Practice problems for Econ 1530 Sections C and D
York University
Fall 2005

Problem 1. A quantity K grows by 2% per year (with annual compounding). After 4 years the quantity has increased to:

(a) $K(1.02)^4$
(b) $K(1.04)^2$
(c) $K(1.02)^2$
(d) $K(1.04)^4$

Problem 2. A quantity K grows by $p\%$ per year, every year for 110 years (with annual compounding). At the end of this 110 year period the quantity has tripled (that is, it has grown to $3K$). The growth rate is closest to which of the below options?

(a) $p = 0$
(b) $p = 1$
(c) $p = 2$
(d) $p = 3$

Problem 3. The expression $ax^2 + 4y\left(y - x\sqrt{a}\right)$ can be written as which of the following?

(a) $[ax - 2y]^2$

(b) $[x\sqrt{a} + 2y]\,[x\sqrt{a} - 2y]$

(c) $[x\sqrt{a} - 2y]^2$

(d) none of the above

Problem 4. The inequality $a - b > -1$ can be written as which of these inequalities?
(a) $b - a > -1$
(b) $b - a < 1$
(c) $b + a > -1$
(d) none of the above

Problem 5. The solution to the equation $ax + b = a\,(1 - x) + 2$ can be written as:

(a) $x = \frac{1}{a} + \frac{1}{2}\frac{a-b}{a}$

(b) $x = \frac{2+a-b}{2}$

(c) $x = \frac{2+a-b}{a}$

(d) none of the above

Problem 6. The solution(s) to the equation $x^2 + bx = 0$ is (are):
(a) $x_1 = 1/b$ and $x_2 = 0$
(b) $x_1 = -b$ and $x_2 = 0$
(c) $x = 0$ is the only solution
(d) the equation has no solution

Problem 7. Let the mean of a population be $\overline{x} = \frac{1}{T}\sum_{i=1}^{T} x_i$, and the sum of squared deviations from the mean be $S = \sum_{i=1}^{T}(x_i - \overline{x})^2$. Then S can be written as:
(a) $\left[\sum_{i=1}^{T}(\overline{x} - x_i)\right] - T\overline{x}$

(b) $\left(\sum_{i=1}^{T} x_i^2\right) - \overline{x}^2$

(c) $\left(\sum_{i=1}^{T} x_i^2\right) + \overline{x}^2$

(d) $\left(\sum_{i=1}^{T} x_i^2\right) - T\overline{x}^2$

Problem 8. Let $f(x) = \frac{1}{1-\sqrt{x}}$. Which one of the below numbers belongs to the domain of f?
(a) -2
(b) -1
(c) 0
(d) 1

Problem 9. The graph of the linear function $f(x)$ passes through the points $(x_1, y_1) = (0, 1)$ and $(x_2, y_2) = (1, \alpha)$. Which of the below is true?
(a) $f(x) = 1 + (1 - \alpha)x$
(b) $f(x) = 1 + \alpha x$
(c) $f(x) = 1 + cx$
(d) $f(x) = 1 - (1 - \alpha)x$

Problem 10: Rewriting $200y - 120x - 100 = 0$ on the form $y = ax + b$, gives:
(a) $a = 0; b = 200$
(b) $a = 3/5; b = 1/2$
(c) $a = 100; b = 5$
(d) $a = 1; b = 1 - a$

Problem 11: The line in Figure 1 is the graph of which function?
(a) $y = 2 - x$
(b) $y = x + 1$
(c) $y = 2x + x$
(d) $y = 2x + 1$

Problem 12: The graph in Figure 2 refers to a function $f(x) = ax^2 + bx + c$. Which option below is true about a, b, and c?
(a) $a > 0$, $b^2 > 4ac$
(b) $a > 0$, $b^2 < 4ac$
(c) $a < 0$, $b^2 > 4ac$
(d) $a < 0$, $b^2 < 4ac$

Problem 13: Which are the coordinates of the intersection of the two lines in Figure 3?
(a) $x = 0.25$, $y = 1.6$
(b) $x = 0.324$, $y = 1.623$
(c) $x = 1/3$, $y = 5/3$
(d) $x = 12/33$, $y = 5/3$

Problem 14: Let $f(x) = B^{1-\alpha}e^{\alpha x}$, for $B > 0$. Then $f(\ln B)$ equals what of the below?

