Some mathematical prerequisites for ECON4010

Some exponential and logarithmic equalities:

$$\ln(xy) = \ln(x) + \ln(y)$$

$$\ln(\tfrac{x}{y}) = \ln(x) - \ln(y)$$

$$\ln(x^a) = a\ln(x)$$

$$e^{x+y} = e^x e^y$$

$$(e^x)^y = e^{xy}$$

To differentiate some common functions:

$$f(x) = x^2 \Rightarrow f'(x) = 2x$$

$$f(x) = x^3 \Rightarrow f'(x) = 3x^2$$

$$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$$

$$f(x) = e^{ax} \Rightarrow f'(x) = ae^{ax}$$

$$f(x) = \ln(x) \Rightarrow f'(x) = \tfrac{1}{x} = x^{-1}$$

$$f(x) = \tfrac{1}{x} = x^{-1} \Rightarrow f'(x) = \tfrac{-1}{x^2} = -x^{-2}$$

Some rules:

$$f(x) = g\left[h(x)\right] \Rightarrow f'(x) = g'\left[h(x)\right]h'(x)$$

$$f(x) = g(x)h(x) \Rightarrow f'(x) = g'(x)h(x) + g(x)h'(x)$$

$$f(x) = \tfrac{g(x)}{h(x)} \Rightarrow f'(x) = \tfrac{g'(x)h(x) - g(x)h'(x)}{\left[h(x)\right]^2}$$

Some tricks:

$$\frac{f'(x)}{f(x)} = \frac{\partial}{\partial x}\left[\ln(f(x))\right]$$

$$f(x) = g(x)h(x) \Rightarrow \frac{f'(x)}{f(x)} = \frac{g'(x)}{g(x)} + \frac{h'(x)}{h(x)}$$

Solving a simple differential equation:

$$f'(x) = af(x) \Rightarrow f(x) = f(0)e^{ax}$$

First-order Taylor approximation: The function

$$g(x) = f(x^*) + f'(x^*)\left[x - x^*\right],$$

which is linear (or affine) in x, is a first-order Taylor approximation of $f(x)$, around x^*.