Econ 2400 slides

September 3, 2025

Preliminary and evolving

A Introduction

Modern macroeconomics is *empirical*, meaning it is interested in measures of things that occur in the real world (e.g., inflation, unemployment, interest rates, government budget deficits)

The most commonly used measure is *Gross Domestic Product (GDP)*

Often more interested in *GDP per capita*. Why?

- GDP measures total production of an economy (details below)

- But some countries produce more because they have more people (e.g., Luxembourg vs. Nigeria)

- Living in a country with large total production does not make you better off if there are more people to share it

- Better measure of (average) living standards: GDP per capita (GDP divided by population)

- Can also use GDP per worker (GDP divided by labor force); similar to GDP per capita, but more a measure of productivity than living standards

Interested in how GDP (or GDP per capita) changes over time

Figure 1 in Williamson's book plots Canada's GDP per capita against time 1870-2021

Long run growth, with short-run fluctuations

The short-run fluctuations are called *business cycles*

In Canada (and most rich countries) the biggest decline (or dip) in GDP per capita happened in the 1930's: called the *Great Depression*

Note also more recent events: the Great Financial Crisis (GFC) around 2008-2009 and Global Pandemic (GP) around 2020-2022

We often prefer to plot the *(natural) logarithm* of GDP per capita, and/or plot the *growth rate* of GDP per capita

Let y_t denote GDP per capita in year t

Then the *net* growth rate of y_t in year t is

$$g_t = \frac{y_t - y_{t-1}}{y_{t-1}}$$

The *gross* growth rate is just 1 plus the net growth rate:

$$1 + g_t = \frac{y_t}{y_{t-1}}$$

(Sometimes we use g to denote the gross growth rate; then $g - 1$ is the net growth rate)

The natural logarithm of y_t is written $\ln(y_t)$, which is such that

$$e^{\ln(y_t)} = y_t$$

where $e \approx 2.71828$

Rule(s) to remember: if x is "small" (close to zero), then $\ln(1+x) \approx x$, $\ln(1-x) \approx -x$, $e^x \approx 1+x$, $e^{-x} \approx 1-x$

Also recall that for any $x > 0$ and $z > 0$ it holds that $\ln(x/y) = \ln(x) - \ln(y)$

We now see that

$$g_t \approx \ln(1 + g_t) = \ln\left(\frac{y_t}{y_{t-1}}\right) = \ln(y_t) - \ln(y_{t-1})$$

If we plot $\ln(y_t)$ against t, then g_t can be interpreted as the slope of the plotted path

See Figure 2 in Williamson's book

B Measurement

In Canada, GDP data are made available by Statistics Canada through the *National Income and Expenditure Accounts* (NIEA)

Broadly speaking, GDP measures the market value of all goods and services produced within a country's borders in a given year

But GDP can also be measured for other territorial units and time periods: e.g., subnational units, like provinces; quarters rather than years

Next: explore how to calculate GDP

B.1 Measuring GDP

Three ways to calculate GDP: product (or value-added) approach; expenditure approach; income approach

Product approach:

- GDP = sum of value added across all sectors of the economy
  - Value added = value of *final* goods output, minus value of *intermediate* goods inputs
  - Example: car could be final output; steel, aluminum, electronic components intermediate inputs
  - Note: some goods can be used as both final output and intermediate inputs

Expenditure approach

- GDP = C + I + G + NX

- C = expenditures on consumption by households

- I = expenditures on investment (goods produced, not consumed same year)

- G = expenditures by government (consumption and investment)

- NX is exports (EX) minus imports (IM)

  - IM is value of (final and/or intermediate) goods that came from overseas; EX is value of goods shipped overseas

  - Implies GDP + IM = C + I + G + EX

Income approach

- GDP = sum of all incomes earned by those who contributed to production (typically workers and capital owners)

B.1.1 Williamson's coconut example

Hypothetical economy with 3 sectors: one coconut producer, one restaurant, and a government

