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Tobin's q (Acemoglu Sec. 7.7)

Installation costs of capital, $\phi(i)$

Firm's problem: max pres. value of profit flows

$$\boxed{PV \text{ of profits} = \int_0^\infty e^{-rt}\left[f(k(t)) - i(t) - \phi(i(t))\right]dt}$$

r = constant interest rate

$\phi'(i)>0$, $\phi''(i)>0$

$$\boxed{\dot{k}(t) = i(t) - \delta k(t)}$$

Present-value Hamiltonian

$$H(k,i,\lambda) = e^{-rt}\left[f(k) - i - \phi(i)\right] + \lambda\left[i - \delta k\right]$$

Let $q = e^{rt}\lambda$

Current-value Hamiltonian

$$H(k,i,q) = \left[f(k) - i - \phi(i)\right] + q\left[i - \delta k\right]$$

$$\boxed{\begin{aligned} H_i(k,i,q) &= -1 - \phi'(i) + q = 0 \\ H_k(k,i,q) &= f'(k) - \delta q = rq - \dot{q} \end{aligned}}$$

Transversality:

$$\lim_{t \to \infty} e^{-rt} q(t) k(t) = 0$$

Using Present-Value Hamiltonian

$$\boxed{\begin{aligned} H_{i} &= e^{-rt}\left[-1-\phi'(i)\right]+\lambda = 0 \\ H_{k} &= e^{-rt} f'(k) - \delta\lambda = -\dot{\lambda} \end{aligned}}$$

Transversality:

$$\lim_{t\to\infty} \lambda(t)k(t) = 0$$

$$e^{-rt}\left[1+\phi'(i)\right] = \lambda$$

$$\boxed{1+\phi'(i) = e^{rt}\lambda \equiv q}$$

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$$f'(k) - \delta e^{rt}\lambda = -\dot{\lambda} e^{rt}$$

$$\boxed{f'(k) - \delta q = rq - \dot{q}}$$

<u>Check:</u>

$$q = e^{rt}\lambda$$

$$\dot{q} = r e^{rt}\lambda + e^{rt}\dot{\lambda}$$

$$\dot{q} = r q + e^{rt}\dot{\lambda}$$

$$\boxed{-\dot{\lambda} e^{rt} = r q - \dot{q}}$$

From $H_i(\cdot) = 0$

$$q = 1 + \phi'(i)$$

$$\dot{q} = \phi''(i)(\dot{i})$$

Note:

$$(\dot{i}) = \frac{\partial i}{\partial t}$$

Use $H_k(\cdot) = rq - \dot{q}$

$$f'(k) - \delta q = rq - \dot{q}$$

$$\dot{q} = (r+\delta)q - f'(k) = \phi''(i)(\dot{i})$$

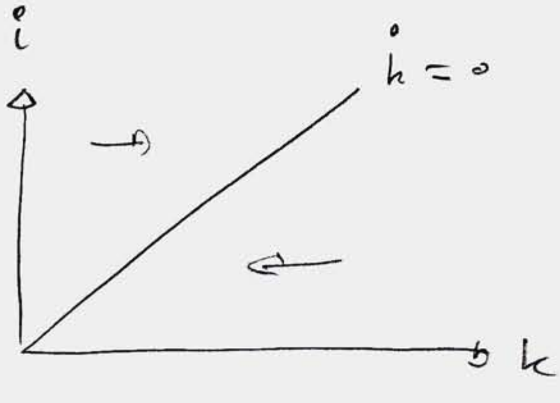
$$(\dot{i}) = \left[\frac{1}{\phi''(i)}\right]\left[(r+\delta)[1+\phi'(i)] - f'(k)\right]$$

Dynamical System

$$\boxed{\dot{(i)} = \frac{1}{\phi''(i)}\left[(r+\delta)\left[1+\phi'(i)\right] - f'(k)\right]}$$

$$\boxed{\dot{k} = i - \delta k}$$

$\underline{\dot{k} = 0}$ when $i = \delta k$



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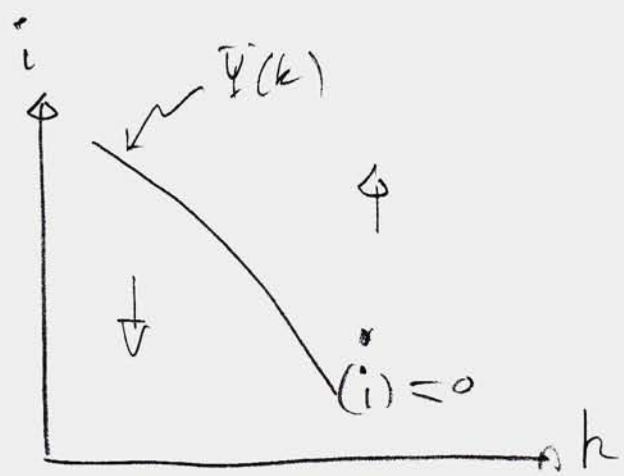
$\underline{\dot{(i)} = 0}$ when $i = \Psi(k)$