(a) $(\ln B)\, e^{\alpha B}$

(b) $B^{1-\alpha}e^{\alpha B}$

(c) B^{α}

(d) None of the above

Problem 15: What does $e^a e^b + \ln(ab)$ equal?
(a) $e^{a+b} + \ln a + \ln b$
(b) $e^{a+b}(\ln a + \ln b)$
(c) $(e^a)^b + \ln a + \ln b$
(d) None of the above

Problem 16: Which of the following best describes a one-to-one function f?
(a) x never equals $f(x)$
(b) $f(x)$ never takes the same value for different values of x
(c) $f(1) = 1$
(d) $f^{-1}(x)$ is strictly increasing over its domain

Problem 17: Let $f(x) = a - \ln x$. Which one of the below functions is inverse of f?
(a) $g(y) = [a - \ln y]^{-1}$
(b) $g(y) = -a + \ln y$
(c) $g(y) = e^a/e^y$
(d) none of the above

Problem 18: Let $g(x)$ be the inverse of $f(x)$. Which of the below is always true?
(a) $g(f(x)) = x$ for all x in the domain of f
(b) $g'(f(x)) > 0$ for all x in the domain of f
(c) $g'(f(x))f'(x) = 0$ for all x in the domain of f
(d) none of the above

Problem 19: Let $U = x^{\alpha}y^{1-\alpha}$, where $0 < \alpha < 1$ and $U > 0$. Which of the below is true?

(a) $\frac{dy}{dx} = (1-\alpha)y^{-\alpha}x^{\alpha}$

(b) $\frac{dy}{dx} = 1$

(c) $\frac{dy}{dx} = \left(\frac{1}{1-\alpha}\right)\left(\frac{x-1}{x}\right)^{\frac{1}{1-\alpha}}$

(d) $\frac{dy}{dx} = -\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{U}{x}\right)^{\frac{1}{1-\alpha}}$

Problem 20: Which are the functions of the graphs shown in Figure 4?
(a) $f(x) = x(x-1)+2$ and $g(x) = 2(1+x)$
(b) $f(x) = x(x-1)+2$ and $g(x) = 2(1-x)$
(c) $f(x) = x(x-1)+2$ and $g(x) = 6(1-x/3)$
(d) $f(x) = -x(x+1)+2$ and $g(x) = 2-2x$

Problem 21: Figure 5 shows the graphs of f and f^{-1}. What is the function of the third graph?
(a) $y = x-1$
(b) $y = 1-x$
(c) $y = x$
(d) $y = ax$ for any $a > 0$

Problem 22: Panel A in Figure 6 shows the graph of $f(x)$. Which one of the other panels shows the graph of $f'(x)$? (The intervals on the x-axes are the same for all panels.)
(a) Panel B
(b) Panel C
(c) Panel D
(d) none of the above

Problem 23: Let $f(x) = e^{g(x)}$. Which of the below is true?
(a) $f'(x)/g'(x) = f(x)$
(b) $f'(x)g'(x) = 1$
(c) $f'(x) = e^{g(x)}$
(d) none of the above

Problem 24: Let $f(x) = 1-x^2$. Which of the below is true?
(a) f is decreasing and convex on the interval [0,1]
(b) f is decreasing and concave on the interval [0,1]
(c) f is increasing and concave on the interval [0,1]
(d) none of the above

Problem 25: Let $f(x) = ax^2 + bx + c$. If f is concave over its whole domain, which of the below *must* be true?
(a) $a \leq 0$ and $c \geq 0$
(b) $a \geq 0$ and $c \geq 0$
(c) $a \leq 0$ and $b \leq 0$
(d) none of the above

Problem 26: Let $f(x) = \frac{ax^2}{2} + \ln(x^a)$. Which option below gives a correct expression for $f'(x)$?

(a) $f'(x) = \frac{a(1+x^2)}{x}$

(b) $f'(x) = ax + \ln(ax^{a-1})$

(c) $f'(x) = ax + \frac{\ln(ax^{a-1})}{x}$

(d) none of the above

Problem 27: Let $F(x) = f(\frac{1}{1+e^{\alpha x}})$. Which option below gives a correct expression for $F'(x)$?

(a) $F'(x) = f'(\frac{\alpha e^{\alpha x}}{1+e^{\alpha x}})$

(b) $F'(x) = -\frac{\alpha e^{\alpha x}}{1+e^{\alpha x}}$

(c) $F'(x) = -\frac{\alpha e^{\alpha x}}{(1+e^{\alpha x})^2} f'(\frac{1}{1+e^{\alpha x}})$

(d) none of the above

Problem 28: Let $f(x) = \left(\frac{x}{a+x}\right)^b$. Which option below gives a correct expression for $f'(x)$?

(a) $f'(x) = b\left(\frac{x}{a+x}\right)^{b-1}\left(\frac{a}{a+x}\right)$

(b) $f'(x) = b\left(\frac{x}{a+x}\right)^{b-1}$

(c) $f'(x) = \frac{abf(x)}{x(a+x)}$

(d) none of the above

Problem 29: Let $f(x) = \sqrt{x}$. Which option below gives a linear approximation of $f(x)$ about $x = 1$?