Closed economy: no exports or imports

10 million coconuts produced, sold for \$2 each (\$20M)

4 million coconuts consumed directly by households (\$8M);
6 million nuts used as intermediate input by restaurant (\$12M)

Coconut producer pays \$5M in wages, \$0.5M in interest on loans, \$1.5M in taxes

Coconut producer's profits=\$20M-\$5M-\$0.5M-\$1.5M=\$13M (income for households who own coconut plantation)

Restaurant sells \$30M worth of meals, pays \$12M to coconut producer, \$4M in wages, \$3M in taxes

Restaurant's profits=\$30M-\$12M-\$4M-\$3M=\$11M (income for households who own restaurant)

Government production equals its total spending (=tax revenue); paid as income to gov't workers

Gov't production (and expenditure and income) = \$1.5M +\$3M + \$1M = \$5.5M

Households earn income from coconut producer, restaurant, and government; also pay \$1M in taxes

Product approach

- GDP = value added coconut producer + value added restaurant + value added government

- GDP = \$20M+(\$30M-\$12M)+\$5.5=**\$43.5**

Expenditure approach

- C = \$8M+\$30M = \$38M (coconuts and restaurant meals)
- I = \$0 (no investment)
- G = \$5.5M (government consumption equals tax revenue)
- NX = \$0 (closed economy)
- GDP = \$38M + \$5.5M = **\$43.5M**

Income approach

- Households (consumers) earn income from (pre-tax) wages ($5M+$4M+$5.5M=$14.5M), interest ($0.5M), and (after-tax) profits ($13M+$11M=$24M), and pay taxes of $1M

- After-tax household income = $14.5M+$0.5M+$24M-$1M=$38M

- Government income = $5.5M

- GDP = $38M+$5.5M=**$43.5M**

Example with trade Now let restaurant import 2 million nuts at \$2 per nut (\$4M)

Rest same as before

Restaurant sells \$30M in meals; now pays \$12M+\$4M=\$16M to (domestic and foreign) coconut producers, \$4M in wages, \$3M in taxes

Restaurant's profits reduced by \$4M; now \$30M-\$16M-\$4M-\$3M=\$7M

Task: recalculate GDP using each of the three approaches

Product approach: GDP = \$20M+(\$30M-\$16M)+\$5.5=**\$39.5**

Expenditure approach: GDP = C + I + G + NX = \$38M + \$5.5M - \$4M=**\$39.5**

Income approach:

- (After-tax) profits paid to households reduced by \$4M; now \$13M+\$7M=\$20M
- Thus after-tax household income = \$14.5M+\$0.5M+\$20M-\$1M=\$34M
- Gov't income = \$5.5M like before
- GDP = \$34M+\$5.5M=**\$39.5M**

B.2 Problems with GDP (and some solutions)

GDP is a useful measure of economic activity, but has potential drawbacks, depending on context and application

Examples

- Does not capture underground economy (e.g., drugs, gambling)
- Does not capture household production (e.g., care for dependants, baking/cooking at home)
- Does not take into account value of leisure
- Does not take into account depletion of natural resources

- Does not take into account externalities (e.g., pollution, congestion, traffic fatalities)

- Hard to value some non-market production (e.g., government expenditures)

- Some (poor) countries lack reliable data

Possible solutions:

- Using GDP per hour worked (if data available) addresses issue of not accounting for leisure

- Alternative measures that correlate with GDP per capita can address lack of data, and some of the other issues:
  - Mortality, life expectancy
  - Population density
  - Night lights

B.3 Nominal and real GDP

Want to measure changes in GDP over time

Tricky, because both the quantity of the goods produced changes over time, and the prices they are valued at

B.3.1 One-good economy

Easy with just one (final) good

2 years: 1 and 2

P_1 and P_2 are prices in years 1 and 2

Q_1 and Q_2 are quantities in years 1 and 2

$GDP_1 = P_1 Q_1$ is nominal GDP in year 1

$GDP_2 = P_2 Q_2$ is nominal GDP in year 2

Want to calculate real GDP (*RGDP*) in years 1 and 2

Must choose which year to use as *base year*

Let the base year be denoted by super-index (top right), and let year in which GDP is measured be indicated by a sub-index (bottom right)

