Midterm Exam – Econ 5700
12 October 2016
Department of Economics
York University

Notes: For Problems 1 and 2 you must explain how you arrived at your answers, unless the answer is just one word. For Problem 3 you only need to state the correct answers, not explain or motivate anything.

1. The Malthus Model [10 marks]

Consider a Malthusian model with a utility function given by

$$U_t = (1-\beta)\ln(c_t - \widehat{c}) + \beta \ln(n_t),$$

where c_t is adult consumption, $\widehat{c} > 0$ subsistence consumption, n_t the number of children, and β a parameter (where $0 < \beta < 1$). The adult population, L_t, evolves according to $L_{t+1} = n_t L_t$. Income per agent is $y_t = (A/L_t)^\alpha$, where A and α are parameters (such that $A > 0$ and $0 < \alpha < 1$). The cost per child is $q > 0$.

(a) Derive optimal fertility in terms of y_t and some parameter(s) of the model. [3 marks]

(b) Find a function ϕ, such that $L_{t+1} = \phi(L_t)$. Illustrate in a 45°-diagram that it is *possible* for population dynamics to display oscillatory cycles. [3 marks]

(c) Find one, or more, necessary and sufficient conditions in terms of the model parameters α, β, q, and $\widehat{c}$, under which the steady state is (locally) stable and *non*-oscillatory. (Hint: the derivative of ϕ evaluated in steady state must be between 0 and 1.) [4 marks]

2. The Solow Model and Barro Regressions [10 marks]

Recall the Solow model discussed in class. With full depreciation ($\delta = 1$), we can write the capital stock in period $t+1$ as

$$k_{t+1} = \frac{s y_t}{1+n}. \tag{1}$$

where y_t denotes per-capita GDP, n is population growth, and s is the rate of saving and investment.

(a) Let $y_t = A[\alpha k_t^\rho + 1 - \alpha]^{1/\rho}$, where $A > 0$, $0 < \alpha < 1$, and $\rho < 1$. Rewrite (1) as a difference equation expressing y_{t+1} in terms of y_t and exogenous parameters. [2 marks]

(b) Now instead let $y_t = A k_t^\alpha$, where $A > 0$ and $0 < \alpha < 1$. Write the growth rate of y_t, here defined as $g_t = \ln(y_{t+1}) - \ln(y_t)$, in terms of $\ln(y_t)$ and exogenous parameters. [2 marks]

(c) Suppose you run a growth regression based on this specification:

$$g_{i,t} = \gamma + \beta \ln(y_{i,t}) + \varepsilon_{i,t}, \tag{2}$$

where the notation is standard (i denoting country, and t the year or period). What set of parameters appearing in your answer to (b) would correspond to γ and β in (2)? [1 mark]

(d) When each observation in the data is represented by one country we usually call it a cross section (meaning variables do not carry a time index). What do we call a data set that is structured such that each observation represents one country *and* one year (or one period), as assumed in (2)? [1 mark]

(e) In column (1) of the attached regression table from Barro's paper, the democracy index takes values from 0 to 1. Approximately what value of the democracy index would predict the fastest growth rate, all else equal? Show how you found that value. (If you cannot compute even an approximate value on paper, you may answer with an expression that defines it.) [2 marks]

(f) What effects on growth from female schooling did Barro find? Explain briefly. [2 marks]

3. Stata Coding [10 marks]

Consider the code below, with segments hidden by "[#]" in six places.

```
#delimit;
set obs 20; egen x = seq(), f(1) t(20);
gen y=x^2;
gen z=[1]+100;
label var y "Variable A";
label var z "Variable B";
label var x "Variable [2]";
xtile [3] = x, n(4);
twoway
(scatter y x if w<=[4], msymbol(+) mcolor(green) )
(scatter y x if w>=[5], msymbol(x) mcolor(red) )
(scatter z x, msymbol(o) mcolor(blue) mlabel([6]));
```

The two commands on the second line are new compared to what we learned in class, but for this problem all you need to know is that they produce a variable, x, containing 20 observations, taking values 1, 2,...,20 (i.e., all integers from 1 to 20). The rest of the code produces the attached figure. (You may not tell the colors, but that should not be needed to solve this problem.)

(a) Which are the hidden codes? Answer with the exact code and the numbered segment it replaces, **[1]** to **[6]**. [6 marks]

(b) Next add the following code: `xtset w x, delta(1); gen q=F.y-y;` Which are the first four values of the generated variable `q` (for which `x` takes values 1, 2, 3 and 4)? (If you cannot compute the numbers by hand you may answer with expressions that define them.) [4 marks]

Answer sheet for Problem____Econ 5700, Midterm 12 October 2016

Student Name: SID Number:

Write your answers below. Do **not** fold the answer sheets or write on the back.