(a) $(x+1)/2$
(b) x^2
(c) $x - 0.5$
(d) none of the above

Problem 30: Which option below equals $\lim_{x\to 1}\left(\frac{\ln x}{1-x}\right)$?
(a) 1
(b) 0
(c) ln(2)
(d) −1

Problem 31: Let $f(x) = a + 2xb - b^2 - x^2$. Which of the below gives the maximum point for $f(x)$?
(a) $x = b$
(b) $x = 0$
(c) $x = b \pm \sqrt{a}$
(d) $x = a$

Problem 32: Let $f(x) = \frac{x}{1+x^2}$. Which of the below gives the minimum point for $f(x)$?
(a) $x = 1$
(b) $x = -1$
(c) $x = 0$
(d) none of the above

Problem 33: Let the function $f(x) = 2 - x$ be defined on the interval $[0, 4]$. Which of the below gives the maximum point for $f(x)$?
(a) $x = -1$
(b) $x = 2$
(c) $x = 0$
(d) none of the above

Problem 34: Let the utility function $U(c) = a\sqrt{c}$, where $a > 0$, be defined for all $c \geq 0$. Which of the below gives the maximum point of the function $F(c) = U(c)+U(1-c)$? (Note: for what values of c is F defined?)
(a) $c = 0$
(b) $c = 1$
(c) $c = a$
(d) none of the above

Problem 35: Let the function $f(x)$ have a first derivative $f'(x) = a - x$, where a is a constant. Which of the below is true?
(a) $x = a$ is a local maximum point for f
(b) $x = a$ is a local minimum point for f
(c) $x = 0$ is a local maximum point for f
(d) $x = 0$ is a local minimum point for f

Problem 36: Let the twice differentiable function $f(x)$ be defined for all real numbers x. Assume that $f''(x) < 0$ for all x and that $f(x)$ has a global maximum point at $x = b$. Which of the below is true?
(a) $f'(b) > 0$
(b) $f'(b + a) > 0$ for $a > 0$
(c) $f'(b + a) > 0$ for $a < 0$
(d) none of the above

Problem 37: Let $f(x) = 3x^2$. Which of the below gives $\int f(x)dx$? (C is a constant.)
(a) $x^3 + C$
(b) $3x^2 + C$
(c) $6x + C$
(d) none of the above

Problem 38: Which of the below equals $\int_1^e \ln(x)dx$? *Hint:* what is the derivative of $x\ln(x) - x$?
(a) -1
(b) 1
(c) e
(d) none of the above

Problem 39: Which of the below equals $\int_0^1 [1 - x^2]dx$?
(a) 0
(b) 1/3
(c) 2/3
(d) none of the above

Problem 40: Let $\Omega(z) = \int_a^z \phi(x)dx$. Which of the below gives $\Omega'(z)$?
(a) $\Omega'(z) = \phi'(z)$
(b) $\Omega'(z) = \phi(z)$
(c) $\Omega'(z) = \phi(z - a)$
(d) none of the above