Then

$$\begin{aligned}
RGDP_1^1 = &\quad GDP_1 && = P_1 Q_1 \\
RGDP_1^2 = &\quad GDP_1 \left(\frac{P_2}{P_1}\right) && = P_2 Q_1 \\
RGDP_2^1 = &\quad GDP_2 \left(\frac{P_1}{P_2}\right) && = P_1 Q_2 \\
RGDP_2^2 = &\quad GDP_2 && = P_2 Q_2
\end{aligned}$$

Note that the (gross) growth rate in real GDP from year 1 to year 2 is the same regardless of which base year we use:

$$\frac{RGDP_2^1}{RGDP_1^1} = \frac{RGDP_2^2}{RGDP_1^2} = \frac{Q_2}{Q_1}$$

That is, real GDP growth is the same as growth in quantity produced; does not matter which base year we use

B.3.2 Two-good economy

With more than one good, much harder

Prices can change between years in ways that vary across goods: relative prices change

Practical solution: use *chain-weighted* index

Two goods (A and B); two years (1 and 2)

Both goods are final goods

Eight variables to keep track of:

- P_1^A = price of good A in year 1
- P_1^B = price of good B in year 1
- P_2^A = price of good A in year 2
- P_2^B = price of good B in year 2
- Q_1^A = quantity of good A in year 1
- Q_1^B = quantity of good B in year 1
- Q_2^A = quantity of good A in year 2
- Q_2^B = quantity of good B in year 2

Now nominal GDP in years 1 and 2 can be written

$$GDP_1 = P_1^A Q_1^A + P_1^B Q_1^B$$

$$GDP_2 = P_2^A Q_2^A + P_2^B Q_2^B$$

Real GDP level in year 1 with base years 1 and 2:

$$RGDP_1^1 = P_1^A Q_1^A + P_1^B Q_1^B \quad = GDP_1$$

$$RGDP_1^2 = P_2^A Q_1^A + P_2^B Q_1^B$$

Real GDP level in year 2 with base years 1 and 2:

$$RGDP_2^1 = P_1^A Q_2^A + P_1^B Q_2^B$$

$$RGDP_2^2 = P_2^A Q_2^A + P_2^B Q_2^B \quad = GDP_2$$

Now there are two ways to calculate the real growth rate from year 1 to year 2, and they typically produce different results

Let g_1 be the (gross) real GDP growth rate with *year 1 as base year*:

$$g_1 = \frac{RGDP_2^1}{RGDP_1^1} = \frac{P_1^A Q_2^A + P_1^B Q_2^B}{P_1^A Q_1^A + P_1^B Q_1^B}$$

Then let g_2 be the same with *year 2 as base year*:

$$g_2 = \frac{RGDP_2^2}{RGDP_1^2} = \frac{P_2^A Q_2^A + P_2^B Q_2^B}{P_2^A Q_1^A + P_2^B Q_1^B}$$

No reason why g_1 and g_2 should be the same!

This only happens when the change in price from year 1 to year 2 is the same for both goods (A and B); i.e., if the relative price between goods A and B are the same for both years (see problem)

What if relative price changes from year 1 to year 2?

