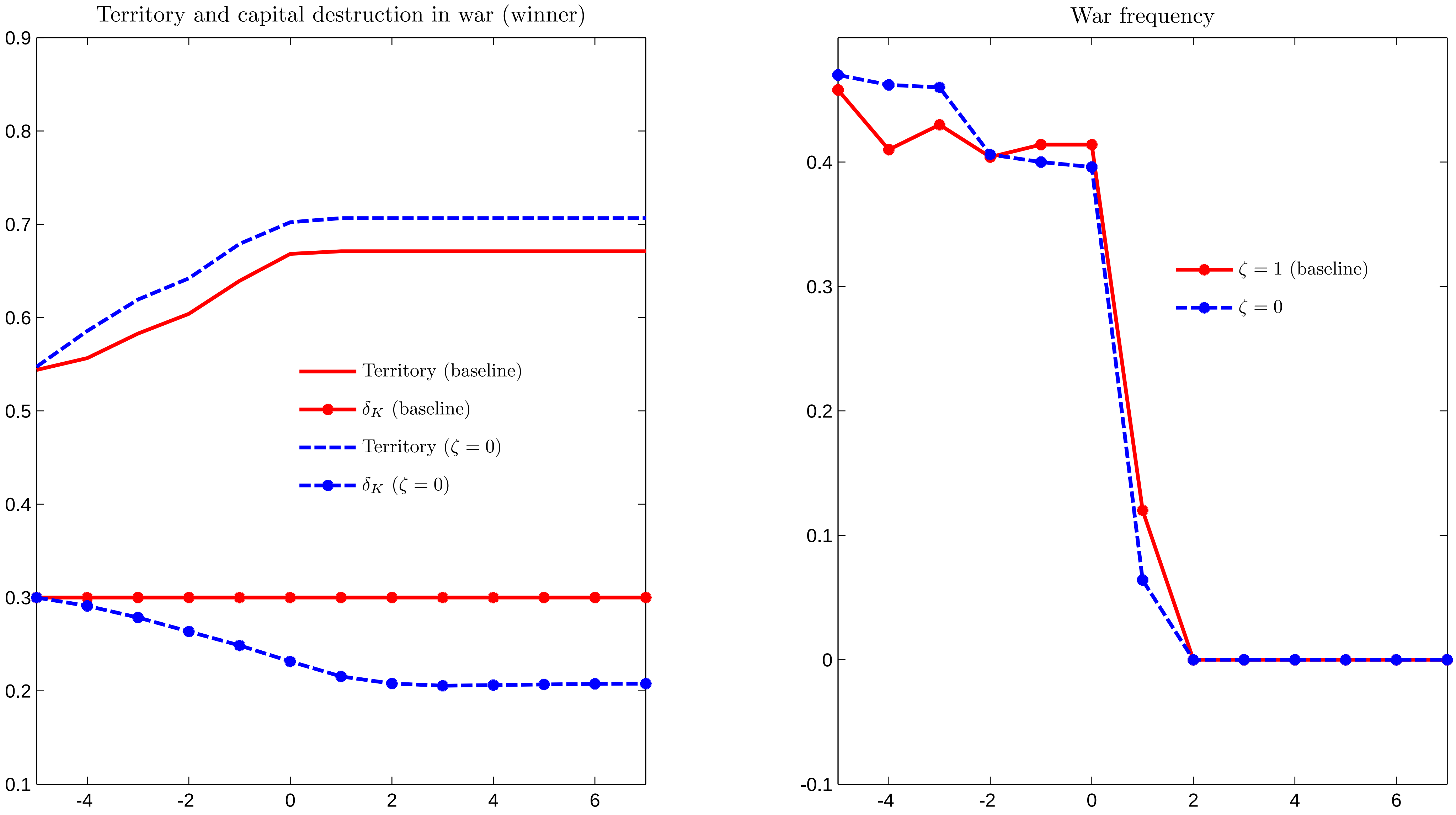

Figure 6: letting the costs of war depend in K and N by setting $\zeta = 0$.