Figure 1

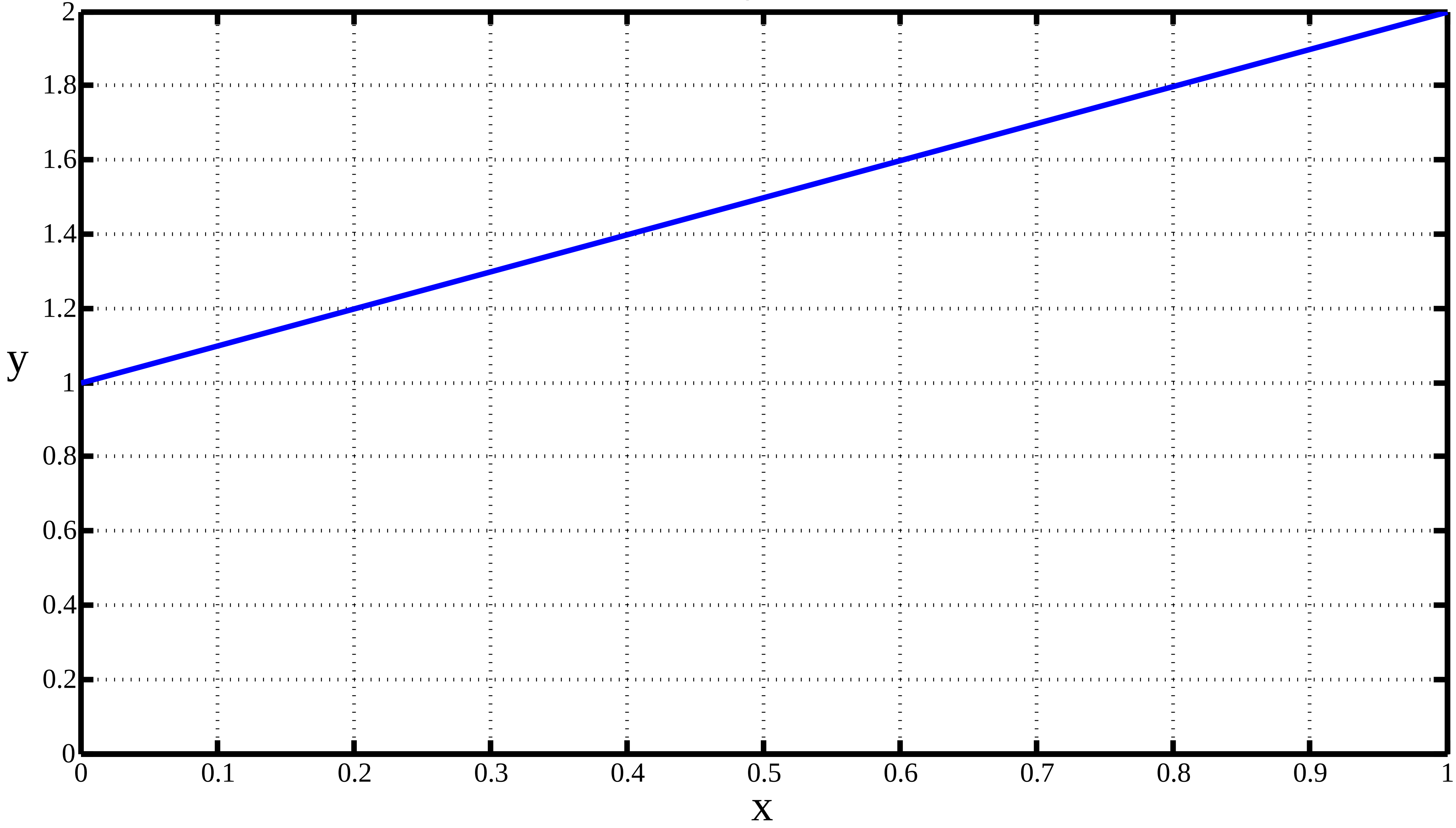


Figure 2

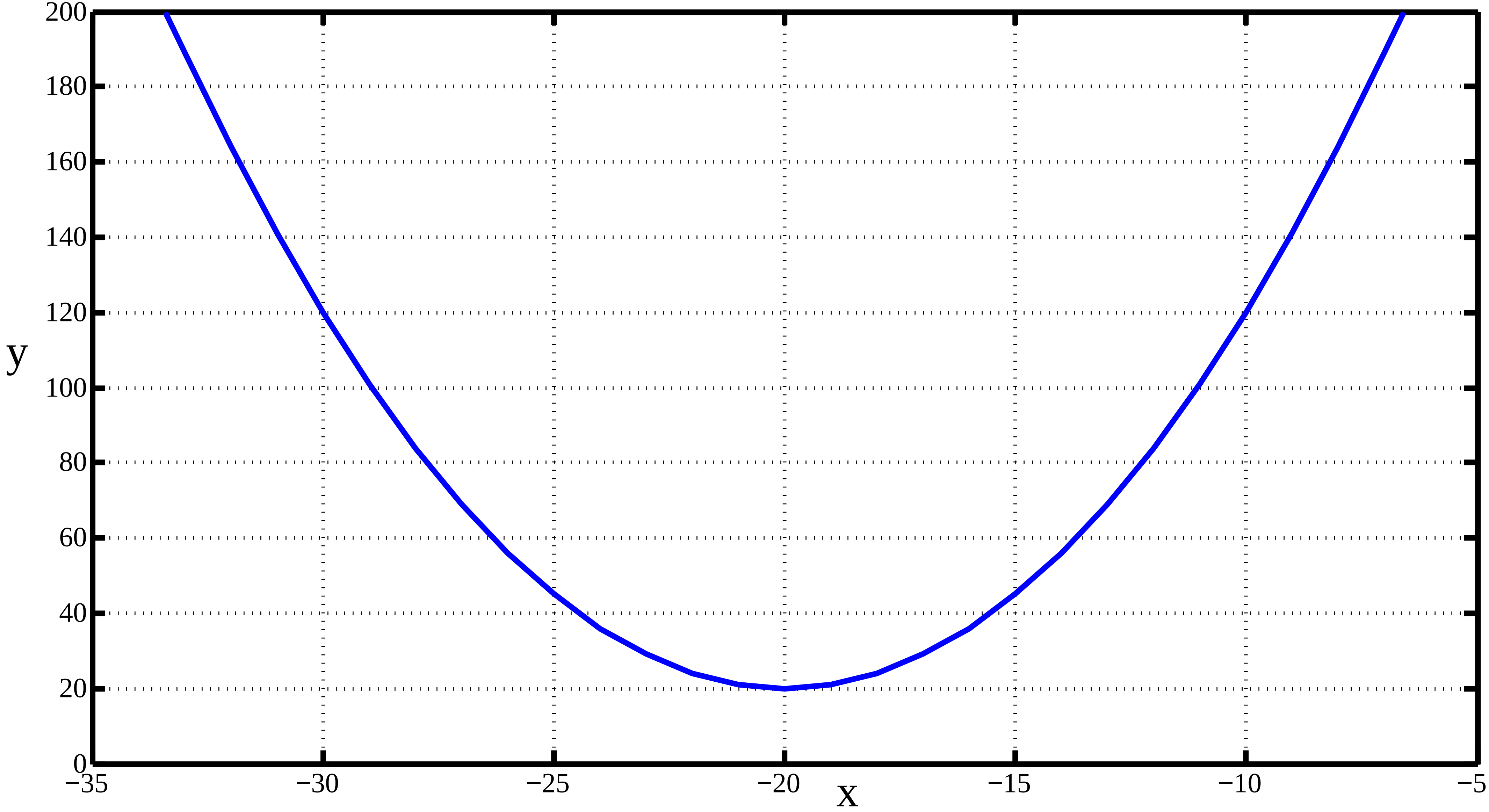


Figure 3

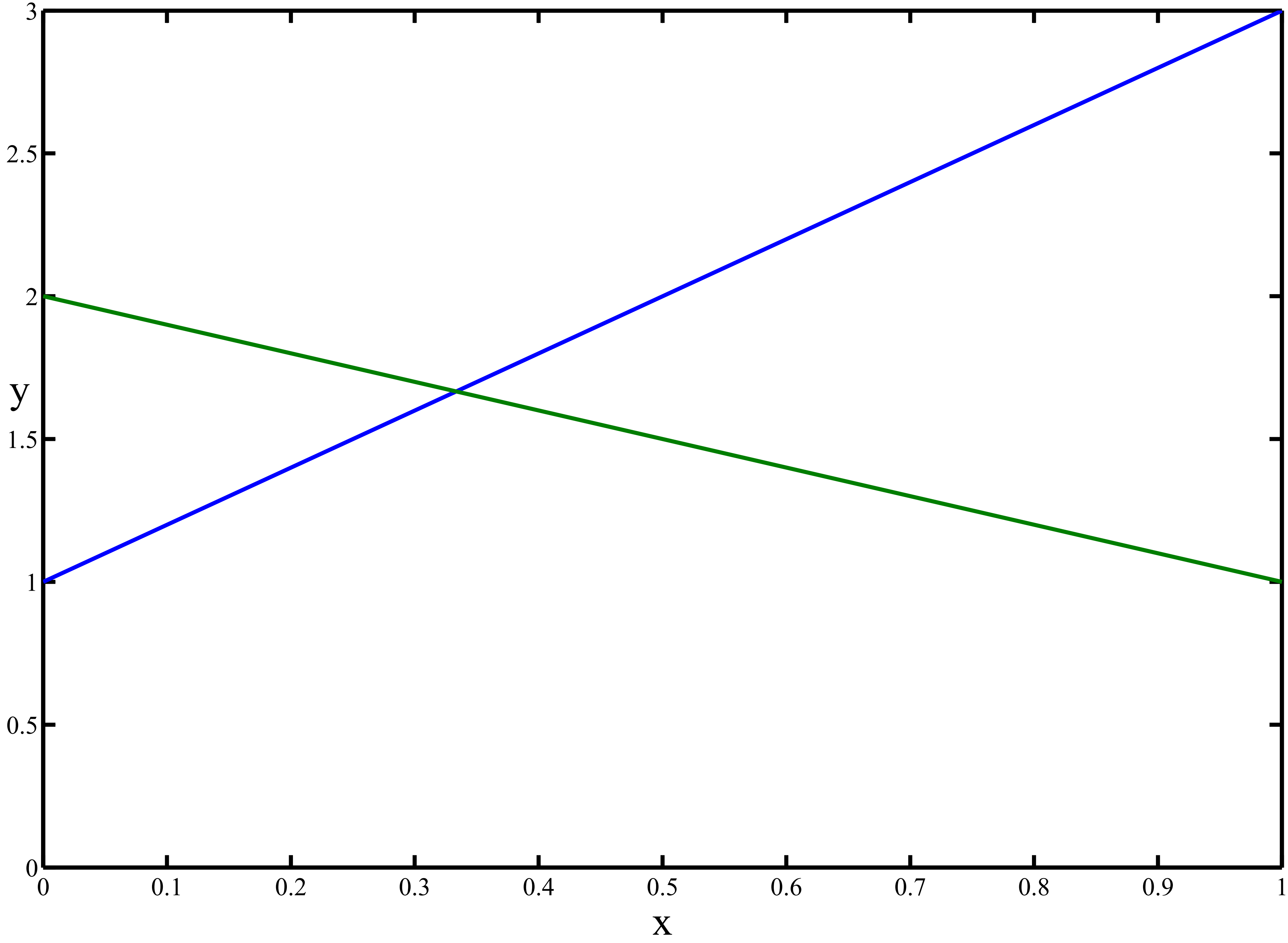


Figure 4

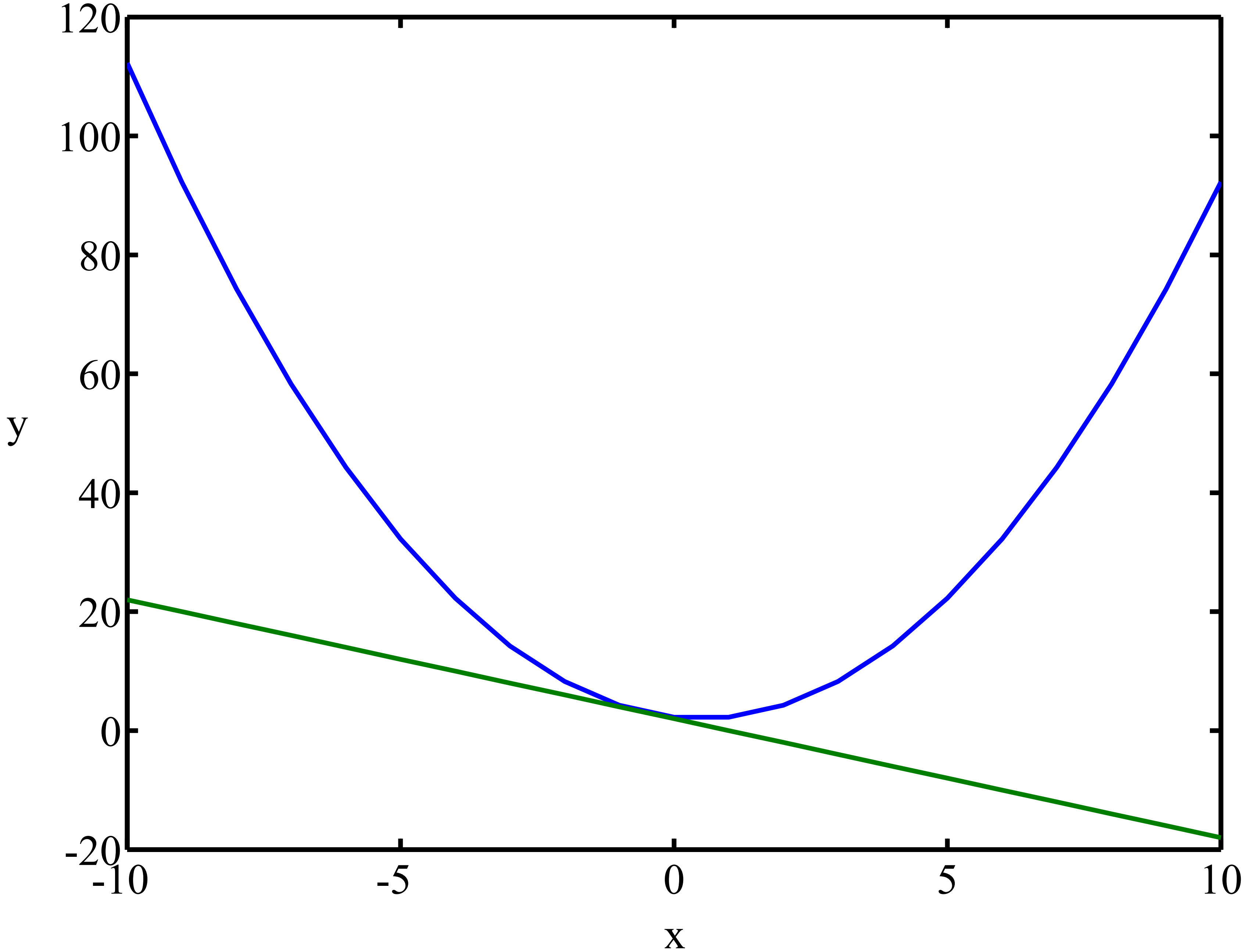


Figure 5

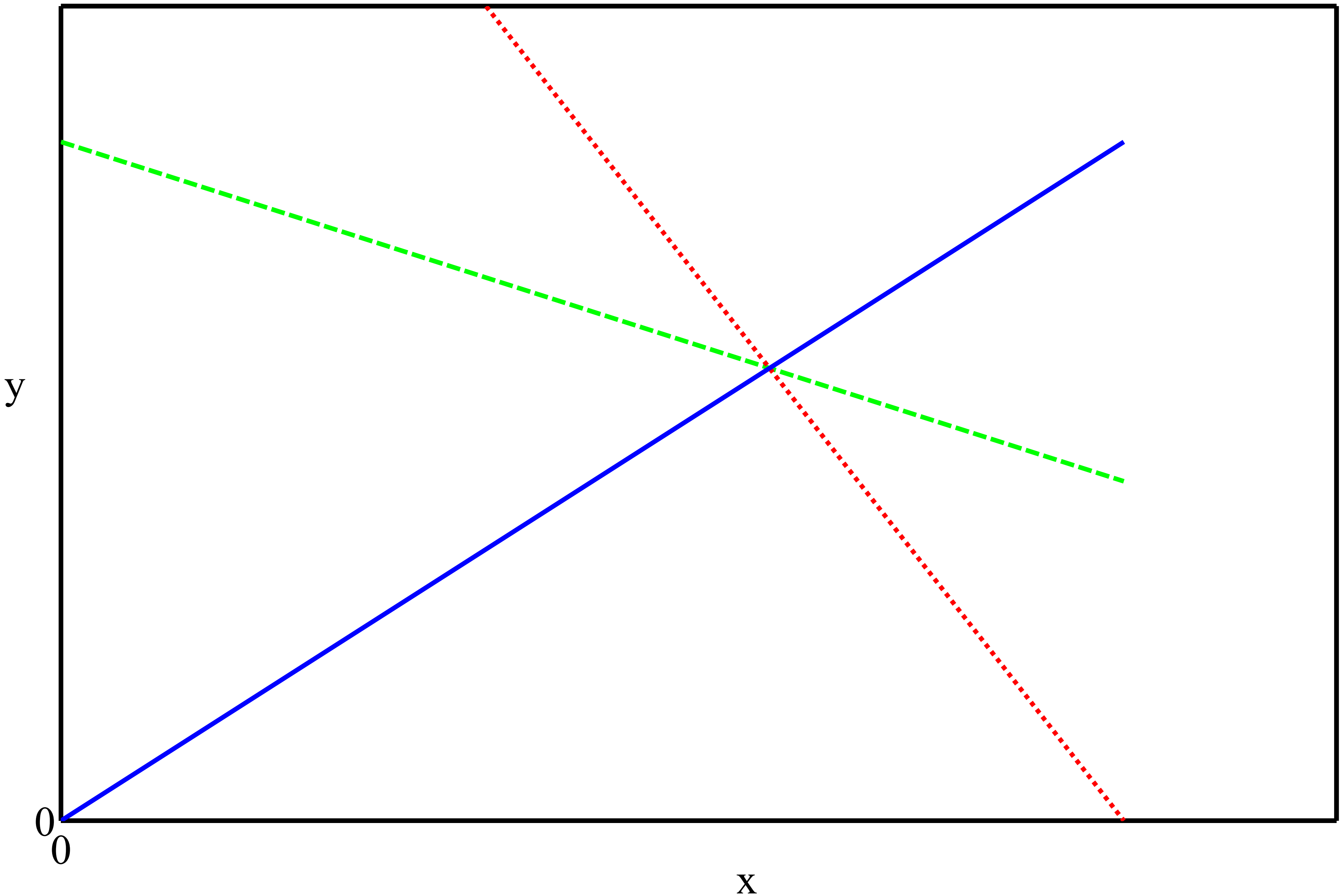


Figure 6: Panel A

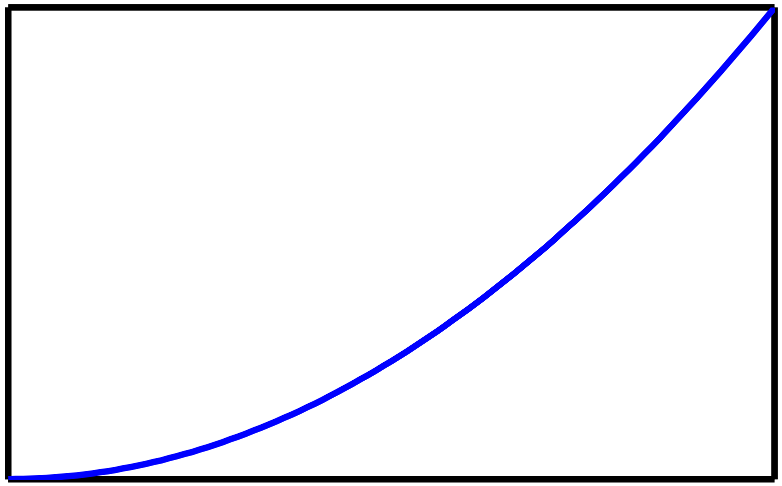


Figure 6: Panel B

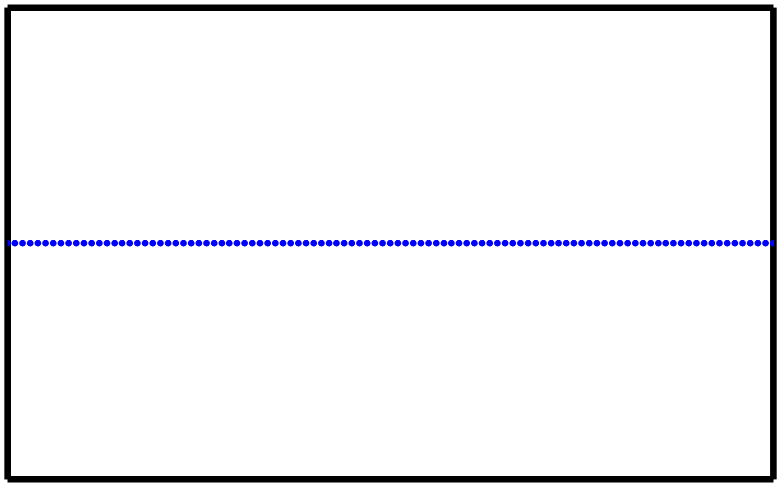