Construct this *chain-weighted* growth rate:

$$g_c = (g_1 g_2)^{1/2} = \left[\frac{RGDP_2^1}{RGDP_1^1}\frac{RGDP_2^2}{RGDP_1^2}\right]^{1/2}$$

We call g_c the geometric average of g_1 and g_2

Let *RCGDP* denote real chain-weighted GDP, with super-index indicating base year and sub-index indicating year in which real GDP is measured

Real chain-weighted GDP in years 1 and 2 with *base year 1*:

$$RCGDP_1^1 = \quad = GDP_1$$

$$RCGDP_2^1 = \quad = g_c GDP_1$$

Real chain-weighted GDP in years 1 and 2 with *base year 2*:

$$RCGDP_1^2 = \quad = \frac{GDP_2}{g_c}$$

$$RCGDP_2^2 = \quad = GDP_2$$

Note that

$$\frac{RCGDP_2^1}{RCGDP_1^1} = \frac{RCGDP_2^2}{RCGDP_1^2} = g_c$$

So which base year we use does not affect real growth rate from year 1 to year 2

B.4 Comparing GDP across countries

Often want to assess which countries are relatively "rich" or "poor"

Difficult for reasons related to comparing GDP in different points of time in the same country (different prices)

Also: prices measured in different currencies

Naive approach: use current exchange rates

But consumers in poor countries do not pay as much for the same consumption basket as rich countries after exchange rate conversion

Particularly true for services

Called the "Penn problem"

Solution: adjust GDP measures for cost of buying (roughly) the same basket

Also called Purchasing Power Parity (PPP) adjusted measure of GDP

B.5 Inflation and the price level

Inflation means broad-based increase in prices

Easy to measure with only one good

With many goods, we need to construct an index that measures some general price level

Two such indices are the *(implicit) GDP price deflator* and the *Consumer Price Index (CPI)*

Main difference is that CPI involves other (fewer) goods than the GDP price deflator

Here focus on GDP price deflator

In Williamson's text, the GDP price deflator is written in words as

$$\text{GDP price deflator} = \frac{\text{Nominal GDP}}{\text{Real GDP}} \times 100$$

To be more precise, let *DEF* denote the GDP price deflator, with super-index indicating base year and sub-index the year in which the deflator is measured

Suppose we let year 1 be the base year. Then

$$DEF_1^1 = \frac{GDP_1}{\underbrace{RGDP_1^1}_{GDP_1}} \times 100 = 100$$

$$DEF_2^1 = \frac{GDP_2}{RGDP_2^1} \times 100$$

The (gross) growth rate of the GDP price deflator thus becomes

$$\frac{DEF_2^1}{DEF_1^1} = \frac{\frac{GDP_2}{GDP_1}}{\frac{RGDP_2^1}{RGDP_1^1}} = \frac{\text{Nominal GDP growth}}{\text{Real GDP growth}}$$

More than one way to calculate real GDP growth; thus more than one way to calculate growth in the GDP price deflator

But the unique chain-weighted real GDP measure produces one unique inflation measure

Consider simple case: one (final) good, 2 periods

Recall: means real GDP growth does not depend on base year

P_1, P_2 = prices in years 1 and 2; Q_1, Q_2 = quantities in years 1 and 2

$GDP_1 = P_1 Q_1$ is nominal GDP in year 1; correspondingly for year 2

For this one-good economy, we know that the (gross) growth rate in real GDP from year 1 to year 2 is

$$\frac{RGDP_2^1}{RGDP_1^1} = \frac{RGDP_2^2}{RGDP_1^2} = \frac{Q_2}{Q_1}$$

And the (gross) growth rate in nominal GDP from year 1 to year 2 is

$$\frac{GDP_2}{GDP_1} = \frac{P_2 Q_2}{P_1 Q_1}$$

So the (gross) growth rate in the GDP price deflator becomes

$$\frac{DEF_2^1}{DEF_1^1} = \frac{\frac{P_2 Q_2}{P_1 Q_1}}{\frac{Q_2}{Q_1}} = \frac{P_2}{P_1}$$

B.6 Saving and the current account

We already know (most of) these variables:

- Y = GDP
- C = consumption expenditures
- I = investment expenditures
- G = government expenditures
- NX = net exports; same as *trade balance*

From the expenditure approach to calculating GDP we know that

$$Y = C + I + G + NX$$

Now consider a few more variables:

