Midterm Exam – Econ 2450
21 November 2016
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Instructions: Unless otherwise explicitly stated, you must show how you arrived at your answer to get full mark. However, where the answer is just one, or a few, words no further motivation is needed. Answers must be given on the answer sheets provided. Ask for extra answer sheets if needed. Do not fold the answer sheets or write on the back.

Problem 1 [**4 marks**]

This problem derives labor demand from the firm's profit maximization problem, but with a different production function than that discussed in class. Total output of a firm that hires L workers is $A(B+L)^{\gamma}$, where $B>0$, $A>0$ and $0<\gamma<1$. The (real) wage is w, and there are N firms.

(a) Draw the graph of output, $A(B+L)^{\gamma}$, in a diagram with L on the horizontal axis and $A(B+L)^{\gamma}$ on the vertical. Consider only non-negative levels of L. Show how an increase in B shifts the graph. [1 mark]

(b) The representative firm maximizes profits subject to a non-negativity constraint on the number of workers hired, $L \geq 0$. Find a level of w, call it $\widehat{w}$, such that the firm's profit-maximizing choice of L is zero if $w \geq \widehat{w}$, and positive if $w < \widehat{w}$. Your answer should be in terms of (some or all of) the variables A, B, and γ. [1 mark]

(c) Draw aggregate demand for labor, L^D, in a diagram with w on the vertical axis, and L^D on the horizontal. Indicate the vertical intercept of L^D, if any. [1 mark]

(d) What do the words G and E in DSGE model stand for? [1 mark]

NOTE: In Problems 1 (a) and (c) you do not need to motivate anything, just draw everything correctly.

Problem 2 [**4 marks**]

Consider a two-period model, where agents earn incomes Y in the first period, and $(1+g)Y$ in the second, where g is income growth between the periods. This is a small open economy, and the world interest rate, r^*, is exogenous. Agents face the interest rate

$$1+r=\begin{cases} 1+r^* & \text{if} \quad S \geq 0, \\ (1+r^*)\left(\frac{Y}{Y+S}\right) & \text{if} \quad S<0, \end{cases} \tag{1}$$

where S is the agent's saving. In words, this means that agents who save ($S \geq 0$) face the world interest rate, r^*, while agents who borrow ($S<0$) face the interest rate $(1+r^*)\left(\frac{Y}{Y+S}\right)$, which is greater than $1+r^*$, since $Y/(Y+S)>1$ when $S<0$. (In this problem, it will always hold that $Y+S>0$, but you do not need to show that here.)

A representative agent has logarithmic utility, with weight β on second-period utility, where $0<\beta<1$. This implies that an agent facing an interest rate r saves as amount S, given by

$$S=\left[\beta-(1-\beta)\left(\frac{1+g}{1+r}\right)\right]Y. \tag{2}$$

An equilibrium is here defined as levels of S and $1+r$ for which both (1) and (2) hold.

(a) Draw the graph of (1) in a diagram with S on the horizontal axis and $1+r$ on the vertical. Let S vary from (close to) $-Y$ to plus infinity. [1 mark]

(b) Assume that $1+r^* < (1+g)(1-\beta)/\beta$. Draw the graph of (2) in the same diagram as under (a), with S on the horizontal axis and $1+r$ on the vertical. Let S vary from minus infinity to (close to) βY. [1 mark]

(c) Continue to assume that $1+r^* < (1+g)(1-\beta)/\beta$, which can be seen to imply that $S < 0$ in equilibrium. Find an expression for the equilibrium level of S. Your answer should be in terms of some, or all, of r^*, g, Y, and β. [1 mark]

(d) What do we call property that can be seized by the lender if the borrower is unable to pay back the loan? (Hint: it starts with a C.) [0.5 marks]

(e) Sometimes governments fail to pay back what they have borrowed. What is this called? (Hint: two words, starting with S and D.) [0.5 marks]

NOTE: In Problems 2 (a) and (b) you do not need to motivate anything, just draw everything correctly.

Problem 3 [4 marks]

Consider a two-period model, where a representative agent earns an incomes Y_1 and Y_2 in the first and second periods, respectively. Consumption is C_1 in the first period, and C_2 in the second. The budget constraints of the representative agent are $C_1 = Y_1 - S$, and $C_2 = Y_2 + S(1+r)$, where S is saving and r is the (real) interest rate. The agent maximizes this utility function:

$$U = (1-\beta)\ln(C_1 - \overline{C}) + \beta \ln(C_2)$$

where $\beta \in (0,1)$ and $\overline{C} > 0$. The exogenous variable $\overline{C}$ is called subsistence consumption, meaning that the marginal utility of first-period consumption goes to infinity as C_1 approaches $\overline{C}$. We also assume that $Y_1 > \overline{C}$, which means that first-period income is sufficient to cover subsistence consumption.

