The Mulligan-Tsui (2008, 2010) model

Idea:

Rent extracting regime(s) face potential challengers

Can erect barriers to entry and/or limit taxation to pacify the populace

Two types of policy

- entry barriers, b
- rents, r

In 2008 version, also public good (social/economic policies etc.)

Time structure: continuous time, but discrete regimes

Regime t replaced by regime $t+1$, etc.

Notation: x refers to regime t, and $\widehat{x}$ to regime $t+1$

The (incumbent) regime

$v(R)$ = value for a regime of governing over an interval of length R with certainty

r = "rents" (=revenue collected by the regime)

b = barriers to entry for opponents

β = exogenous cost of buying one unit of b

i = exogenous interest (or discount) rate

$$v(R) = (r - \beta b) \int_0^R e^{-is} ds = (r - \beta b) \frac{1 - e^{-iR}}{i} \tag{1}$$

$n =$ number of challengers facing incumbent

$h =$ success hazard for each challenger

$\overline{h} =$ average h, taken as given by each potential challenger

Then the probability (density) that the regime lasts for an interval of length R becomes

$$n\overline{h}e^{-n\overline{h}R} \tag{2}$$

Exercise: check that

$$\int_0^\infty n\overline{h}e^{-n\overline{h}R}dR = 1 \tag{3}$$

$V =$ expected value of being incumbent regime

Integrate over possible outcomes of regime length, R

$$V = \int_0^\infty n\overline{h}e^{-n\overline{h}R}v(R)dR = \frac{r - \beta b}{i + n\overline{h}} \tag{4}$$

The public

Representative citizen (consumer)

Cares about disposable income $(y - r)$

$$u = u(y - r) \tag{5}$$

$y =$ exogenous pre-tax income per citizen

The challengers

Each of the challengers' success hazard is

$$h = s\frac{u(y-\widehat{r})}{u(y-r)} = \frac{s\widehat{u}}{u} \tag{6}$$

Idea: success more likely if public is disgruntled; incentive for incumbent regime to tax less

$s =$ exogenous parameter $=$ success hazard in steady state

$w =$ payoff if not challenging

$b =$ entry barrier, i.e., cost of challenging (recall: set by incumbent at cost βb)

The potential challenger's optimality condition states that

$$\underbrace{\frac{h\widehat{V}}{i + n\overline{h}} - b}_{\text{payoff if challenging}} \quad \leq \quad \underbrace{w}_{\text{payoff if not}} \tag{7}$$

Must hold with equality if $n > 0$

Equilibrium

All challengers identical, $\overline{h} = h$

Incumbent regime maximizes payoff

$$V = \max_{b \geq 0, r \geq 0, n \geq 0} \frac{r - \beta b}{i + nh} \tag{8}$$

subject to

$$\frac{h\widehat{V}}{i + nh} - b - w \leq 0 \tag{9}$$

$$h = \frac{su(y - \widehat{r})}{u(y - r)} \tag{10}$$

$$\begin{aligned} r - \beta b &\geq 0 \\ b &\geq 0 \\ n &\geq 0 \\ r &\leq \tau y \end{aligned} \tag{11}$$

Exogenous parameters: $\tau > 0$ (max tax rate); and y, w, i, β and s (explained above)

Note: incumbent regime takes as given actions by a potentially successful challenger (the "hat" variables)

Below: focus on stationary equilibria (steady states), meaning $V = \widehat{V}$, $u = \widehat{u}$, $h = s$

Seven types of stationary equilibria can arise

Here look at three where $n = 0$

Can be shown to always hold in equilibrium if $s < i^2/\beta \equiv s_c$

- "unthreatened leviathan" ($r = \tau y$, $n = b = 0$)
- "threatened leviathan" ($r = \tau y$, $n = 0$, $b > 0$)
- "contestable market" ($r < \tau y$, $n = 0$, $b > 0$)

Next: finding parametric conditions under which each of these three equilibria exists

Assume

$$\frac{u(y(1-\tau))}{u'(y(1-\tau))} < \tau y \tag{12}$$

Explained later

(a) Unthreatened leviathan

Given $n = 0$, incumbent regime maximizes

$$V = \frac{r - \beta b}{i} \tag{13}$$

subject to

$$r \leq \tau y \tag{14}$$

$$\frac{h\widehat{V}}{i} - b - w \leq 0 \tag{15}$$

For $b = 0$ and $r = \tau y$ to be optimal to the incumbent, in equilibrium it must hold that

$$\frac{sV}{i} - w = \frac{s(\frac{\tau y}{i})}{i} - w \leq 0 \tag{16}$$

or

$$s \leq \frac{i^2 w}{\tau y} \equiv s_\pi \tag{17}$$

Means that if, and only if, $s \leq s_\pi$, then there exists an equilibrium where $r = \tau y$, $n = b = 0$