- Y^d = disposable income

- T = taxes paid from private sector to government

- TR = transfers paid from government to private sector

- NFP = net factor payments from overseas

- INT = interest on government debt

- $CA = NFP + NX$ = current account

Relationship between GDP and disposable private sector income:

$$Y^d = Y + \underbrace{NFP + INT + TR}_{\text{non-production income}} - T$$

Define private sector saving as $S^p = Y^d - C$:

$$S^p = Y^d - C = \underbrace{Y - C}_{I+G+\mathit{NX}} + \mathit{NFP} + \mathit{INT} + \mathit{TR} - T$$

Rewrite as

$$S^p = I + \underbrace{\mathit{NFP} + \mathit{NX}}_{\mathit{CA}} - \underbrace{[T - \mathit{TR} - G - \mathit{INT}]}_{S^g}$$

where $S^g =$ government saving:

$$S^g = \underbrace{T}_{\text{gov't revenue}} - \underbrace{(\mathit{TR} + G + \mathit{INT})}_{\text{gov't spending}}$$

($D = -S^g =$ government deficit)

Now defined total saving in the economy:

$$S = S^p + S^g$$

Result:

$$S - I = \mathit{CA}$$

Insights:

- A current account surplus means that the economy saves more than it invests, which builds up national wealth (assets held overseas)

- A current account deficit means that the economy invests more than it saves, which runs down national wealth

- Investing less at home (for given savings) implies higher current account balance, more accumulation of assets overseas

- For given domestic investment and given private saving, a higher current account balance requires higher taxes and less spending (higher S^g)

- *NFP* is payment from assets accumulated overseas through previous current account surpluses

- If $NFP > 0$, then an economy can run a current account surplus ($CA > 0$) with a trade deficit ($NX < 0$)

C Correlations in the data

Real GDP tends to grow over time at a relatively stable rate (sort of linear when logged)

Fluctuates over time around long-run trend

Same for many other macro variables

Useful to remove the trend to get a "detrended" measure of (logged) real GDP

Usually constructed as (percentage) deviation from trend

$$\text{deviation}_t = \frac{\text{actual}_t - \text{trend}_t}{\text{trend}_t}$$

Illustrated by time series plot (time on horizontal axis, deviation on vertical)

Deviations of (logged) real GDP from trend are called *business cycles*

Points where detrended time series farthest from zero called *peaks* and *troughs*

Above trend moving towards the peak: *boom* or *expansion*

Below trend moving towards the through: *recession* or *contraction*

Figure 3.2 in Williamson's book: four Canadian recessions (74-75; 81-82; 90-92; 08-09; 20-21)

Detrended time series are *persistent*: current deviation shows positive correlation with deviation in previous period

Other variables also fluctuate around trends and can be detrended the same way

Correlations between different detrended variables

- Can be positive, negative, or (close to) zero
  - If positive correlation with the business cycle: called *procyclical*
  - If negative correlation with the business cycle: called *counter cyclical*
  - If zero correlation with the business cycle: called *acyclical*

- Variables can also be *leading* or *lagging* the business cycle

Examples: see figures in Ch. 3 in Williamson's book

D Competitive equilibrium and social optimum

What determines the size of GDP (Y) and its expenditure components (C, I, G, *NX*)?

In most economies markets play an important role, but never *only* markets

G (and T) depend on policy, but so do C, I, *NX* to some extent through, e.g., taxes, regulations

Common question asked by macroeconomists: do markets generate socially optimal results?

To answer that we need a *model*

D.1 Models and variables

All models involve variables

Represent quantifiable entities that we can (usually) measure in the real world

Example: output (or GDP) is usually denoted by Y

More general than using numbers

D.2 Exogenous and endogenous variables

Two types of variables

- *Endogenous variables:* determined within the model
- *Exogenous variables:* determined outside the model

Typical task: learn how an endogenous variable changes when changing some exogenous variable

Next: look at simple model of how C and Y are determined (endogenous)