Total output in the second period is produced by a representative firm, and equals AK^α, where K is the capital stock, $A > 0$ and $\alpha \in (0,1)$. Second-period labor income is given by:

$$Y_2 = (1-\alpha)AK^\alpha.$$

Profits for a representative firm, π, are given by

$$\pi = AK^\alpha - (1+r)K.$$

(a) Derive an expression for optimal S in terms of Y_1, Y_2, r, $\overline{C}$, and β. (Hint: to check that you are on the right track, note that $\overline{C} = 0$ should bring you back to the case discussed in class.) [1 mark]

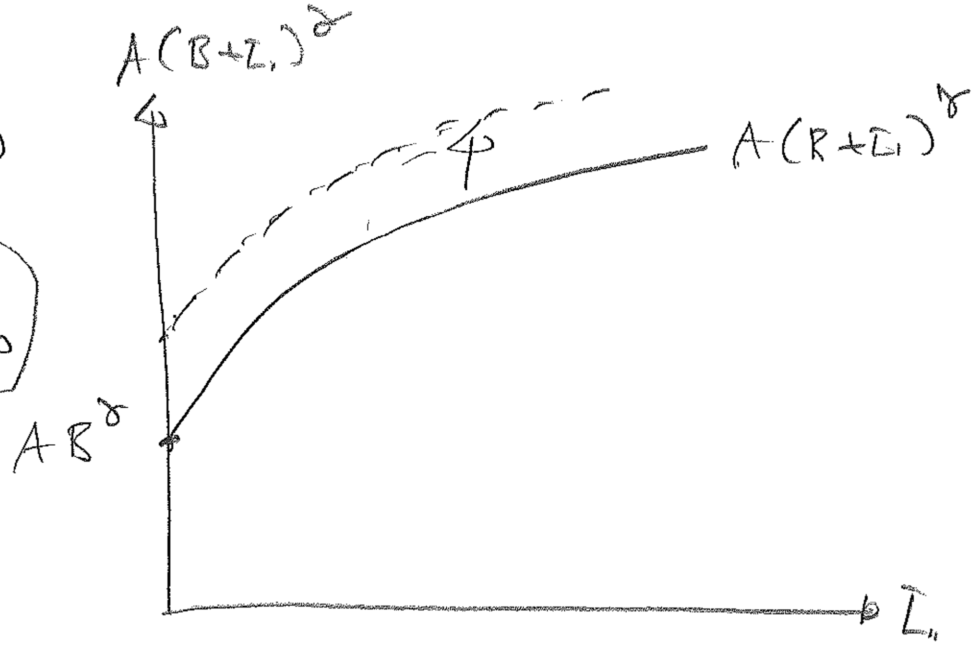
(b) This is a closed economy, where all capital investment is made up of household saving. Use your answer to (a) to derive an expression for equilibrium investment, K. Your answer should be an expression for K involving (some or all of) Y_1, $\overline{C}$, α, β, and A. Show each step. If you could not solve (a), you may try setting $\overline{C} = 0$ for possible partial marks. [2 marks]

(c) What do we call the theoretical result that an agent's intertemporal consumption choices do not depend on the allocation of taxes across periods? (Hint: two words, starting with R and E.) [0.5 marks]

(d) What do we call the phenomenon of poor countries growing faster than rich? (Hint: it starts with a C.) [0.5 marks]

1 (a)

