Midterm Exam – Econ 4020 – v2
18 March 2019
Department of Economics
York University

Instructions: On this exam, you should *not* show how you arrived at your answer, only write the answer in the space indicated on the relevant answer sheet. Answers must be given on the correct answer sheet, one for each problem. If you make a mistake, ask for a new answer sheet. Do not fold the answer sheets or write on the back, or outside of the indicated space.

Student name: SID number:

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1. The Solow model [4 marks]

Consider a standard Solow model, where the capital stock per effective worker, k, evolves over time according to

$$\dot{k} = sf(k) - (n + g + \delta)k,$$

where $f(k)$ is the intensive-form production function. We let $y = f(k)$ denote income per effective worker, and $c = (1 - s)y$ consumption per effective worker. Note that k, y, and c are dependent on time but we suppress the time argument, while s, n, g, and δ are strictly positive constants (and $0 < s < 1$). As usual, the steady-state levels of k, y, and c are denoted k^*, y^*, and c^*, respectively.

(a) Let $f(k) = Z\left[\beta k^\rho + 1 - \beta\right]^{1/\rho}$, where $0 < \beta < 1$, and $\rho < 1$. Find an expression for k^* in terms of (some or all of) s, Z, β, ρ, n, g, and δ (and possibly numbers). (It is assumed that these parameters are as such that k^* exists.) [1 mark]

(b) Now let $f(k) = k^\beta$, where $0 < \beta < 1$. Find an expression for y^* in terms of (some or all of) s, β, n, g, and δ (and possibly numbers). [1 mark]

(c) Now assume that $f(k) = Dk^\beta$, where $D > 0$ and $0 < \beta < 1$. Find an expression for c^* in terms of (some or all of) s, β, D, n, g, and δ (and possibly numbers). [1 mark]

(d) Now assume a general production function, such that $f'(k) > 0$ and $f''(k) < 0$. Find an expression for c^*/k^* in terms of (some or all of) s, n, g, and δ (and possibly numbers). [1 mark]

2. The Ramsey model [4 marks]

Consider a standard Ramsey model with Cobb-Douglas production, meaning output per effective worker equals Zk^α, where k denotes capital per effective worker, $0 < \alpha < 1$, and $Z > 0$. The dynamics of consumption per effective worker, c, is given by the so-called Euler equation:

$$\frac{\dot{c}}{c} = \frac{\alpha Z k^{\alpha-1} - \rho - \theta g}{\theta},$$

where $\alpha Z k^{\alpha-1}$ is the marginal product of capital. Similarly, the dynamics of k is given by

$$\dot{k} = Zk^{\alpha} - c - (n+g)k.$$

Here k and c are dependent on time but we suppress the time argument, while α, Z, ρ, n, g, and θ are all strictly positive constants.

(a) In the phase diagram provided, draw both the ($\dot{c} = 0$)-locus, and the ($\dot{k} = 0$)-locus, for two values of g, denoted g_0 and g_1, where $g_1 > g_0$. You should *not* show any saddle path or other trajectories, only the relevant loci, and clearly indicate which one of them refers to g_0 and g_1, respectively. Also indicate the associated steady-state levels of c and k, denoted c_0^* and c_1^*, and k_0^* and k_1^*, for c and k respectively. [2 marks]

(b) Consider an economy which is initially in a steady state associated with $g = g_0$. At some point in time, $\widehat{t}$, the level of g increases to $g_1 > g_0$. Draw the time path of k. For full mark, you need to draw the path correctly and indicate $\widehat{t}$, k_0^*, and k_1^* on relevant axes. (Hint: the path of c may be ambiguous but here you are asked about the path of k.) [2 marks]