For now: ignore G (and T), I, and *NX*

Closed economy; one period, no investment or capital; no government

D.3 Competitive market economy

Households buy goods from firms, firms hire labor from households

D.3.1 Householdsand labor supply

$w =$ real wage rate

$x =$ leisure; $1 - x =$ labor supply

available time endowment $= 1$

$(1 - x)w =$ labor income

$C =$ consumption

$\pi =$ non-labor income (firm profit $=$ dividend paid to owner of firm)

Budget constraint:

$$C = (1 - x)w + \pi,$$

Illustrate budget constraint in diagram with x on horizontal axis, C on vertical axis

- Slope $= -w$

- Vertical intercept $= w + \pi$

Utility function assumed to logarithmic

$$U = (1 - \beta)\ln(C) + \beta \ln(x)$$

$\beta \in (0, 1)$ is weight on leisure

We want to draw indifference curves along which U is constant

Use same diagram as for budget line: x on horizontal axis, C on vertical axis

Find slope of indifference curve at given (x, C)-coordinate (assuming $x < 1$ for now)

Total differentiation of U gives

$$\left(\frac{\partial U}{\partial C}\right)\frac{dC}{dx} + \left(\frac{\partial U}{\partial x}\right) = 0$$

Solve for dC/dx

$$\frac{dC}{dx} = -\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial C}} = -\text{MRS}$$

Slope of indifference curve $= -$MRS

MRS $=$ Marginal Rate of Substitution between C and x

Back to case with log utility:

$$\text{MRS} = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial C}} = \frac{\beta\left(\frac{1}{x}\right)}{(1-\beta)\left(\frac{1}{C}\right)} = \left(\frac{\beta}{1-\beta}\right)\frac{C}{x}$$

Find x and C at which utility is maximized

One of two U-max solutions possible

- $x = 1$ and $C = \pi$ (corner solution)

- $x < 1$ and $C = (1 - x)w + \pi > \pi$ (interior solution)

Interior solution such that slope of indifference curve = slope as budget line

$$-\text{MRS} = -\left(\frac{\beta}{1-\beta}\right)\frac{C}{x} = -w$$

Illustrate in indifference curve diagram

Finding interior solution in log utility case: substitute for $C = (1 - x)w + \pi$ into MRS $= w$

$$\underbrace{\left(\frac{\beta}{1-\beta}\right)\frac{\overbrace{(1-x)w+\pi}^{C}}{x}}_{\text{MRS}} = w$$

Solve for labor supply, $L^S = 1 - x$

$$L^S = 1 - x = 1 - \underbrace{\frac{\beta(w+\pi)}{w}}_{x} = \frac{(1-\beta)w - \beta\pi}{w}$$

Corner solution if MRS $> w$ at $x = 1$

$$w < \underbrace{\frac{\beta\pi}{1-\beta}}_{\text{MRS if } x=1} = \widehat{w}$$

Both cases together

$$L^S = 1 - x = \begin{cases} \frac{(1-\beta)w - \beta\pi}{w} & \text{if} \quad w \geq \widehat{w} \\ 0 & \text{if} \quad w \leq \widehat{w} \end{cases}$$

Illustrate in labor market diagram with w on vertical axis, L^S on horizontal axis

D.3.2 Firms and labor demand

Y = total output (=GDP)

L = labor input

Production function

$$Y = ZL^{1-\alpha}$$

Z = (total factor) productivity

$\alpha \in (0, 1)$ is an exogenous parameter ("capital share" of output, but here no capital)

Similar to a *Cobb-Douglas* production fuction, but here ignoring the capital input (more later)

$wL =$ cost of labor

$\pi =$ representative firm's profit (dividend to households)

$$\pi = Y - wL = ZL^{1-\alpha} - wL$$

Firm chooses L to maximize π, taking w as given

First-order condition

$$\frac{\partial \pi}{\partial L} = (1-\alpha)ZL^{-\alpha} - w = 0$$

Solving for L gives labor demand, L^D

$$L^D = \left[\frac{(1-\alpha)Z}{w}\right]^{\frac{1}{\alpha}}$$

D.3.3 Labor market equilibrium

Equilibrium

- Employment and wages determined by $L^D = L^S$
  - Being on L^S and L^D curves means household maximizes utility, firm maximizes profit