Sketches of solutions

1. (a) The first-order condition for utility maximization says

$$(1-\beta)[y_t - \widehat{c} - qn_t]^{-1}q = \beta[n_t]^{-1},$$

which gives

$$n_t = \frac{\beta}{q}(y_t - \widehat{c}).$$

(b)

$$\begin{aligned} L_{t+1} &= n_t L_t = \frac{\beta}{q}\left[y_t - \widehat{c}\right] L_t, \\ &= \frac{\beta}{q}\left[y_t L_t - \widehat{c} L_t\right] = \frac{\beta}{q}\left[A^\alpha L_t^{1-\alpha} - \widehat{c} L_t\right] = \phi(L_t). \end{aligned}$$

To draw the graph of $\phi(L_t)$, note that

$$\begin{aligned} \phi'(L_t) &= \frac{\beta}{q}\left[(1-\alpha)A^\alpha L_t^{-\alpha} - \widehat{c}\right], \\ \phi''(L_t) &= -\frac{\alpha\beta(1-\alpha)}{q}A^\alpha L_t^{-(1+\alpha)} < 0, \end{aligned}$$

implying an inversely U-shaped graph. To show that this may imply oscillatory dynamics, you must draw the graph such that the steady state is to the right of the peak of $\phi(L_t)$, in the region where $\phi'(L_t) < 0$.

(c) Let the steady-state level of L_t be denoted $\overline{L}$. From $\overline{L} = \phi(\overline{L})$ we get

$$\begin{aligned} \overline{L} &= \frac{\beta}{q}\left[A^\alpha \overline{L}^{1-\alpha} - \widehat{c}\overline{L}\right], \\ 1 &= \frac{\beta}{q}\left[\left(\frac{A}{\overline{L}}\right)^\alpha - \widehat{c}\right], \\ 1 + \left(\frac{\beta}{q}\right)\widehat{c} &= \frac{\beta}{q}\left(\frac{A}{\overline{L}}\right)^\alpha, \\ \frac{q+\beta\widehat{c}}{\beta} &= \left(\frac{A}{\overline{L}}\right)^\alpha. \end{aligned}$$

We can use the expressions for $\phi'(L_t)$ and $\left(A/\overline{L}\right)^\alpha$ above to see that

$$\begin{aligned} \phi'(\overline{L}) &= \frac{\beta}{q}\left[(1-\alpha)\left(\frac{A}{\overline{L}}\right)^\alpha - \widehat{c}\right], \\ &= \frac{\beta}{q}\left[(1-\alpha)\left(\frac{q+\beta\widehat{c}}{\beta}\right) - \widehat{c}\right], \\ &= 1 - \alpha\left(\frac{q+\beta\widehat{c}}{q}\right). \end{aligned}$$

We now see that $\phi'(\overline{L}) < 1$ always holds. The condition for $\phi'(\overline{L}) > 0$ can be written

$$\alpha < \frac{q}{q + \beta\widehat{c}}.$$

2. (a) Forward the expression for y_t to $t+1$, i.e., $y_{t+1} = A[\alpha k_{t+1}^{\rho} + 1 - \alpha]^{1/\rho}$, and substitute for k_{t+1} using (1). This gives

$$y_{t+1} = A\left[\alpha\left(\frac{sy_t}{1+n}\right)^{\rho} + (1-\alpha)\right]^{\frac{1}{\rho}}.$$

(b) Forward the expression for y_t to period $t+1$, i.e., $y_{t+1} = Ak_{t+1}^{\alpha}$. Then use (1) to get

$$\frac{y_{t+1}}{y_t} = A\left(\frac{sy_t}{1+n}\right)^{\alpha}\left(\frac{1}{y_t}\right) = A\left(\frac{s}{1+n}\right)^{\alpha} y_t^{-(1-\alpha)}.$$

Logging gives

$$g_t = \ln(y_{t+1}) - \ln(y_t) = \underbrace{\ln(A) + \alpha\ln\left(\frac{s}{1+n}\right)}_{\gamma}\underbrace{-(1-\alpha)}_{\beta}\ln(y_t).$$