3. Endogenous growth [4 marks]

Consider a two-sector endogenous growth model, with a goods sector and an "ideas" (or R&D) sector. Total goods output, Y, is produced with the production function

$$Y = \left([1-a_K]\,K\right)^{1-\rho}\left([1-a_L]\,AL\right)^{\rho},$$

where ρ is an exogenous parameter, such that $0 < \rho < 1$; A is the level of technology (the number of ideas); L is the total labor force; K is the capital stock; and a_L and a_K are the fractions of L and K, respectively, allocated to ideas production, with the remainder allocated to goods production. (We assume that $0 < a_L < 1$ and $0 < a_K < 1$.) New ideas, $\dot{A}$, are produced with the production function

$$\dot{A} = D(a_K K)^{\beta}(a_L L)^{\gamma} A^{\theta},$$

where θ, γ, β, and D are strictly positive. The dynamic evolution of K is given by

$$\dot{K} = sY,$$

where $0 < s < 1$. Note that Y, L, K, and A depend on time (t) but we suppress the time argument, while D, a_L, a_K, s, ρ, γ, θ, and β are exogenous and do not depend on time.

For (a)-(c) below we assume that $\theta + \beta < 1$ and that L grows at the constant rate $n > 0$ (i.e., $\dot{L}/L = n$).

(a) Find an expression for the steady-state growth rate of A (denoted g_A^*) in terms of (some or all of) D, a_L, a_K, s, ρ, γ, θ, n, and β. [1 mark]

(b) Find an expression for the steady-state growth rate of K (denoted g_K^*) in terms of (some or all of) D, a_L, a_K, s, ρ, γ, θ, n, and β. [1 mark]

(c) Find an expression for the steady-state growth rate of Y (denoted g_Y^*) in terms of (some or all of) D, a_L, a_K, s, ρ, γ, θ, n, and β. [1 mark]

(d) Now assume that $\theta + \beta = 1$ and that L is constant and equal to one (i.e., $L = 1$ and $n = 0$). Furthermore, assume that the fractions of labor and capital allocated to ideas production are the same, and denoted just a (i.e., $a_L = a_K = a$). Find an expression for the ratio K/A on the balanced growth path (i.e., when the growth rates of A and K are at their steady-state levels). Your answer should be in terms of (some or all of) the exogenous variables D, a, s, ρ, γ, and β. [1 mark]

Midterm Exam – Econ 4020 – v1
18 March 2019
Department of Economics
York University

Instructions: On this exam, you should *not* show how you arrived at your answer, only write the answer in the space indicated on the relevant answer sheet. Answers must be given on the correct answer sheet, one for each problem. If you make a mistake, ask for a new answer sheet. Do not fold the answer sheets or write on the back, or outside of the indicated space.

Student name: SID number:

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where $f(k)$ is the intensive-form production function. We let $y = f(k)$ denote income per effective worker, and $c = (1 - s)y$ consumption per effective worker. Note that k, y, and c are dependent on time but we suppress the time argument, while s, n, g, and δ are strictly positive constants (and $0 < s < 1$). As usual, the steady-state levels of k, y, and c are denoted k^*, y^*, and c^*, respectively.

(a) Let $f(k) = [\beta k^\rho + 1 - \beta]^{1/\rho}$, where $0 < \beta < 1$, and $\rho < 1$. Find an expression for k^* in terms of (some or all of) s, β, ρ, n, g, and δ (and possibly numbers). (It is assumed that these parameters are as such that k^* exists.) [1 mark]

(b) Now let $f(k) = Zk^\beta$, where $Z > 0$ and $0 < \beta < 1$. Find an expression for y^* in terms of (some or all of) s, β, Z, n, g, and δ (and possibly numbers). [1 mark]

(c) Assume the same production function as under (b), i.e., $f(k) = Zk^\beta$. Find an expression for c^* in terms of (some or all of) s, β, Z, n, g, and δ (and possibly numbers). [1 mark]

(d) Now assume a general production function, such that $f'(k) > 0$ and $f''(k) < 0$. Find an expression for c^*/k^* in terms of (some or all of) s, n, g, and δ (and possibly numbers). [1 mark]

2. The Ramsey model [4 marks]

Consider a standard Ramsey model with Cobb-Douglas production, meaning output per effective worker equals Zk^α, where k denotes capital per effective worker, $0 < \alpha < 1$, and $Z > 0$. The dynamics of consumption per effective worker, c, is given by the so-called Euler equation:

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Here k and c are dependent on time but we suppress the time argument, while α, Z, ρ, n, g, and θ are all strictly positive constants.