- Output determined by $Y = ZL^{1-\alpha}$

- Firm's profit determined by $\pi = Y - wL$

- Leisure is determined by $L^S = 1 - x$

- Consumption determined by agent's budget constraint, $C = (1 - x)w + \pi$

Use first-order condition for profit maximum and production function

$$wL = (1 - \alpha)Y$$

Interpretation: $1 - \alpha$ is the labor share of output

Profit for firm's owner

$$\pi = Y - wL = \alpha Y$$

Gives

$$\frac{\pi}{w} = \left(\frac{\alpha}{1 - \alpha}\right) L$$

Let L^* be equilibrium level of L

From expression for L^S

$$L^S = L^* = (1-\beta) - \beta \underbrace{\left(\frac{\alpha}{1-\alpha}\right) L^*}_{\pi/w}$$

Solving for L^* gives

$$L^* = \frac{(1-\beta)(1-\alpha)}{1-\alpha(1-\beta)}.$$

Notes:

- Allowing for capital input ($Y = ZK^{\alpha}L^{1-\alpha}$) does not change results
  - Zero profit for firm
  - Replaced by payments to capital owners (i.e., the rep agent), which becomes αY
- Time endowment can be different than one, say h
  - Then $1 - x$ is the *fraction* of h that the agent works
  - Agent's labor supply becomes $(1 - x)h$, leisure becomes xh
- We can let there be more agents than one, say N
  - Each agent works $1 - x$; labor supply becomes $(1 - x)N$

D.4 Social optimum

Let all decisions be made by a *social planner*

Not representing anything we can observe in the real world

Just a tool (or “construct") to find the socially optimal outcome

Social planner maximizes representative agent’s utility subject to the economy’s *resource constraints*

No prices, wages

In this example, resource constraints are:

Labor supply = available time minus leisure

$$L = 1 - x$$

Consumption = production

$$C = ZL^{1-\alpha}$$

Together

$$C = Z(1 - x)^{1-\alpha}$$

Draw in diagram with C on vertical axis and x on horizontal axis

Called *Production Possibility Frontier*

Find x that maximizes utility subject to resource constraint

Substitute $C = Z(1-x)^{1-\alpha}$ into log utility function

$$U = (1-\beta)\ln \underbrace{\left[Z(1-x)^{1-\alpha}\right]}_{C} + \beta \ln(x)$$

First-order condition w.r.t x

$$\frac{\partial U}{\partial x} = -(1-\beta)(1-\alpha)\left(\frac{1}{1-x}\right) + \beta\left(\frac{1}{x}\right) = 0$$

Gives

$$x = \frac{\beta}{1-\alpha(1-\beta)}$$

$$L^{\text{soc}} = 1 - x = \frac{(1-\beta)(1-\alpha)}{1-\alpha(1-\beta)}$$

Same outcome in social optimum as competitive equilibrium: $L^{\text{soc}} = L^*$

D.5 The first and second welfare theorems

A social optimum is also called a *Pareto efficient resource allocation*

Means that no one can be made better off without someone being made worse off

In a single-agent model, this just means that the agent cannot be made better off by changing the allocation of resources (i.e., changing the choice between leisure and consumption)

In other contexts, more complicated (e.g., if only some agents own the firm)

Usually *many* Pareto efficient allocations (e.g., one guy gets all consumption)

Is the competitive equilibrium always socially optimal?

Can any Pareto efficient allocation be sustained as a competitive equilibrium? (E.g., by redistributing resources and/or using taxes?)