(c) See above
(d) Panel, or panel data
(e) The regression results say, roughly, that

$$\text{growth}=...0.09 \times \text{democracy} - 0.088 \times \text{democracy}^2...$$

implying that growth is maximized at

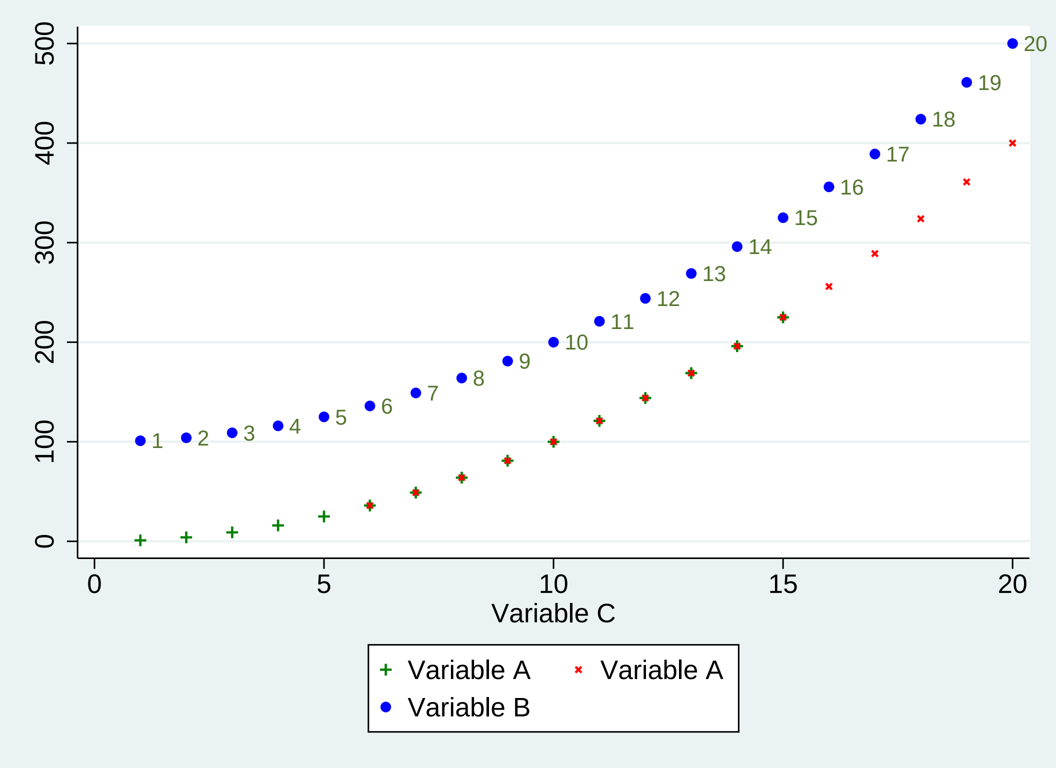
$$\text{democracy}=\frac{0.09}{2 * 0.088} \approx \frac{1}{2}$$

(f) Barro finds no significant effects on growth from levels of female schooling either at the secondary or primary level.

3. The complete code should be:
(a)

```
#delimit;
set obs 20; egen x = seq(), f(1) t(20);
gen y=x^2;
gen z=y+100; /* or z=x^2+100; */
label var y "Variable A";
label var z "Variable B";
label var x "Variable C";
xtile w = x, n(4);
twoway
(scatter y x if w<=3, msymbol(+) mcolor(green) )
(scatter y x if w>=2, msymbol(x) mcolor(red) )
(scatter z x, msymbol(o) mcolor(blue) mlabel(x));
```

(b) 3, 5, 7, 9. To find the first value, note that $x = 1$ gives $y = 1$, and $x = 2$ gives $y = 4$, the difference becoming 3. Similarly for the others.


0
100
200
300
400
500
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
0
5
10
15
20
Variable C
Variable A
Variable A
Variable B

6-20-96 Table 1 Regressions for Per Capita Growth Rate

independent variable	(1)	(2)
log(GDP)	-.0254 (.0031)	-.0225 (.0032)
male secondary and higher schooling	.0118 (.0025)	.0098 (.0025)
log(life expectancy)	.0423 (.0137)	.0418 (.0139)
log(GDP)*male schooling	-.0062 (.0017)	-.0052 (.0017)
log(fertility rate)	-.0161 (.0053)	-.0135 (.0053)
government consumption ratio	-.136 (.026)	-.115 (.027)
rule-of-law index	.0293 (.0054)	.0262 (.0055)
terms-of-trade change	.137 (.030)	.127 (.030)
democracy index	.090* (.027)	.094 (.027)
democracy index squared	-.088 (.024)	-.091 (.024)
inflation rate	-.043 (.008)	-.039 (.008)
Sub Saharan Africa dummy	--	-.0042** (.0043)
Latin America dummy	--	-.0054 (.0032)
East Asia dummy	--	.0050 (.0041)
R^2	.58, .52, .42	.60, .52, .47
number of observations	80, 87, 84	80, 87 84

*p-value for joint significance of two democracy variables is 0.0006 in column 1 and 0.0004 in column 2.

**p-value for joint significance of three dummy variables is 0.11.