curve shifts up when $B\uparrow$



(b) $\pi = A(B+L_1)^\gamma - wL_1$

$$\frac{\partial \pi}{\partial L_1} = A\gamma(B+L_1)^{\gamma-1} - w$$

If $\frac{\partial \pi}{\partial L_1} \leq 0$ when $L_1 = 0$,

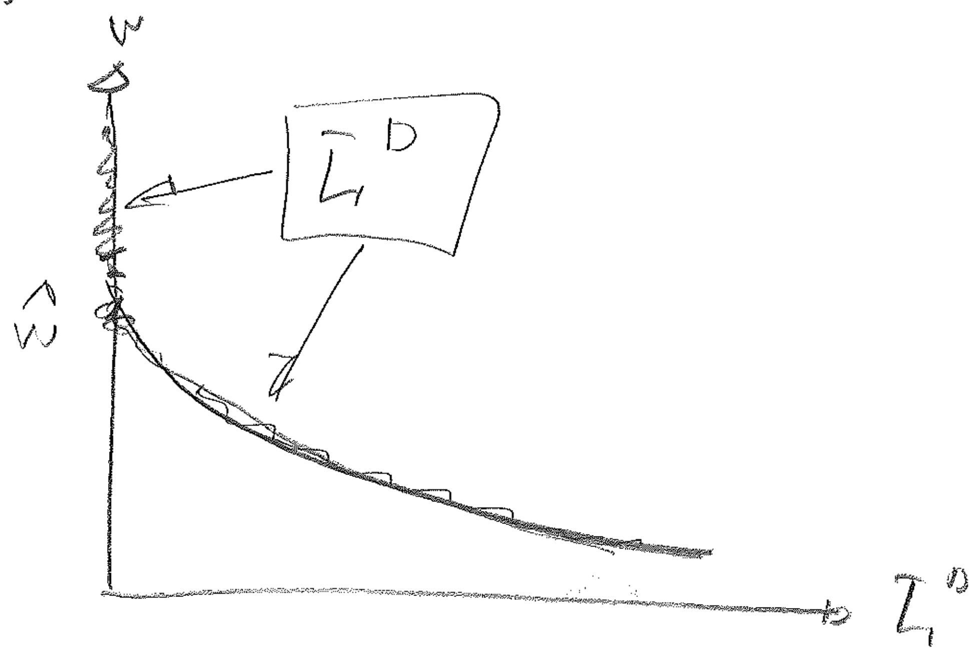
then optimal to set $L_1 = 0$

This happens when

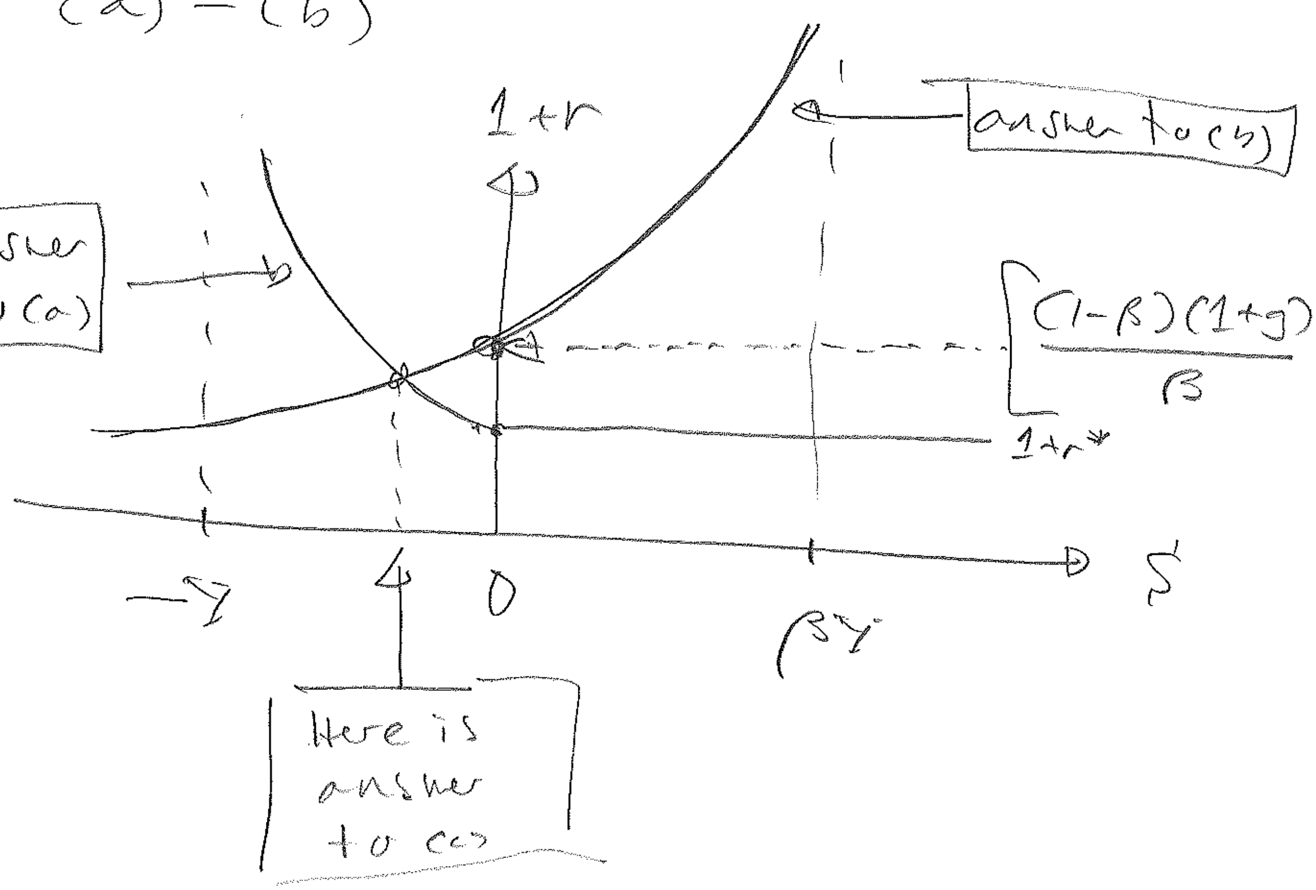
$$\frac{\partial \pi}{\partial L_1} = A\gamma(B)^{\gamma-1} - w \leq 0$$

