Final Exam for Econ 1530 Sections C and D
York University
20 December 2005

First name:

Last name:

SID number:

Instructions: Write your name and SID number above; circle your answer below, *and* fill out the same answer on the bubble sheet (as well as SID number and name).

Problem 1. A quantity of K increases by 1% per year (with annual compounding). After approximately how many years has the quantity increased to $2K$?
(a) 70 years
(b) 20 years
(c) 2 years
(d) 1/2 a year

Problem 2. Let $S = \sum_{i=1}^{n} i$. Then S equals:
(a) $n(n-1)/2$
(b) $n(n+1)/2$
(c) n
(d) none of the above

Problem 3. The graph of the linear function $f(x)$ passes through the points $(x_1, y_1) = (0, 2)$ and $(x_2, y_2) = (1, 1)$. Which is the function?
(a) $f(x) = 2 - x$
(b) $f(x) = \alpha - x$
(c) $f(x) = 1 - x$
(d) none of the above

Problem 4. If $f'(x) \geq 0$ and $f''(x) \leq 0$, what of the below describes f?
(a) f is increasing and convex
(b) f is decreasing and concave
(c) f is decreasing and convex
(d) f is increasing and concave

Problem 5. The function $f(x)$ is such that $f'(x) = -2$ for all x, and passes through the point $(x_1, y_1) = (1, 4)$. Which is the function?
(a) $f(x) = 1 - x$
(b) $f(x) = -x^2$
(c) $f(x) = 2(3 - x)$
(d) none of the above

Problem 6. Let $f(x) = \sqrt{x}$. Which of the below gives a linear approximation of $f(x)$ about $x = 1$?[1]
(a) $f(a) + \sqrt{a}(x - a)$
(b) $(1 - x)/2$
(c) $(1 + x)/2$
(d) none of the above

Problem 7: Let $F(x) = G(y)$. Which one of the below options gives $\frac{dy}{dx}$?
(a) $dy/dx = G'(y)/F'(x)$
(b) $dy/dx = -F'(y)/G'(x)$
(c) $dy/dx = F'(x)/G'(y)$
(d) none of the above

Problem 8: Let $f(x) = x(\ln x - 1)$. Which of the below gives the second derivative of $f(x)$?

(a) $f''(x) = \frac{1}{(1-x)^2}\left(\frac{1-x}{x} + \ln x\right)$

(b) $f''(x) = \frac{1}{x}$

(c) $f''(x) = \frac{-1}{x^2}$

(d) none of the above

Problem 9: Let $U(C) = -e^{-\gamma C}$, where $\gamma > 0$ is a constant. What below gives $A(C) = \frac{-U''(C)}{U'(C)}$?
(a) $A(C) = 1/(1 - \gamma C)$
(b) $A(C) = \gamma/(1 - \gamma C)$
(c) $A(C) = \gamma$
(d) none of the above

Problem 10: Let $F(x) = 1 - e^{-ax}$, where $a > 0$ is a constant. Let $G(z) = F'(\ln z)$. Which of the below gives $G'(z)$?
(a) $G'(z) = -a^2 z^{-(1+a)}$
(b) $G'(z) = az^{-a}$
(c) $G'(z) = -a^2$
(d) none of the above

Problem 11: Let $f(x) = \frac{1 - \sqrt{1 - \gamma x}}{x}$, where $\gamma > 0$ is a constant. Which one of the below options gives $\lim_{x \to 0} f(x)$?
(a) γ
(b) $1/\gamma$
(c) $\gamma/2$
(d) none of the above

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[1]Recall that a linear approximation of $f(x)$ about $x = a$ is given by $f(a) + f'(a)[x - a]$.

Problem 12: Let the function $f(x) = -(1+x)^2$ be defined on the interval $[0, 1]$. Which of the below gives the maximum point for $f(x)$?
(a) $x = -1$
(b) $x = 0$
(c) $x = 1$
(d) none of the above

Problem 13: Figure 1 shows the graph of $f'(x)$. Which of the below statements is the correct one? [Note that the graph shows $f'(x)$, not $f(x)$.]
(a) $x = 16$ is a local maximum point for $f(x)$
(b) $x = 0$ is a local maximum point for $f(x)$
(c) $x = 20$ is a local minimum point for $f(x)$
(d) $x = 8$ is a local maximum point for $f(x)$

Problem 14: Let $f(x) = 2x - \frac{x^2}{2}$. Which of the below gives the maximum point for $f(x)$?
(a) $x = 2$
(b) $x = 1$
(c) $x = 0$
(d) none of the above

Problem 15: Let the function $f(x)$ be such that $f''(a) = -2$ and $f'(a) = 0$ for some a. Which of the below is true?
(a) $x = 0$ is a local maximum point for $f(x)$
(b) $x = 0$ is a local minimum point for $f(x)$
(c) $x = a$ is a local maximum point for $f(x)$
(d) $x = -2$ is a local minimum point for $f(x)$