(a) In the phase diagram provided, draw both the ($\dot{c} = 0$)-locus, and the ($\dot{k} = 0$)-locus, for two values of Z, denoted Z_0 and Z_1, where $Z_1 > Z_0$. You should *not* show any saddle path or other trajectories, only the relevant loci, and clearly indicate which one of them refers to Z_0 and Z_1, respectively. Also indicate the associated steady-state levels of c and k, denoted c_0^* and c_1^*, and k_0^* and k_1^*, for c and k respectively. [2 marks]

(b) Consider an economy which is initially in a steady state associated with $Z = Z_0$. At some point in time, $\widehat{t}$, the level of Z increases to $Z_1 > Z_0$. Draw the time path of k. For full mark, you need to draw the path correctly and indicate $\widehat{t}$, k_0^*, and k_1^* on relevant axes. (Hint: the path of c may be ambiguous but here you are asked about the path of k.) [2 marks]

3. Endogenous growth [10 marks]

Consider a two-sector endogenous growth model, with a goods sector and an "ideas" (or R&D) sector. Total goods output, Y, is given by the production function

$$Y = ([1-a_K]\,K)^{1-\lambda}\,([1-a_L]\,AL)^{\lambda},$$

where λ is an exogenous parameter, such that $0 < \lambda < 1$; A is the level of technology (the number of ideas); L is the total labor force; K is the capital stock; and a_L and a_K are the fractions of L and K, respectively, allocated to ideas production, with the remainder allocated to goods production. (We assume that $0 < a_L < 1$ and $0 < a_K < 1$.) New ideas, $\dot{A}$, are given by the production function

$$\dot{A} = B(a_K K)^{\beta}(a_L L)^{\gamma} A^{\theta},$$

where θ, γ, β, and B are strictly positive. The dynamic evolution of K is given by

$$\dot{K} = sY,$$

where $0 < s < 1$. Note that Y, L, K, and A depend on time (t) but we suppress the time argument, while B, a_L, a_K, s, λ, γ, θ, and β are exogenous and do not depend on time.

For (a)-(c) below we assume $\theta + \beta < 1$ and that L grows at the constant rate $n > 0$ (i.e., $\dot{L}/L = n$).

(a) Find an expression for the steady-state growth rate of A (denoted g_A^*) in terms of (some or all of) B, a_L, a_K, s, λ, γ, θ, n, and β. [1 mark]

(b) Find an expression for the steady-state growth rate of K (denoted g_K^*) in terms of (some or all of) B, a_L, a_K, s, λ, γ, θ, n, and β. [1 mark]

(c) Find an expression for the steady-state growth rate of Y (denoted g_Y^*) in terms of (some or all of) B, a_L, a_K, s, λ, γ, θ, n, and β. [1 mark]

(d) Now assume that $\theta + \beta = 1$ and that L is constant and equal to one (i.e., $L = 1$ and $n = 0$). Furthermore, assume that the fractions of labor and capital allocated to ideas production are the same, and denoted just a (i.e., $a_L = a_K = a$). Find an expression for the ratio K/A on the balanced growth path (i.e., when the growth rates of A and K are at their steady-state levels). Your answer should be in terms of (some or all of) the exogenous variables B, a, s, λ, γ, and β. [1 mark]

V2

Econ 4020 Midterm Exam 18 March 2019 – Answer sheet for Problem 1

Student Name:	SID Number:

Answer to 1 (a):

$$k^* = \left[\frac{(1-\beta)(sz)^\rho}{(n+g+\delta)^\rho - \beta(sz)^\rho}\right]^{\frac{1}{\rho}}$$

Answer to 1 (b):

$$y^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{\beta}{1-\beta}}$$

Answer to 1 (c):

$$c^* = (1-s)\, D^{\frac{1}{1-\beta}} \left(\frac{s}{n+g+\delta}\right)^{\frac{\beta}{1-\beta}}$$

Answer to 1 (d):

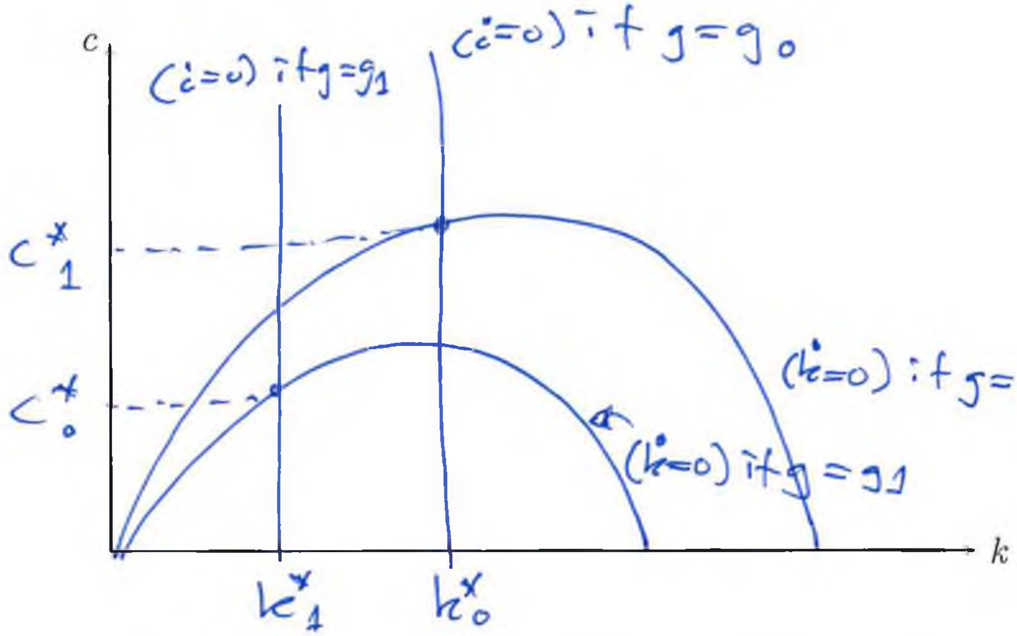
$$\frac{c^*}{k^*} = \left(\frac{1-s}{s}\right)(n+g+\delta)$$

(v2)