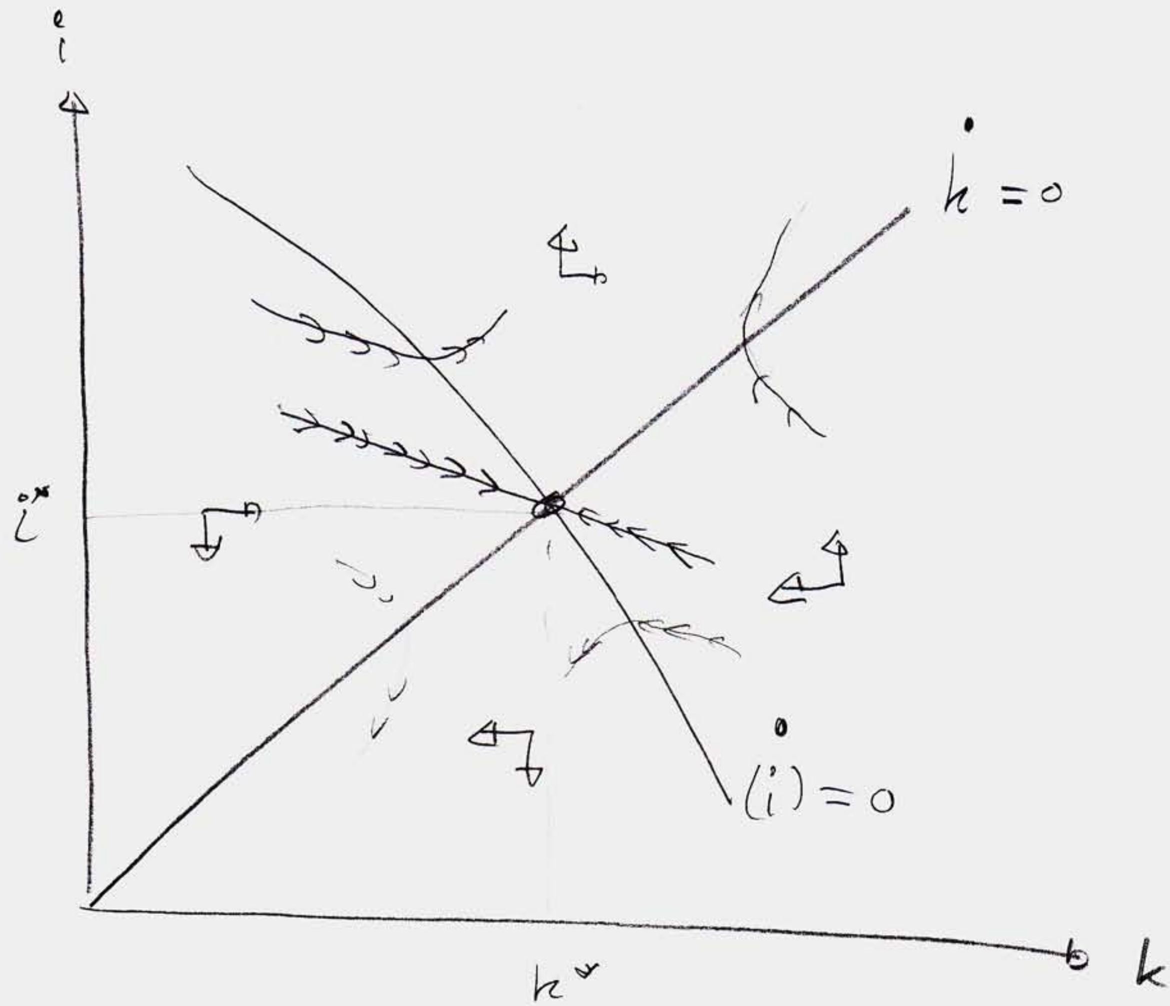
$$(r+\delta)\left[1+\phi'(\Psi(k))\right] = f'(k)$$

$$\phi'(\Psi(k)) = \frac{f'(k)}{r+\delta} - 1$$

$$\phi''(i)\,\Psi'(k) = \frac{f''(k)}{r+\delta}$$

$$\Psi'(k) = \left[\frac{1}{\phi''(i)}\right]\frac{f''(k)}{r+\delta}$$





Transversality Condition

$\Rightarrow$ given $k(0)$, set $i(0)$ to be on saddle path (stable arm)