Practice problems for midterm 1

1. Consider the difference equation

$$x_{t+1} = 2x_t(2 - x_t).$$

(a) Use your favorite software for simple numerical simulations (like Excel or Matlab) to generate a time path for x_t from this difference equation. Run the simulation for 20-30 periods. Hand in a plot showing the time paths for three different values of x_0. (You do not need to show any code.)

(b) With at least 6 decimals, find some value of x_0 that makes $x_5 = 0.15$. Explain how you went about. (Trial and error is not cool.) Is x_0 unique?

2. Consider the difference equation for k_t generated by the discrete-time Solow model and shown in eq. (5) in the 5750 Lecture Notes (Part I). Find the steady state level of k_t when the production function is:
(a) Cobb-Douglas, and
(b) CES.

3. Starting from eq. (15) in the 5750 Lecture Notes (Part I), write eq. (16) correctly (without existing typos, if any).

4. Let

$$R(t) = \int_0^t r(\tau)d\tau.$$

(a) Use the Euler equation from a continuous-time Ramsey model, as discussed in class, but with constant technology and population ($n = g = 0$) and log utility [$u(c) = \ln(c)$] to derive an expression for $c(t)$ in terms of $c(0)$, $R(t)$, t, and ρ.

Consider an agent who owns only initial capital of $k(0)$ and has no wage income, so that $w(t) = 0$ for all $t \geq 0$. The agent's budget constraint states that the present value of all future consumption streams must sum up to $k(0)$. This can be written

$$\int_0^\infty e^{-R(t)}c(t)dt = k(0).$$

(b) Use this budget constraint and your answer under (a) to derive an expression for $c(0)$.

Practice problems for midterm 2

1. Consider a standard Malthusian framework similar to e.g. the model in Ashraf and Galor (2010), but with different notation. This is an OLG model where agents live in two period, as dependent children and working adults. Adults active in period t earn y_t and consume c_t; they have n_t children and each child consumes/costs an exogenous q units of the consumption good. Total adult population in period t is P_t.

(a) Write the budget constraint for an adult in period t.

(b) Let utility be

$$U_t = (1-\beta)\ln c_t + \beta \ln n_t,$$

where $\beta \in (0,1)$ is an exogenous preference parameter. Find fertility as a function of the per-adult income, y_t.

(c) Total output (and income) in period t is

$$Y_t = (A_t L)^{\alpha} P_t^{1-\alpha},$$

where L is an exogenous amount of land and A_t is land productivity. What is income per adult in period t?

(d) At given A_t, what is the dynamic equation for P_t? Illustrate the dynamics in a 45-degree diagram.

(e) If A_t is constant at some level A, what is the steady state level of adult population?

(f) Let $A_{t+1} = (1+g)A_t$, for some $g > 0$. What is the growth rate of population on the balanced growth path? Find per-adult income on the balanced growth path as a function of exogenous parameters. (Hint: P_t/A_t is constant on the balanced growth path.)

2. This problem explains the Method of Undetermined Coefficients, using a simpler, one-sex version of the model in Fernández (2010) discussed in class. The mechanics are also similar to Problem 13 in the 5750 problem sets (referring to the Lucas property rights model).

Consider the Bellman equation

$$V(k_t) = \max_{k_{t+1}\geq 0} \left\{ \ln(Ak_t - nk_{t+1}) + \beta V(k_{t+1}) \right\},$$

where n is the number of children, and the other notation is just standard.

Guess

$$V(k_t) = a + b\ln(k_t).$$

(a) Find b.

(b) Find a.

(c) Find a condition in terms of β, n and A that ensures sustained growth in k_t.

3. Consider a version of the Hansen-Prescott model, different from the one set up in class and closer to the one in the original paper, by letting there be property rights to land and letting the land price in period t be q_t per unit. Young agents buy land to rent out when old and then sell.

(a) Write the representative agent's budget constraints, determining consumption when young and old, letting l_t be the amount of land (s)he buys.

(b) Show that $q_{t+1} = q_t(1 + r_{K,t+1}) - r_{L,t+1}$ must hold in equilibrium if agents choose to hold both land and capital.

4. Consider the Galor-Weil (2000) model.

(a) Show how to derive the expression for $G(e_{t+1}, g_{t+1})$ and explain in your own words why optimal e_{t+1} is zero when $G(0, g_{t+1}) < 0$? (Note typo in paper and slides.)

(b) Why do Galor and Weil assume that $G(0, 0) < 0$? Explain in your own words.

(c) Show how to derive the solution to Problem 15 (c) in the 5750 problems from winter 2004. You may use the solutions to 15 (a) and (b).

(d) Name two differences in the assumptions underlying the Hansen-Prescott and Galor-Weil models.

Solution to Problem 1 under Practice problems for midterm 2

(a) $c_t = y_t - qn_t$

(b) $n_t = (\beta/q)y_t$

(c) $y_t = (A_tL/P_t)^\alpha$

(d) $P_{t+1} = n_tP_t = (\beta/q)(A_tL)^\alpha P_t^{1-\alpha}$

(e) $\overline{P} = AL(\beta/q)^{1/\alpha}$

(f) With P_t/A_t being constant and $A_{t+1}/A_t = 1 + g$, it follows that $A_{t+1}/A_t = P_{t+1}/P_t = n_t = (\beta/q)y_t = 1 + g$. This gives steady state y_t as $\overline{y} = q(1 + g)/\beta$.