Interpretation: no potential challenger undertakes a challenge ($n = 0$), even when $b = 0$ and $r = \tau y$

(b) Threatened leviathan

If $b > 0$ and $n = 0$ it must hold that the barrier is set high enough to deter challengers

$$\frac{h\widehat{V}}{i} - b - w = 0 \tag{18}$$

Incumbent regime maximizes

$$V = \left(\frac{1}{i}\right)\left\{ r - \beta \underbrace{\left[\frac{1}{i}\left(\frac{su(y-\widehat{r})}{u(y-r)}\right)\widehat{V} - w\right]}_{b} \right\} \tag{19}$$

Check that $dV/dr > 0$ at conjectured equilibrium; means $r \leq \tau y$ is binding

$$\frac{dV}{dr} = \left(\frac{1}{i}\right)\left\{1 - \beta\frac{s}{i}\frac{\widehat{u}u'}{u^2}\widehat{V}\right\} > 0 \tag{20}$$

Set $u = \widehat{u}$, $V = \widehat{V}$, $r = \tau y$

$$\frac{u(y-r)}{u'(y-r)} = \frac{u(y(1-\tau))}{u'(y(1-\tau))} > \frac{s\beta V}{i} \tag{21}$$

To find V use incumbent's objective function, imposing equilibrium

$$V = \left(\frac{1}{i}\right)\left\{\underbrace{\tau y}_{r} - \beta\underbrace{\left[\frac{sV}{i} - w\right]}_{b}\right\} \tag{22}$$

Solve for V

$$V = \frac{i(\tau y + \beta w)}{i^2 + \beta s} \tag{23}$$

Now (21) and (23) give this condition:

$$\frac{u(y(1-\tau))}{u'(y(1-\tau))} > \frac{s\beta}{i}\frac{i(\tau y + \beta w)}{i^2 + \beta s} = \underbrace{\left(\frac{s\beta}{i^2 + \beta s}\right)}_{<1}(\tau y + \beta w) \tag{24}$$

Note: RHS of (24) increasing in s, and always less than $\tau y + \beta w$

Note that $\tau y + \beta w > u(y(1-\tau))/u'(y(1-\tau))$ holds given (12) above (Assumption 3 in the paper)

Thus there exists some s at which (24) takes equality (draw figure to check); call that s_τ

$$s_\tau = \left[\underbrace{\frac{u'(y(1-\tau))}{u(y(1-\tau))} (\tau y + \beta w)}_{>1} - 1 \right] \left(\frac{i^2}{\beta} \right) > 0 \tag{25}$$

Thus, $s < s_\tau$ must hold for incumbent to set $r = \tau y$

Next verify that incumbent sets $b > 0$; requires that

$$\frac{sV}{i} \geq w \tag{26}$$

Using expression for V above:

$$s \geq \frac{w(i^2 + \beta s)}{\tau y + \beta w} \tag{27}$$

Solving for s:

$$s \geq \frac{wi^2}{\tau y} \equiv s_\pi \tag{28}$$

Reverse of condition for equilibrium of type (a)

(c) Contestable market

Since $b > 0$ and $n = 0$ we can use (18) and (19)

Check that $dV/dr < 0$ at $r = \tau y$

$$\frac{dV}{dr} = \left(\frac{1}{i}\right)\left\{1 - \beta\frac{s}{i}\frac{\widehat{u}u'}{u^2}\widehat{V}\right\} < 0 \tag{29}$$

Gives same condition as in (24) except reversed

Overview of results

Use (25) and (28) to check that $s_\pi < s_\mathcal{T}$; hinges on (12)

Assumed $s < i^2/\beta = s_\mathcal{C}$; means $n = 0$ (not shown here)

Illustrate the three equilibria on $[0, s_\mathcal{C}]$

- $s < s_\pi$: unthreatened leviathan
- $s \in (s_\pi, s_\mathcal{T})$: threatened leviathan
- $s \in (s_\mathcal{T}, s_\mathcal{C})$: contestable market

Moving from low s to higher; increasing threats to regime

Resulting transitions:

- First rise in barriers ($b > 0$), while maintained maximum taxation $r = \tau y$
- Eventually lower taxation ($r < \tau y$)