or $\boxed{w \geq A\gamma B^{\gamma-1} \equiv \hat{w}}$

(c)


w
L_1^D
$\hat{w}$
L_1^D

② (a) – (b)


1+r
answer to (b)
answer to (a)
$\frac{(1-\beta)(1+g)}{\beta}$
1+r*
−Y
0
βY
S
Here is answer to (c)

② (c) $\bar{S} < 0 \Rightarrow 1+r = (1+r^*)\left[\frac{\bar{Y}}{\bar{Y}+\bar{S}}\right]$

$$\frac{1+g}{1+r} = \frac{1+g}{\underbrace{(1+r^*)\left[\frac{\bar{Y}}{\bar{Y}+\bar{S}}\right]}_{1+r}}$$

$$\frac{1+g}{1+r} = \left(\frac{1+g}{1+r^*}\right)\left[\frac{\bar{Y}+\bar{S}}{\bar{Y}}\right] = \left(\frac{1+g}{1+r^*}\right)\underbrace{\left[1+\frac{\bar{S}}{\bar{Y}}\right]}_{\frac{\bar{Y}+\bar{S}}{\bar{Y}}}$$

From eq, (2)

$$\frac{\bar{S}}{\bar{Y}} = \beta - (1-\beta)\overbrace{\left(\frac{1+g}{1+r^*}\right)\left[1+\frac{\bar{S}}{\bar{Y}}\right]}^{\frac{1+g}{1+r}}$$

$$\frac{\bar{S}}{\bar{Y}}\left[1+(1-\beta)\left(\frac{1+g}{1+r^*}\right)\right] = \underbrace{\beta-(1-\beta)\left(\frac{1+g}{1+r^*}\right)}_{>0 \text{ since } 1+r^* < (1-\beta)(1+g)/\beta.}$$

② (c) cont'd

$$\frac{S^1}{Y} = \frac{\beta - (1-\beta)\left(\frac{1+g}{1+r^*}\right)}{1+(1-\beta)\left(\frac{1+g}{1+r^*}\right)}$$

$$\boxed{S^1 = \left[\frac{\beta - (1-\beta)\left(\frac{1+g}{1+r^*}\right)}{1+(1-\beta)\left(\frac{1+g}{1+r^*}\right)}\right] Y}$$

② (d) Collateral

② (e) Sovereign Default

③ (a)

$$\bar{U} = (1-\beta)\ln\Big[\overbrace{(\bar{Y}_1 - \bar{c}_i)}^{c_{i1} - \bar{c}_i} - s_i\Big] + \beta\ln\Big[\overbrace{\bar{Y}_2 + s_i(1+r)}^{c_{i2}}\Big]$$

FOC:

$$\frac{\partial U}{\partial s} = -(1-\beta)\left(\frac{1}{(\bar{Y}_1 - \bar{c}_i) - s_i}\right) + \beta\left(\frac{1+r}{\bar{Y}_2 + s_i(1+r)}\right) = 0$$

$$(1-\beta)\left[\bar{Y}_2 + s_i(1+r)\right] = \beta(1+r)\left[(\bar{Y}_1 - \bar{c}_i) - s_i\right]$$

$$s_i(1+r)\Big[\overbrace{1-\beta+\beta}^{=1}\Big] = \beta(1+r)(\bar{Y}_1 - \bar{c}_i) - (1-\beta)\bar{Y}_2$$

$$\boxed{s_i = \beta(\bar{Y}_1 - \bar{c}_i) - (1-\beta)\frac{\bar{Y}_2}{1+r}}$$

(3)(b) $\Pi = A\underline{K}^{\alpha} - (1+r)\underline{K}$

FOCi: $\frac{\partial \Pi}{\partial \underline{K}} = \alpha A\underline{K}^{\alpha-1} - (1+1) = 0$