Problem 16: Let $f(x) = e^x - x$. Which of the below gives the minimum point for $f(x)$?
(a) $x = 1$
(b) $x = e$
(c) $x = 0$
(d) none of the above

Problem 17: Let $f(x) = -x + \ln x$. Which one of the below statements is correct?
(a) $x = e$ is a maximum point for $f(x)$
(b) $x = 1$ is a maximum point for $f(x)$
(c) $x = 1$ is a minimum point for $f(x)$
(d) none of the above

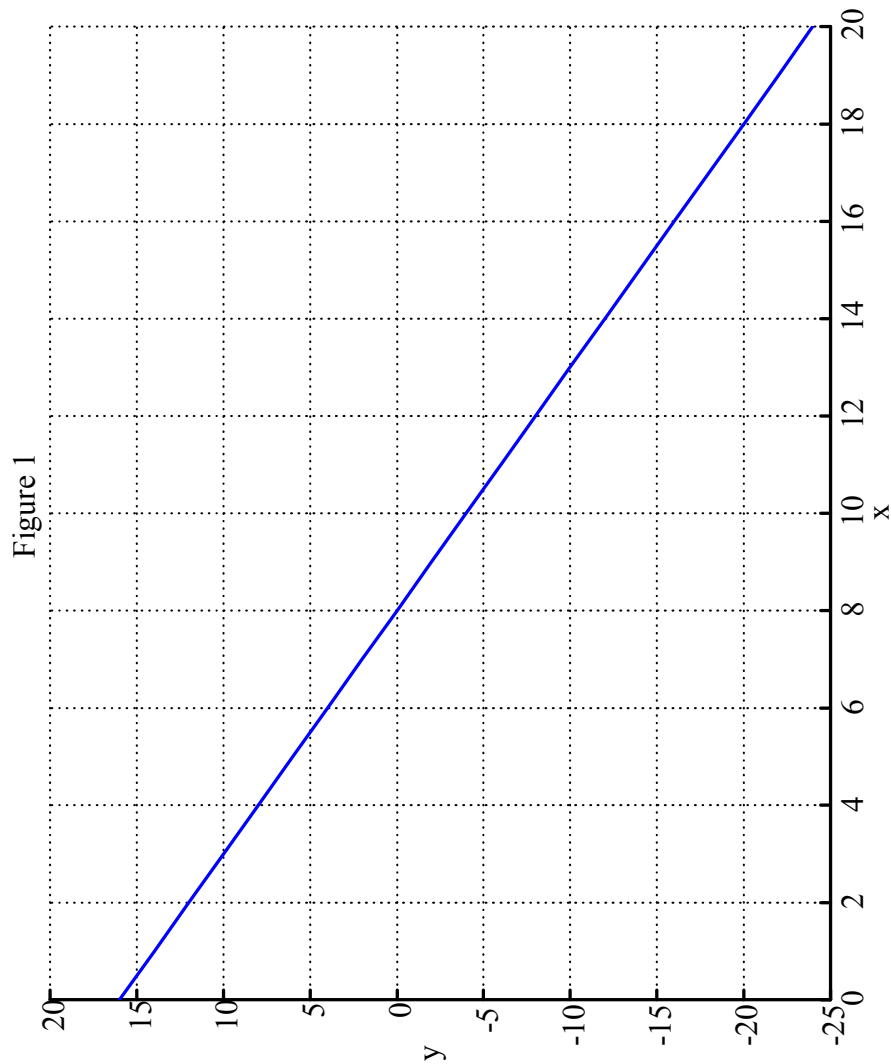**Problem 18:** Which of the below gives $\int x^{a-1} dx$ for $a \neq 0$? (C is a constant.)
(a) $x^a/a + C$
(b) $(a + x)^{a-1} + C$
(c) $(a - 1)x^{a-2} + C$
(d) none of the above

Problem 19: Which of the below equals $\int_0^1 e^x(1+x)dx$? *Hint:* what is the derivative of xe^x?
(a) 0
(b) 1
(c) 23
(d) none of the above

Problem 20: Let $G(a) = \int_1^{1+a} 2(x-1)dx$. Which of the below gives $G'(a)$?
(a) $G'(a) = 0$
(b) $G'(a) = a \ln a$
(c) $G'(a) = 2a$
(d) none of the above

Figure 1

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Solutions

Problem 1: (a)

Problem 2: (b)

Problem 3: (a)

Problem 4: (d)

Problem 5: (c)

Problem 6: (c)

Problem 7: (c)

Problem 8: (b)

Problem 9: (c)

Problem 10: (a)

Problem 11: (c)

Problem 12: (b)

Problem 13: (d)

Problem 14: (a)

Problem 15: (c)

Problem 16: (c)

Problem 17: (b)

Problem 18: (a)

Problem 19: (d) Right answer is e

Problem 20: (c)