Midterm Exam – Econ 5700
13 November 2019
Department of Economics
York University

Student name: ________________ SID number:____________________

Notes: As a rule, you must explain how you arrived at your answers.

1. The Solow Model and Krusell and Smith (2015) [10 marks]

Consider the Solow model, where we let Y_t denote total gross output, K_t the total capital stock, and L_t the labor force, all in period t. The production function is here given by

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}.$$

Let $\delta \in (0,1)$ be the rate at which the capital stock depreciates in each period. Capital thus evolves over time according to

$$K_{t+1} = I_t + (1-\delta)K_t,$$

where I_t is gross investment, and $I_t - \delta K_t$ is net investment. Total consumption is denoted C_t. As always, consumption and gross investment sum up to total gross output, Y_t, i.e.,

$$C_t + I_t = Y_t.$$

The labor force grows at (net) rate n each period, i.e., $L_{t+1} = (1+n)L_t$, and we let capital per worker be denoted $k_t = K_t/L_t$.

Recall that we can make different assumptions about what determines investment in the Solow model. In the standard textbook version, gross investment, I_t, is a constant fraction $s \in (0,1)$ of gross output, and given by

$$I_t = sY_t.$$

In the Piketty version of the Solow model, net investment is a constant fraction $\widetilde{s} \in (0,1)$ of net output, and given by

$$I_t - \delta K_t = \widetilde{s}\left(Y_t - \delta K_t\right).$$

(a) Find a difference equation for capital per worker in the textbook version of the Solow model. Your answer should be an expression for k_{t+1} in terms of (some or all of) k_t, α, s, n, and δ. Show each step. [2 marks]

(b) Find a difference equation for capital per worker in the Piketty version of the Solow model. Your answer should be an expression for k_{t+1} in terms of (some or all of) k_t, α, $\widetilde{s}$, n, and δ. Show each step. [2 marks]

(c) Let the net investment rate in the textbook version of the Solow model be $\widetilde{S}_t^{\text{text}} = \left(I_t - \delta K_t\right) / \left(Y_t - \delta K_t\right)$. Find an expression for $\widetilde{S}_t^{\text{text}}$ in terms of (some or all of) k_t, α, s, n, and δ. Note that $\widetilde{S}_t^{\text{text}}$ is the net investment rate outside of steady state, so the expression for $\widetilde{S}_t^{\text{text}}$ should contain k_t. Show each step. [2 marks]

(d) Find an expression for the steady-state capital stock per worker, k^*, in the Piketty version of the Solow model. Your answer should be an expression for k^* in terms of (some or all of) α, $\widetilde{s}$, n, and δ. Show each step. [*Hint*: one way to solve this problem is to impose steady state on your answer under (b).] [2 marks]

(e) Let consumption per worker be $c_t = C_t/L_t$, and let c^* be the steady-state level of c_t. In the Piketty version of the Solow model, what does c^* equal if $n = 0$? Show how you arrived at your answer. [2 marks]

2. Murphy, Shleifer, and Vishny (1993) [10 marks]

Consider the model of Murphy, Shleifer and Vishny (1993). Agents choose whether to be producers or rent-seekers. If they become producers, they choose whether to work in a market sector earning gross income α, part of which is stolen, or work in "home production" earning (safe) income $\gamma < \alpha$.

Let $S \leq \beta$ be the (endogenous) amount stolen per rent-seeker, where β is the (exogenous) maximum amount a rent-seeker can steal. The amount stolen per market producer thus equals Sn, where n is the ratio of rent-seekers to market producers.

If all producers work in the market sector, then each rent-seeker takes the maximum, β, and each producer earns income $\alpha - \beta n$, since βn is the amount stolen per producer. If some producers work in home production, then each producer earns income $\alpha - Sn$, which must equal γ, since producers must be indifferent between home and market.

At given n, we let income per rent-seeker be denoted by $R(n)$, and income per producer (after theft) by $Y(n)$.

(a) Let n' denote the threshold level of n, such that if $n > n'$, then some producers work in home production. Find an expression for n' in terms of (some or all of) α, β, and γ. [2 marks]

(b) Find an expression for $R(n)$. Other than n, your answer should involve (some or all of) α, β, and γ. Consider both the cases $n \geq n'$ and $n < n'$. [2 marks]

(c) Find an expression for $Y(n)$. Other than n, your answer should involve (some or all of) α, β, and γ. Consider both the cases $n \geq n'$ and $n < n'$. [2 marks]

(d) Now impose the equilibrium condition that agents must be indifferent between being rent-seekers and producers, and assume that $\beta > \alpha$. Find expressions for the equilibrium income per agent (which is the same for rent-seekers and producers), and the equilibrium level of n. Your answers should be in terms of (some or all) of α, β, and γ. [4 marks]

3. Empirical growth and aid [5 marks]

(a) The paper by Gennaioli, La Porta, Lopez-de-Silanes and Shleifer (2014) does not use measures of GDP per capita at the country level, but something else. Describe briefly. Give an example of what could be an observation in their data. [2 marks]

Questions (b) and (c) below refer to the comment on Burnside and Dollar (2000) (BD) by Easterly, Levine, and Roodman (2004) (ELR).