Figure 6: Panel C

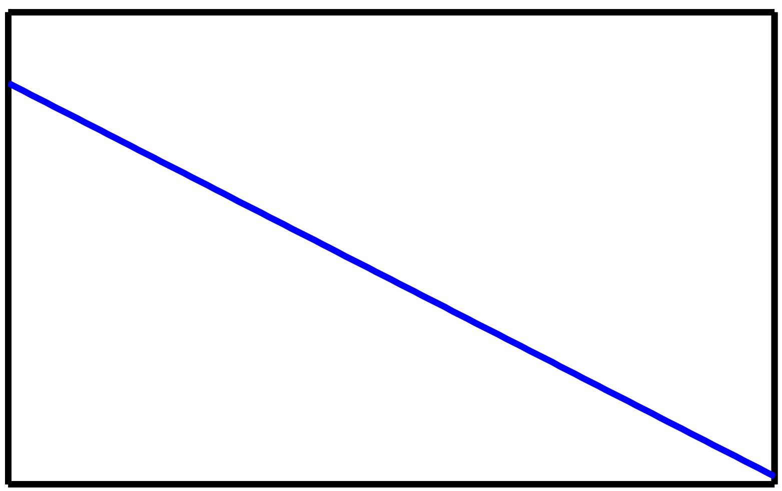
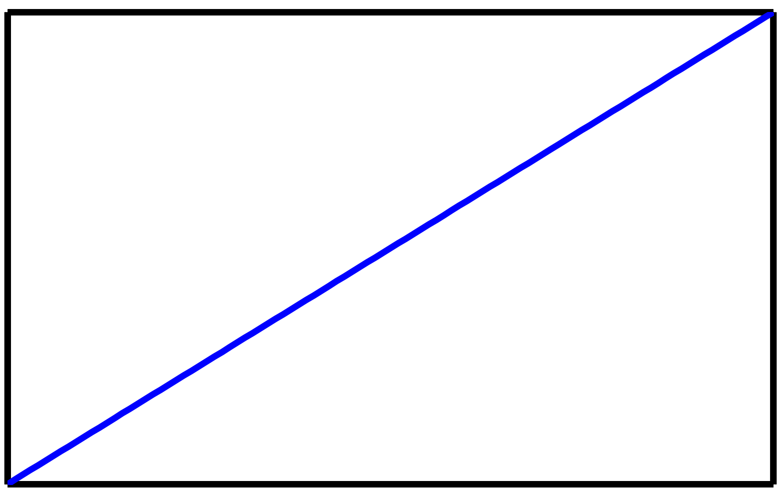


Figure 6: Panel D



Solutions
Problem 1: (a)
Problem 2: (b)
Problem 3: (c)
Problem 4: (b)
Problem 5: (a)
Problem 6: (b)
Problem 7: (d)
Problem 8: (c)
Problem 9: (d)
Problem 10: (b)
Problem 11: (b)
Problem 12: (b)
Problem 13: (c)
Problem 14: (d) [Right answer is $f(\ln B) = B$]
