The Fernández (2012) model

(earlier version 2009)

No Property Rights: women's capital is confiscated by the husband

Budget constrains

$$c_w = \underline{c} \tag{1}$$

$$c_h = A(k_h + k_w) - \underline{c} - nk'_h - nk'_w \tag{2}$$

$\underline{c}$ = woman's consumption

Value functions

$$V_h(k_h + k_w) = \max_{c_h, k'_h, k'_w} \left\{ \ln(c_h) + \frac{\beta}{2} \left[V_h(k'_h + k'_w) + V_w(k'_h + k'_w) \right] \right\} \quad (3)$$

$$V_w(k_h + k_w) = \ln(\underline{c}) + \frac{\beta}{2} \left[V_h(k'_h + k'_w) + V_w(k'_h + k'_w) \right] \quad (4)$$

k_h = capital of man

k_w = capital of woman (his spouse)

k'_h = capital of son

k'_w = capital of daughter

Notes:

Efficient marriage markets; as if marrying siblings

Both spouses get same joy of children's utility

Value functions have only one argument: sum of spouses' capital stocks

Finding optimal $k' = k'_h + k'_w$ and c_h as functions of $k = k_h + k_w$

Method of undetermined coefficients:

guess functional forms for V_h and V_w with coefficients a_h, b_h, a_w, b_w;
take FOC;
substitute optimal choices for c_h, k'_h, k'_w (in terms of a_h, b_h, a_w, b_w) back into V_h and V_w;
check same functional form;
then solve for the undetermined coefficients a_h, b_h, a_w, b_w

See appendix of the paper for details

Solution:

$$c_h^{NR} = \left(\frac{1-\beta}{1-\frac{\beta}{2}}\right) A \left[k - \frac{\underline{c}}{A-n}\right] \tag{5}$$

$$k'_{NR} = \frac{\frac{\beta}{2}\left(Ak - \underline{c}\right) + \left(1-\beta\right)\left[\frac{n\underline{c}}{A-n}\right]}{n\left(1-\frac{\beta}{2}\right)} \tag{6}$$

Assumptions:

$$k_0 > \frac{\underline{c}}{A}\left[\frac{1-\frac{\beta}{2}}{1-\beta}+\frac{A}{A-n}\right] \tag{7}$$

$$\beta A > (2-\beta)n \tag{8}$$

(8) ensures sustained growth: $k'_{NR} > k_{NR}$

(7) ensures that $c_h^{NR} > \underline{c}$ in the first period; thus $k'_{NR} > k_{NR}$ ensures the same in all subsequent periods

Equal Property Rights: husband and wife maximize equally weighted sum of welfare

Budget constraint

$$c_h + c_w = A(k_h + k_w) - nk'_h - nk'_w \tag{9}$$

Value functions

$$V_h(k_h + k_w) + V_w(k_h + k_w) = \max_{c_h, c_w, k'_h, k'_w} \left\{ \ln(c_h) + \ln(c_w) + \beta \left[V_h(k'_h + k'_w) + V_w(k'_h + k'_w) \right] \right\} \tag{10}$$

Method of undetermined coefficients: see appendix again

Solution:

$$c_h^{ER} = c_w^{ER} = \left(\frac{1-\beta}{2}\right) Ak \tag{11}$$

$$k'_{ER} = \frac{\beta Ak}{n} \tag{12}$$

Note that $\beta A > n$ holds given assumption in (8); thus sustained growth ensured, $k'_{ER} > k_{ER}$

Value functions under NR and ER

$$V_h^{ER}(k) = V_w^{ER}(k) = \phi + \left(\frac{1}{1-\beta}\right) \ln(k) \tag{13}$$

where ϕ is complicated function of exogenous parameters (β, A, n)

$$\begin{aligned} V_h^{NR}(k) &= a_h + \left(\frac{1-\frac{\beta}{2}}{1-\beta}\right) \ln\left(k - \frac{\underline{c}}{A-n}\right) \\ V_w^{NR}(k) &= a_w + \left(\frac{\frac{\beta}{2}}{1-\beta}\right) \ln\left(k - \frac{\underline{c}}{A-n}\right) \end{aligned} \tag{14}$$

where a_h and a_w are complicated functions of exogenous parameters (β, A, n)

Let the husband's gain from being in a NR regime be denoted

$$\Delta(k) = V_h^{NR}(k) - V_h^{ER}(k) \tag{15}$$

Note that $\Delta''(k) < 0$

Setting $\Delta'(k) = 0$ shows that $\Delta(k)$ is maximized at $k = \widehat{k}$, where

$$\widehat{k} = \frac{2\underline{c}}{\beta(A - n)} \tag{16}$$

Also: there exists that k^* such that for $k > k^*$ it holds that $\Delta(k) < 0$, and for $\widehat{k} < k < k^*$ it holds that $\Delta(k) > 0$

Assume that men choose property rights regime in the next period; since k grows over time also under the NR regime, this implies an endogenous transition to an ER regime at the point when k exceeds k^*

Also: k^* increasing in n; low fertility means earlier transition from NR to ER