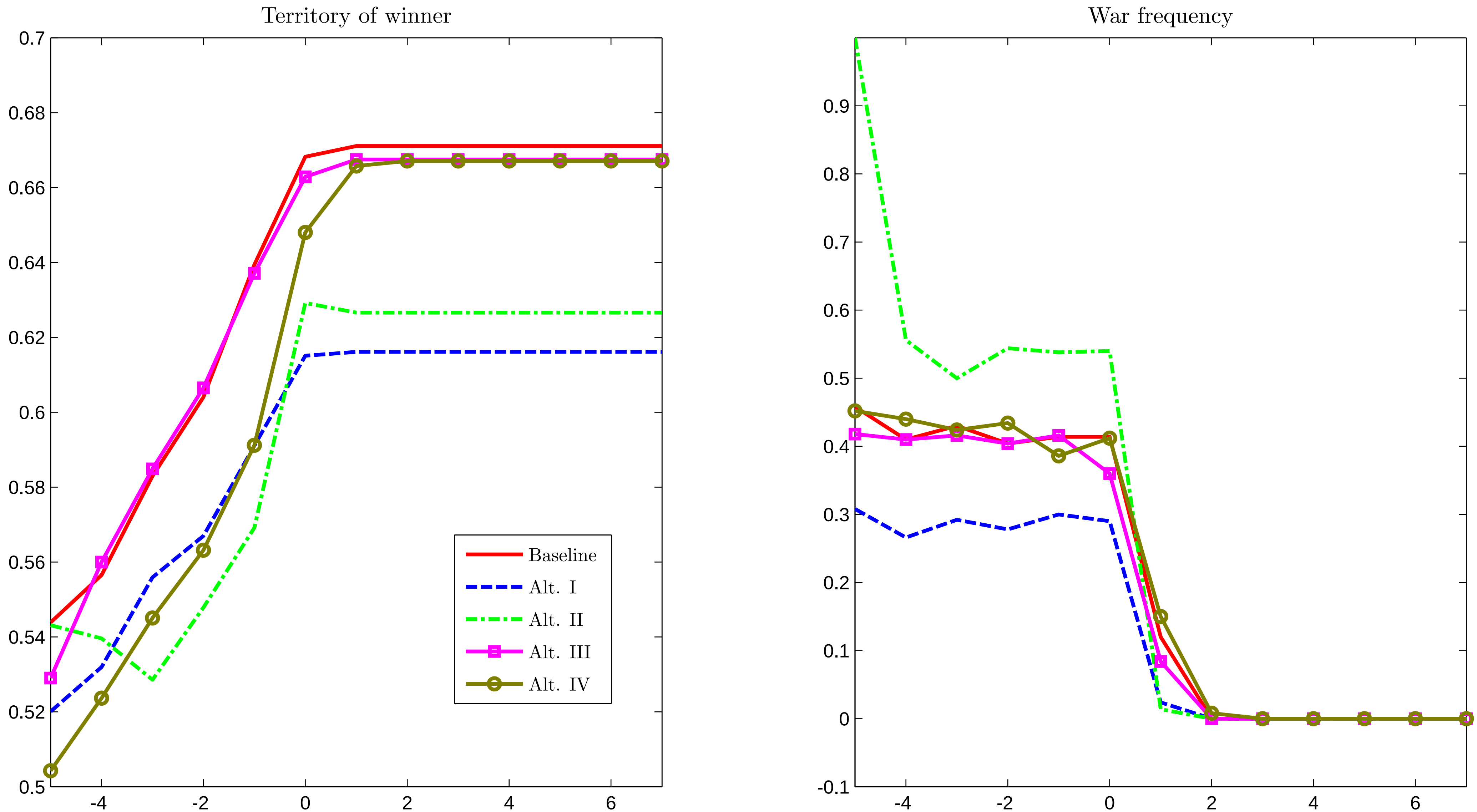

Figure 7: Different forms for the conquest function.