Problem 15: (a)
Problem 16: (b)
Problem 17: (c)
Problem 18: (a)
Problem 19: (d)
Problem 20: (b) [The non-linear graph is ∪-shaped, ruling out (d). The linear graph has negative slope, ruling out (a). To rule out (c) we can either note that $g(-10)$ looks like being less than 26 in the figure. Alternatively, we can use the observation that g is tangent to f. Setting $g'(x) = f'(x)$ gives $x = 1/2$ for both (b) and (c), but only for (b) it holds that $g(1/2) = f(1/2)$]
Problem 21: (c)
Problem 22: (c)
Problem 23: (a) [Note that $f'(x) = g'(x)e^{g(x)}$]
Problem 24: (b)
Problem 25: (d) [f being concave means $f''(x) = 2a \leq 0$; this requires that $a \leq 0$, but implies nothing about b or c]
Problem 26: (a)
Problem 27: (c)
Problem 28: (c)
Problem 29: (a)
Problem 30: (d)
Problem 31: (a)
Problem 32: (b)
Problem 33: (c)
Problem 34: (d) [Right answer is $c = 1/2$; the first-order condition gives the interior candidate: $F'(c) = U'(c) - U'(1-c) = 0$ gives $c = 1/2$; the end points are 1 and 0 and $F(1/2) = a\sqrt{2} > F(1) = F(0) = a$]
Problem 35: (a)
Problem 36: (c) [Draw the graph of $f'(x)$ to see]
Problem 37: (a)
Problem 38: (b)

Problem 39: (c)
Problem 40: (b)