First welfare theorem

- Under certain conditions, a competitive equilibrium is a Pareto efficient allocation of resources

Second welfare theorem

- Under certain conditions, a Pareto efficient allocation is such that there exists a competitive equilibrium (prices, wages, etc.) that generates that same outcome

The conditions do not always hold, e.g., when we have externalities, or other market failures

E Growth models

E.1 Continuous and discrete time

Growth models are dynamic models: involve time, t

Two types of dynamic models:

Discrete-time models: the variable t (time) is a (non-negative) integer: $t \in \{0, 1, 2, ...\}$

Discrete-time variables usually written x_t

Continuous-time models: t is a (non-negative) real number: $t \in [0, +\infty)$

Continuous-time variables usually written $x(t)$

Can be differentiated: $\frac{\partial x(t)}{\partial t}$

Here: emphasis on discrete-time (versions of) growth models

E.2 The Solow Growth Model

Discrete time setting: the time variable t is a (non-negative) integer: $t \in \{0, 1, 2, ...\}$

Notation:

K_t = capital in period t

Y_t = output in period t

δ = depreciation rate

s = rate of saving/investment out of income, Y_t

Evolution of capital stock:

$$K_{t+1} = sY_t + (1 - \delta)K_t$$

L_t = population/labor force in period t

$n =$ (net) growth rate of population

$$L_{t+1} = (1 + n)L_t$$

Assume $n \geq 0$, $\delta \in (0, 1]$; $s \in (0, 1]$

E.2.1 Production function

$$Y_t = ZK_t^{\alpha} L_t^{1-\alpha}$$

$Z =$ total factor productivity

$\alpha =$ capital share of output

Assume that $Z > 0$, $\alpha \in (0, 1)$

$\partial Y_t / \partial K_t > 0$ implies *positive* MPK: more capital can produce more output (at a given level of labor input)

$\partial^2 Y_t / \partial K_t^2 < 0$ implies *decreasing* MPK: increasing capital has a smaller effect on output when the initial capital input is already large

More notation:

$y_t = Y_t/L_t =$ output per worker at time t

$k_t = K_t/L_t =$ capital per worker at time t

Now we see from production function that

$$y_t = \frac{Y_t}{L_t} = ZK_t^{\alpha}L_t^{-\alpha} = Z\left(\frac{K_t}{L_t}\right)^{\alpha} = Zk_t^{\alpha}$$

Then note that

$$\frac{K_{t+1}}{L_t} = \overbrace{\left(\frac{K_{t+1}}{L_{t+1}}\right)}^{k_{t+1}} \times \overbrace{\left(\frac{L_{t+1}}{L_t}\right)}^{1+n} = k_{t+1}(1+n)$$

$$= \frac{sY_t + (1-\delta)K_t}{L_t} = sy_t + (1-\delta)k_t = sZk_t^{\alpha} + (1-\delta)k_t$$

This gives k_{t+1} as a function of k_t

$$k_{t+1} = \frac{sZk_t^{\alpha} + (1-\delta)k_t}{1+n} \equiv \phi(k_t)$$

E.2.2 Properties of $\phi(k_t)$

$$\phi'(k_t) = \frac{s\alpha Z k_t^{\alpha-1} + (1-\delta)}{1+n} > 0$$

$$\phi''(k_t) = \frac{s\alpha(\alpha-1) Z k_t^{\alpha-2}}{1+n} < 0$$

$$\phi(0) = \frac{s \times 0 + (1-\delta) \times 0}{1+n} = 0$$

Draw the graph of $k_{t+1} = \phi(k_t)$ in diagram with k_t on horizontal axis and k_{t+1} on vertical axis; add straight line along which $k_{t+1} = k_t$, called 45°-line

E.2.3 Steady state

k_t will converge over time to what we call the steady-state level of k_t, here denoted k^*

Illustrate dynamics and steady state in 45°-diagram

In problem set we learn that

$$k^* = \left(\frac{sZ}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

$$y^* = Z^{\frac{1}{1-\alpha}} \left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

Note exponent on Z greater than 1

Doubling Z more than doubles steady-state GDP per capita

Intuition: makes capital more productive; also leads to accumulation of more capital

E.3 The Malthus Model

To be added (?)