Final Exam – Econ 4020
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Part A: do all three problems below

1. The Solow model [10 marks]

Consider a Solow model, where output (Y) is produced with a neoclassical production function, $Y = F(K, AL)$, which exhibits constant returns to scale, meaning that

$$\lambda F(K, AL) = F(\lambda K, \lambda AL)$$

for all $\lambda > 0$. The notation is standard and as usual we let lower-case variables denote per-efficient-worker units, so that $y = Y/(AL)$, $k = K/(AL)$, etc.

(a) Show that there exists an intensive-form production function. That is, show that there exists a function, f, such that $y = f(k) = F(k, 1)$. [4 marks]

(b) Firms choose K and L to maximize profits, π, given by $\pi = F(K, AL) - rK - wAL$, where w is the wage rate and r is the real interest rate. Use the profit maximization problem to find an expression for r in terms of (some or all of) $f(k)$, $f'(k)$, and k. [3 marks]

(c) Use the profit maximization problem described under (b) to find an expression for w in terms of (some or all of) $f(k)$, $f'(k)$, and k. [3 marks]

2. The Ramsey model [10 marks]

Consider a Ramsey model with a general neoclassical production function. We saw in class that the dynamics of consumption per efficient worker, c, are given by the so-called Euler equation:

$$\frac{\dot{c}}{c} = \frac{r - \rho - \theta g}{\theta},$$

where r is the real interest rate, and ρ, θ, and g are all strictly positive exogenous parameters. We use standard notation. In particular, consumption per efficient worker is given by $c = C/A$, where C is consumption per worker, and A is the number of efficiency units per worker, where $\dot{A}/A = g$.

(a) Find an expression for the growth rate of consumption per worker, $\dot{C}/C$, it terms of (some or all of) r, ρ, θ, and g. (Note that we are not asking for the steady state growth rate of C, but the growth rate outside of steady state.) [5 marks]

(b) In steady state c and r are constant and equal to c^* and r^*. Show how r^* changes in response to an increase in g. That is, find an expression for $\partial r^*/\partial g$ and determine its sign. [5 marks]

3. Dynamic consistency [10 marks]

Consider a central banker (a policy maker) who faces a Lucas supply curve determining the output-inflation trade-off:

$$y = \overline{y} + b(\pi - \pi^e),$$

where y is the (log of the) level of output, $\overline{y}$ is the corresponding output level when $\pi = \pi^e$, π is inflation, and π^e is expected inflation. The central banker controls π, and aims at minimizing the following loss function:

$$L = \frac{1}{2}\left[y - y^*\right]^2 + \frac{a}{2}\left[\pi - \pi^*\right]^2,$$

where y^* and π^* denote the central banker's most desired levels of y, and π, respectively. We assume that $y^* > \overline{y}$.

(a) Derive an expression for the inflation outcome under discretion, as a function of exogenous parameters. (This is the level of π denoted π^{EQ} in the book and in class.) [5 marks]

(b) Find an expression for the central banker's loss, L, when output and inflation equal their outcomes under discretion. Your answer should be the factor $\left(y^* - \overline{y}\right)^2/2$ multiplied by something involving a and b. [5 marks]

Part B: do any one of the two problems below

4. The Diamond model [15 marks]

Consider a Diamond model, where we set the productivity factor A_t to unity (1) in all periods. The working population, L_t, grows at rate n, i.e., $L_{t+1} = (1+n)L_t$. Lower-case letters denote per-worker terms, e.g. $k_t = K_t/L_t$. Agents live for two periods. Income earned (from labor) in the first period of life (w_t) is spent on saving (S_t) and first-period consumption (C_{1t}). The first-period budget constraint can thus be written

$$C_{1t} = w_t - S_t.$$

In retirement, the same agent consumes C_{2t+1}, consisting of savings from working age, with interest:

$$C_{2t+1} = S_t(1 + r_{t+1}),$$

where r_{t+1} denotes the interest rate on savings held from period t to period $t+1$.

The utility function is given by

$$U_t = (1-\beta)\ln C_{1t} + \beta \ln C_{2t+1},$$

where $0 < \beta < 1$. Capital accumulates according to $K_{t+1} = S_t L_t$.

(a) Derive an expression for optimal S_t in terms of w_t and exogenous parameters. [10 marks]

(b) Let $w_t = (1-\alpha)Zk_t^{\alpha}$, where $Z > 0$ and $0 < \alpha < 1$. Find an expression for the steady state level of k_t, denoted k^*, in terms of n, Z, α, and β. [5 marks]

5. The Lucas model [15 marks]

Consider the Lucas model, where the aggregate supply curve can be written as

$$y = b[p - E(p)],$$

and aggregate demand as

$$y = m - p,$$

where p is the general price level, m is money supply, and y is output; p, m and y are all stochastic. The general price level, p, is defined as the mean across all p_i, where p_i is the price of good i. (All variables are in logs.)

(a) Find an expression for y in terms of b, m, and $E(m)$. [10 marks]

(b) Let $r_i = p_i - p$ denote the relative price of good i, and let $q_i^S = b[p_i - E(p)]$ be the amount supplied of good i by the agent that produces it. Derive an expression for q_i^S in terms of y, b and r_i. [5 marks]

SOLUTIONS

1.
(a) Set $\lambda = 1/(AL)$, which gives

$$y = \frac{Y}{AL} = \frac{F(K, AL)}{AL} = F\left(\frac{K}{AL}, \frac{AL}{AL}\right) = F(k, 1) = f(k)$$

(b) The trick is to set $F(K, AL) = ALf(\frac{K}{AL})$. The first-order condition for a profit maximum then gives:

$$r = f'(k)$$

(c)

$$w = f(k) - f'(k)k$$

2.
(a)

$$\frac{\dot{C}}{C} = \frac{\dot{c}}{c} + \frac{\dot{A}}{A} = \frac{\dot{c}}{c} + g = \frac{r - \rho}{\theta}$$

(b)

$$r^* = \rho + \theta g \Longrightarrow \frac{\partial r^*}{\partial g} = \theta > 0$$

3.
(a)

$$\pi^{EQ} = \pi^* + \frac{b(y^* - \overline{y})}{a}$$

(b)

$$L = \frac{(y^* - \overline{y})^2}{2}\left[1 + \frac{b^2}{a}\right]$$

4.
(a)

$$S_t = \beta w_t$$

(b)

$$k^* = \left(\frac{\beta Z(1 - \alpha)}{1 + n}\right)^{\frac{1}{1-\alpha}}$$

5.
(a)

$$y = \left(\frac{b}{1 + b}\right)[m - E(m)]$$

(b)

$$q_i^S = br_i + y$$