Econ 5011 - Final Exam
19 April 2005

Part A – do all 3 problems

Problem 1. Consider a standard Solow model. The capital stock per effective worker evolves over time according to

$$\dot{k}(t) = sf(k(t)) - (n + g + \delta)k(t).$$

We let the intensive form production function be given by $f(k) = Zk^{\alpha}$, where $0 < \alpha < 1$, and Z is an exogenous parameter (called Total Factor Productivity, TFP). Steady state $k(t)$ [where $\dot{k}(t) = 0$] is denoted k^*.
(a) Find an expression for k^* in terms of exogenous parameters. [5 marks]
(b) Consider an economy situated in steady state. At time $\hat{t}$ there is an increase in Z. Thereafter Z stays at its new (higher) level indefinitely. Show the time path of $k(t)$ in a diagram with time (t) on the horizontal axis. (Nothing more than a correctly drawn diagram is needed for full mark.) [5 marks]

Problem 2. Consider a social planner maximizing

$$V_0 = \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\theta}}{1-\theta} \right)$$

subject to $K_{t+1} = AK_t^{\alpha} - C_t$, where AK_t^{α} is output, C_t consumption, and K_t the capital stock. There is no leisure and population is constant and normalized to one. Everything is deterministic so you do not need to worry about the expectations operators.
(a) Set up the Bellman equation associated with this problem. This should take the form: $V(K_t) = \max_{C_t} \Psi\left(C_t, V(AK_t^{\alpha} - C_t)\right)$. [5 marks]
(b) Find the Euler Equation. This should be an expression containing C_t/C_{t+1}, K_{t+1}, and exogenous variables. [5 marks]

Problem 3. Consider the following Fischer model of staggered price adjustment. Agents set prices for two periods ahead and the desired price for period t is given by

$$p_{it}^* = (1 - \phi)p_t + \phi m_t,$$

where p_t is the general price level and m_t is money supply. (All lower-case letters denote logarithms.) Half of the agents set prices every second period, so the general price level is given by

$$p_t = \left(\frac{1}{2}\right)\left[p_t^1 + p_t^2\right],$$

where p_t^1 is the price set in period $t-1$, and p_t^2 is the price set in period $t-2$.
Agents set their prices so that

$$p_t^1 = E_{t-1}\left[p_{it}^*\right] \text{ and } p_t^2 = E_{t-2}\left[p_{it}^*\right].$$

(a) Derive an expression for p_t^1 in terms of p_t^2 and $E_{t-1}[m_t]$. (Note that both p_t^2 and p_t^1 are known in period $t-1$.) [3 marks]
(b) Derive an expression for $E_{t-2}[p_t^1]$ in terms of and $E_{t-2}[m_t]$ and p_t^2. [3 marks]
(c) Derive an expression for p_t^2 in terms of $E_{t-2}[m_t]$. [2 marks]
(d) Derive an expression for y_t in terms of m_t and expectations of m_t. (Hint: $y_t = m_t - p_t$.) [2 marks]

Part B – do either problem 4 or problem 5

Problem 4. Consider the system of differential equations derived in the Ramsey model:

$$\dot{c}(t) = \frac{c(t)}{\theta}\left[f'(k(t)) - \rho - \theta g\right] \equiv \Psi(c(t), k(t))$$
$$\dot{k}(t) = f(k(t)) - c(t) - (n+g)k(t) \equiv \Phi(c(t), k(t)).$$