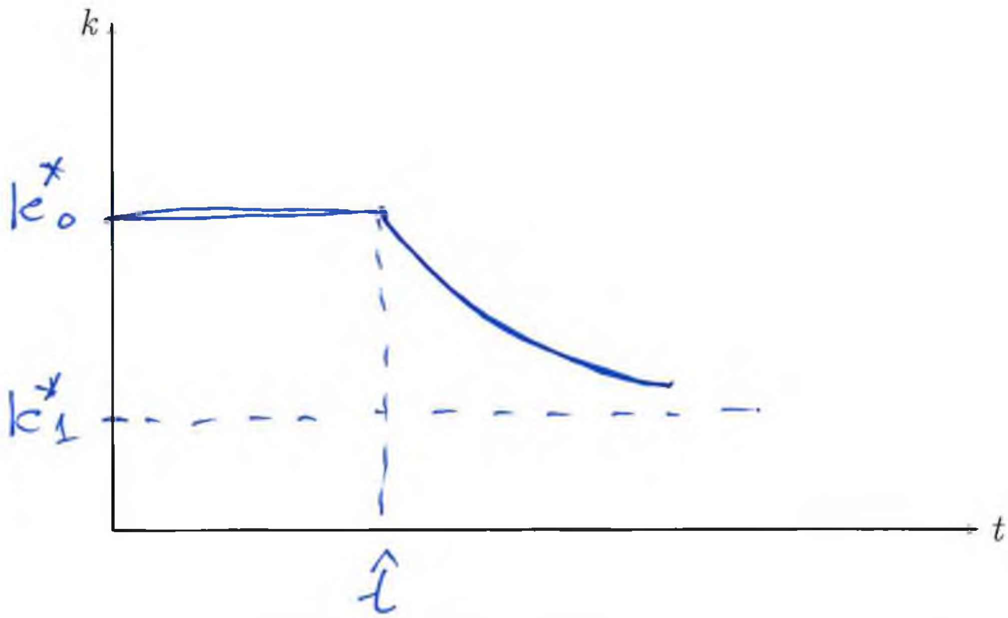
Econ 4020 Midterm Exam 18 March 2019 – Answer sheet for Problem 2

Student Name:	SID Number:

Answer to 2 (a):



Answer to 2 (b):



(V2)

Econ 4020 Midterm Exam 18 March 2019 – Answer sheet for <u>Problem 3</u>

Student Name:	SID Number:

Answer to 3 (a):

$$g_A^* = \frac{n(\gamma+\beta)}{1-\beta-\theta}$$

Answer to 3 (b):

$$g_K^* = \frac{n(1-\theta+\gamma)}{1-\beta-\theta}$$

Answer to 3 (c):

$$g_Y^* = \frac{n(1-\theta+\gamma)}{1-\beta-\theta}$$

Answer to 3 (d):

$$\frac{K}{A} = \left[\frac{s(1-a)}{D\,a^{\beta+\gamma}}\right]^{\frac{1}{\beta+\rho}}$$

V1

Econ 4020 Midterm Exam 18 March 2019 – Answer sheet for Problem 1

Student Name:	SID Number:

Answer to 1 (a):

$$k^* = \left[\frac{(1-\beta)s^\rho}{(n+g+\delta)^\rho - \beta s^\rho}\right]^{\frac{1}{\rho}}$$

Answer to 1 (b):

$$y^* = z^{\frac{1}{1-\beta}} \left(\frac{s}{n+g+\delta}\right)^{\frac{\beta}{1-\beta}}$$

Answer to 1 (c):

$$c^* = (1-s) z^{\frac{1}{1-\beta}} \left(\frac{s}{n+g+\delta}\right)^{\frac{\beta}{1-\beta}}$$

Answer to 1 (d):

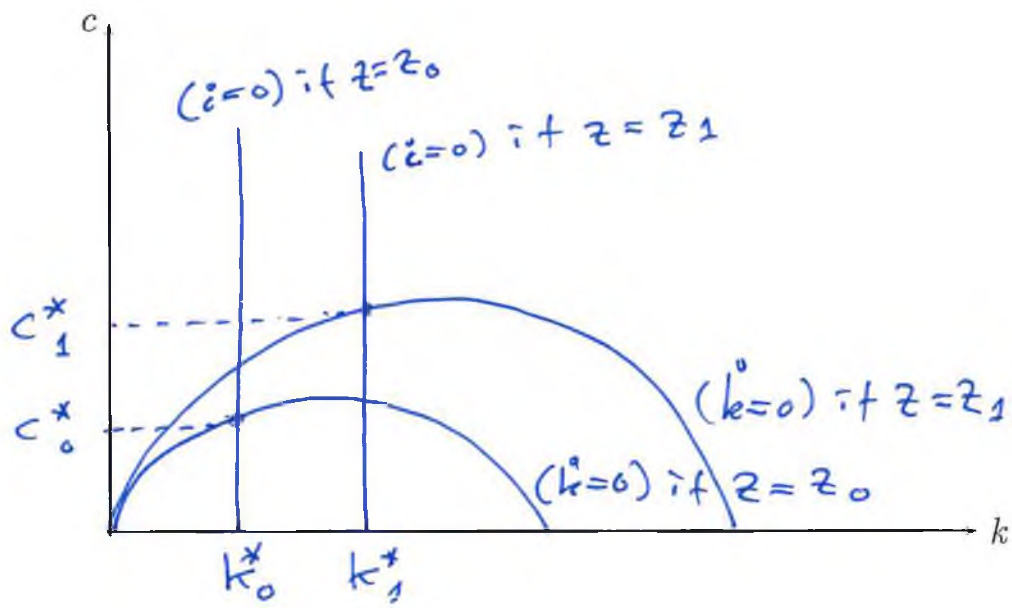
$$\frac{c^*}{k^*} = (1-s)\left(\frac{n+g+\delta}{s}\right)$$

