Aggregation in Ramsey Model

Recall dynamic system ($n=1$, β repl. α)

$$\boxed{\begin{aligned} \frac{u'(c_t)}{u'(c_{t+1})} &= \beta(1+r_{t+1}) \\ k_{t+1} &= w_t + (1+r_t)k_t - c_t \end{aligned}}$$

c_t usually interpreted as cons. of representative agent

Q: What if there are many different agents? Can we still think of world as if it had a rep. agent?

A: Yes, if we impose assumptions.

Assumptions

$$u_i(c_{i,t}) = \begin{cases} \dfrac{[c_{i,t} - \bar{c}_i]^{1-\theta}}{1-\theta} & \text{if } \theta \neq 1, \ \theta > 0 \\ \ln(c_{i,t} - \bar{c}_i) & \text{if } \theta = 1 \end{cases}$$

$$V_{i,t} = \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \quad (\beta \text{ indep. of } i)$$

Fraction of type i, π_i, is constant for all $i \in \{1, 2, \ldots, I\}$

Agents face same r_t, $t \in \{0, 1, \ldots \infty\}$

(But different wages, $w_{i,t}$)

Euler + budget constraint

for each agent i

$$\boxed{\begin{aligned} \frac{u'(c_{i,t})}{u''(c_{i,t+1})} &= \beta(1+r_{t+1}) && (1) \\ k_{i,t+1} &= w_{i,t} + (1+r_t)k_{i,t} - c_{i,t} && (2) \end{aligned}}$$

Let $\sum_{i=1}^{I} \pi_i c_{i,t} = c_t$ etc.

Then (2) becomes

$$\boxed{k_{t+1} = w_t + (1+r_t)k_t - c_t}$$

Examining (1)

$$[c_{i,t} - \bar{c}_i]^{-\theta} = \beta(1 + r_{t+1})[c_{i,t+1} - \bar{c}_i]^{-\theta}$$

$$c_{i,t} - \bar{c}_i = [\beta(1 + r_{t+1})]^{-\frac{1}{\theta}}[c_{i,t+1} - \bar{c}_i]$$

Multiply by π_i, sum from 1 to I,

$$c_t - \bar{c} = [\beta(1 + r_{t+1})]^{-\frac{1}{\theta}}[c_{t+1} - \bar{c}]$$

So: $$\frac{u'(c_t)}{u'(c_{t+1})} = \beta(1 + r_{t+1})$$

still holds!

Note: crucial that β, θ, r_t same for all i, and π_i constant (same for all t)