A first-order Taylor approximation of this system is given by

$$\begin{aligned} \dot{c}(t) &= \Psi_c(c^*, k^*)[c(t) - c^*] + \Psi_k(c^*, k^*)[k(t) - k^*] \\ \dot{k}(t) &= \Phi_c(c^*, k^*)[c(t) - c^*] + \Phi_k(c^*, k^*)[k(t) - k^*]. \end{aligned} \tag{*}$$

where c^* and k^* are the steady state levels of $c(t)$ and $k(t)$, given by $\dot{c}(t) = \dot{k}(t) = 0$. Let

$$\frac{c^*}{\theta} f''(k^*) \equiv \gamma < 0,$$

and recall the notation $\beta = \rho - n - (1-\theta)g > 0$. Let $\widetilde{c}(t) = c(t) - c^*$, and $\widetilde{k}(t) = k(t) - k^*$, and use the vector notation

$$z(t) = \begin{bmatrix} \widetilde{c}(t) \\ \\ \widetilde{k}(t) \end{bmatrix}.$$

(a) Find expressions for $\Psi_c(c^*, k^*)$, $\Psi_k(c^*, k^*)$, $\Phi_c(c^*, k^*)$, and $\Phi_k(c^*, k^*)$ in terms of γ and β (and real numbers). [5 marks]

Your answer under (a) and the linearization in (*) imply that we can write

$$\dot{z}(t) = \begin{bmatrix} \dot{\widetilde{c}}(t) \\ \\ \dot{\widetilde{k}}(t) \end{bmatrix} = \mathbf{B}z(t),$$

where $\mathbf{B}$ is a 2×2 matrix.
(b) The eigenvalues of $\mathbf{B}$ are called μ and given by the characteristic equation: $\mu^2 - \mu \text{tr}(\mathbf{B}) + \det(\mathbf{B}) = 0$.[1] Find both eigenvalues of $\mathbf{B}$ in terms of γ and β (and real numbers). You may denote them μ_1 and μ_2. [10 marks]

[1] Recall that, if

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix},$$

then $\det(\mathbf{X}) = x_{11}x_{22} - x_{12}x_{21}$, and $\text{tr}(\mathbf{X}) = x_{11} + x_{22}$.

5. Dynamic consistency of monetary policy [10 marks].

Consider a two-period environment, where a central banker (a policy maker) faces an output-inflation relationship:

$$y_t = \overline{y} + b(\pi_t - \pi_t^e),$$

y_t is output, $\overline{y}$ is the long-run equilibrium output level, π_t is inflation, and π_t^e is expected inflation, and the sub-index denotes the period ($t = 1, 2$).
The central banker controls π, and can be of either Type 1 or Type 2. Both aim at maximizing a welfare function given by

$$W = w_1 + \beta w_2.$$

The $w's$ are given by:

$$\begin{aligned} \text{For Type 1} \quad &: \quad w_t = y_t - \overline{y} - \frac{a}{2}\pi_t^2 \quad \text{for } t = 1, 2 \\ \text{For Type 2} \quad &: \quad w_t = -\frac{a}{2}\pi_t^2 \quad \text{for } t = 1, 2 \end{aligned}$$

The public cannot observe what type the central banker is. The prior probability assigned to the central banker being of Type 1 is denoted p. We assume that

$$\frac{1}{2} < \beta < \frac{1}{2}\frac{1}{1-p}.$$

Timing is discretionary, i.e., the public forms π_t^e before π_t is set. Since Type 2 always sets $\pi_1 = \pi_2 = 0$, the only way Type 1 can mimic being Type 2 is by setting $\pi_1 = 0$ in the first period.

(a) Find the payoff (the level of W) for a Type-1 central banker who sets $\pi_1 > 0$, and thus "informs" the public of his true type. (This is denoted W^{INF} in the book and in class.) [5 marks]
(b) Consider a mixed-strategy equilibrium where Type 1 mimics Type 2 with probability q. It can be seen that the public then assigns the following probability to the central banker being of Type 1: $\frac{qp}{1-p+qp}$. Use this to find the payoff to the Type-1 banker from mimicking Type 2. [This is denoted $W_0(q)$ in the book and in class.] [5 marks]
(c) Find the equilibrium level of q. [5 marks]

All these problems are from old exams and problems sets (for 4010 and 5011), so solutions should be available there