V1

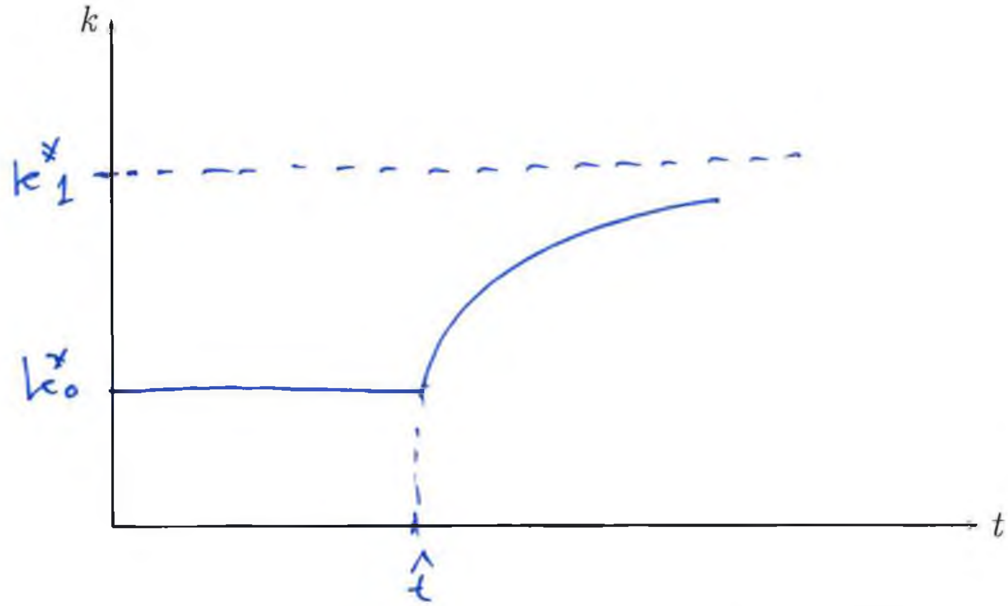
Econ 4020 Midterm Exam 18 March 2019 – Answer sheet for Problem 2

Student Name:	SID Number:

Answer to 2 (a):



Answer to 2 (b):



V1

Econ 4020 Midterm Exam 18 March 2019 – Answer sheet for Problem 3

Student Name: SID Number:

Answer to 3 (a):

$$g_A^* = \frac{n(\beta + \gamma)}{1 - \sigma - \beta}$$

Answer to 3 (b):

$$g_K^* = \frac{n(1 - \sigma + \gamma)}{1 - \sigma - \beta}$$

Answer to 3 (c):

$$g_Y^* = \frac{n(1 - \sigma + \gamma)}{1 - \sigma - \beta}$$

Answer to 3 (d):

$$\frac{K}{A} = \left[\frac{s(1-a)}{B a^{\beta+\gamma}}\right]^{\frac{1}{\beta+\lambda}}$$