From FOCi: $\boxed{1+r = \alpha A\underline{K}^{\alpha-1}}$

Given: $\boxed{Y_2 = (1-\alpha) A\underline{K}^{\alpha}}$

$\Rightarrow \boxed{\frac{\bar{Y}_2}{1+r} = \left(\frac{1-\alpha}{\alpha}\right)\underline{K}}$

From (a), and setting $S' = \underline{K}$

$$S' = \underline{K} = \beta(\bar{Y}_1 - \bar{C}_i) - (1-\beta)\overbrace{\left(\frac{1-\alpha}{\alpha}\right)\underline{K}}^{\bar{Y}_2/(1+r)}$$

Solve for $\underline{K}$

$$\underline{K}\left[\alpha + (1-\beta)(1-\alpha)\right] = \alpha\beta(\bar{Y}_1 - \bar{C}_i)$$

(3) (b) cont'd

$$\boxed{\bar{K} = \left[\frac{\alpha\beta}{\underbrace{\alpha + (1-\beta)(1-\alpha)}_{\substack{= \alpha + 1 - \alpha - \beta(1-\alpha) \\ = 1 - \beta(1-\alpha)}}} \right] (Y_1 - \bar{c}_1)}$$

OK answer

$$\boxed{\bar{K} = \left[\frac{\alpha\beta}{1 - \beta(1-\alpha)} \right] (Y_1 - \bar{c}_1)}$$

More beautiful answer

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(3) (c) Ricardian Equivalence

(3) (d) Convergence

Explaining 1(c); not needed for answer

If $w \geq \hat{w}$, then $L^D = N \times 0 = 0$

If $w < \hat{w}$, then optimal

L_1 given by $\frac{\partial \pi}{\partial L_1} = 0$

$$\frac{\partial \pi}{\partial L_1} = A\gamma(B+L_1)^{\gamma-1} - w = 0$$

Solve for L_1

$$A\gamma(B+L_1)^{\gamma-1} = w$$

$$(B+L_1)^{\gamma-1} = \frac{w}{\gamma A}$$

setting $L_1 = 0$ here gives expression for $\hat{w}$ $\Leftarrow$ $\boxed{L_1 = \left(\frac{\gamma A}{w}\right)^{\frac{1}{1-\gamma}} - B}$

So $L^D = NL_1 = N\left[\left(\frac{\gamma A}{w}\right)^{\frac{1}{1-\gamma}} - B\right]$

if $w < \hat{w}$,

$$\boxed{L^D = \begin{cases} 0 & \text{if } w \geq \hat{w} \\ \underbrace{N\left(\frac{\gamma A}{w}\right)^{\frac{1}{1-\gamma}} - BN}_{\Uparrow \text{ decrease in } w} & \text{if } w > \hat{w} \end{cases}}$$

Explaining 2. (a) – (b) (not needed)

From eq. (1) (for $\hat{S} < 0$)

$$1 + r = (1 + r^*) \left[\frac{\bar{Y}}{\underbrace{\bar{Y} - \hat{S}}_{\substack{\to 0 \\ \text{as } \hat{S} \to -\bar{Y}}}} \right] \to \infty \quad \text{as } \hat{S} \to -\bar{Y}$$

$$1 + r = (1 + r^*) \underbrace{\left[\frac{\bar{Y}}{\bar{Y} - 0} \right]}_{= 1} = (1 + r^*) \text{ if } \hat{S} = 0$$

From eq. (2)

$$(1 - \rho)\left(\frac{1 + g}{1 + r}\right) = \beta - \left(\frac{\hat{S}}{\bar{Y}}\right) = \frac{\beta \bar{Y} - \hat{S}}{\bar{Y}}$$

$$1 + r = \frac{(1 - \rho)(1 + g)\bar{Y}}{\underbrace{\beta \bar{Y} - \hat{S}}_{\substack{\to 0 \\ \text{as } \hat{S} \to \beta \bar{Y}}}} \to \infty \quad \text{as } \hat{S} \to \beta \bar{Y}$$