(b) Before ELR published their comment many others had already commented on BD. What made ELR's comment different from those others? Explain in a few words. [2 marks]

(c) In part of their analysis, ELR use something called the "Hadi method." What is the Hadi method used for? You do not need to describe how it works, only state what it is used for. [1 mark]

4. Sala-i-Martin (2006) [5 marks]

(a) Many have expressed concern about rising levels of income inequality in the world since 1970, or so. Sala-i-Martin (2006) argues that inequality has not increased at all between 1970 and 2000, but rather decreased. How come his conclusion is so different from many others? Explain briefly in your own words. [2 marks]

(b) Consider Figure V from Sala-i-Martin (2006), attached with this exam.[1] Use the information provided there to draw another diagram, where the horizontal axis measures time (years), and the vertical axis measures the fraction of the population (i.e., numbers between 0 and 1). In this diagram, draw two graphs: one graph showing the fraction of the population living on less than \$570 per year, and another graph showing the fraction of the population living on less than \$2000 per year. [3 marks]

---

[1] Note the typo in the legend to the figure: the triangles should refer to 1990, not 1980.

Answer sheet for Problem____Econ 5700, Midterm 13 November 2019

Student Name: SID Number:

____

Write your answers below. Do **not** fold the answer sheets or write on the back.

Answer sheet for Problem____Econ 5700, Midterm 13 November 2019

Student Name: SID Number:

____

Write your answers below. Do **not** fold the answer sheets or write on the back.

Answer sheet for Problem____Econ 5700, Midterm 13 November 2019

Student Name: SID Number:

Write your answers below. Do **not** fold the answer sheets or write on the back.

Answer sheet for Problem____Econ 5700, Midterm 13 November 2019

Student Name: SID Number:

____

Write your answers below. Do **not** fold the answer sheets or write on the back.

Answer sheet for Problem____Econ 5700, Midterm 13 November 2019

Student Name: SID Number:

____

Write your answers below. Do **not** fold the answer sheets or write on the back.

Answer sheet for Problem____Econ 5700, Midterm 13 November 2019

Student Name: SID Number:

____

Write your answers below. Do **not** fold the answer sheets or write on the back.

Answer sheet for Problem____Econ 5700, Midterm 13 November 2019

Student Name: SID Number:

____

Write your answers below. Do **not** fold the answer sheets or write on the back.

Answer sheet for Problem____Econ 5700, Midterm 13 November 2019

Student Name: SID Number:

____

Write your answers below. Do **not** fold the answer sheets or write on the back.

Answer sheet for Problem____Econ 5700, Midterm 13 November 2019

Student Name: SID Number:

Write your answers below. Do **not** fold the answer sheets or write on the back.

Answer sheet for Problem____Econ 5700, Midterm 13 November 2019

Student Name: SID Number:

____

Write your answers below. Do **not** fold the answer sheets or write on the back.

Sketches of solutions, but without some of the required explanatory text:

1.
(a)

$$k_{t+1} = \frac{sk_t^{\alpha} + (1-\delta)k_t}{1+n}$$

(b)

$$k_{t+1} = \frac{\widetilde{s}\,(k_t^{\alpha} - \delta k_t) + k_t}{1+n}$$

(c) First it can be seen from the production function that

$$\frac{K_t}{Y_t} = \frac{K_t/L_t}{Y_t/L_t} = \frac{k_t}{k_t^{\alpha}} = k_t^{1-\alpha},$$

which in the textbook version gives

$$\widetilde{S}_t^{\text{text}} = \frac{I_t - \delta K_t}{Y_t - \delta K_t} = \frac{sY_t - \delta K_t}{Y_t - \delta K_t} = \frac{s - \delta k_t^{1-\alpha}}{1 - \delta k_t^{1-\alpha}}.$$

(d)

$$k^* = \left(\frac{\widetilde{s}}{n + \delta\widetilde{s}}\right)^{\frac{1}{1-\alpha}}$$

(e) Setting $n = 0$ in the answer to (b) and imposing steady state gives $(k^*)^{\alpha} = \delta k^*$. The same can be seen from the answer to (d). Then we see that, with $n = 0$, it follows that

$$c^* = (1 - \widetilde{s})\left[(k^*)^{\alpha} - \delta k^*\right] = 0.$$

2.
(a) $n' = (\alpha - \gamma)/\beta$
(b)

$$R(n) = \min\left\{\beta, (\alpha-\gamma)/n\right\} = \begin{cases} \beta & \text{if } n < n' \\ \frac{\alpha-\gamma}{n} & \text{if } n \geq n' \end{cases}$$

(c)

$$Y(n) = \max\left\{\gamma, \alpha - \beta n\right\} = \begin{cases} \alpha - \beta n & \text{if } n < n' \\ \gamma & \text{if } n \geq n' \end{cases}$$

(d) Equilibrium income equals γ (since producers must be indifferent between home and market production), and equilibrium n is $(\alpha - \gamma)/\gamma$ [from setting $(\alpha - \gamma)/n = \gamma$].

3.
(a) They measure GDP per capita at the subnational level, i.e., across regions within countries. Examples include Canadian provinces and US states. An observation could be, say, the province Ontario (over some period of time, if it's a panel data set).
(b) See p. 774 in ELR. Earlier comments had explored how robust Burnside and Dollar's results were to changes in regression specifications (adding controls, using non-linear specifications, and more). ELR updated the data but used the same regression specifications as

in Burnside and Dollar's original paper. ELR write: "We differentiate our paper from these others by NOT deviating from the BD specification."
(c) To find outliers. On p. 775 in ELR they write that they "adopt the Hadi method for identifying and eliminating outliers."

4.
(a) Sala-i-Martin measures inequality between persons rather than between countries. See the introduction to his paper.
(b) The figure should have one graph referring to \$570 per day, showing a monotonic decline between 1960 to 2000 from 0.2 to 0.07 on the vertical axism and another graph, referring to \$2000 per day, should show a monotonic decline from 0.62 to 0.41 on the vertical axis over the same period.