Midterm Exam – Econ 5700
16 November 2016
Department of Economics
York University

Notes: For Problems 1-3 you must explain how you arrived at your answers, except when you are instructed otherwise, and when the answer is just one word. For Problem 4 you only need to state the correct answer, or draw the correct diagram, not explain or motivate anything.

1. The Diamond Model [10 marks]

Consider the Diamond model, where agents live for two periods, working in the first and retired in the second. Consumption in the two periods are denoted $C_{1,t}$ and $C_{2,t+1}$, respectively, and utility equals

$$U_t = (1-\gamma)\ln(C_{1,t}) + \gamma \ln(C_{2,t+1}),$$

where $0 < \gamma < 1$. The intertemporal budget constraint is given by

$$C_{2,t+1} = R_{t+1}\left(w_t - C_{1,t}\right),$$

where w_t is the wage income in the first period, and R_{t+1} is the gross rate of return on savings held form period t to period $t+1$. The working-age population equals L_t, and grows at (net) rate n each period, i.e., $L_{t+1} = (1+n)L_t$. The total capital stock in period $t+1$, denoted K_{t+1}, is made up of savings by the L_t young agents in the previous period, $K_{t+1} = L_t\left(w_t - C_{1,t}\right)$. Output per worker in period t equals $f(k_t)$, where $k_t = K_t/L_t$ is the capital-worker ratio, and f satisfies $f'(k) > 0$ and $f''(k) < 0$. The wage rate equals $w_t = f(k_t) - f'(k_t)k_t$.

(a) Find a difference equation for capital per worker. Your answer should be a function ϕ, such that $k_{t+1} = \phi(k_t)$. Show each step. [4 marks]

(b) Consider this statement: "The phenomenon of multiplicity of steady states arises only under very unusual assumptions. For example, it could never occur in a standard Diamond overlapping-generations model with logarithmic utility and a neoclassical production function, such that $f'(k) > 0$ and $f''(k) < 0$." Do you agree? Explain why with the help of the model provided in this question. [3 marks]

(c) Now assume Cobb-Douglas production, so that $f(k) = Ak^{\alpha}$, where $A > 0$ and $0 < \alpha < 1$. Find the steady-state level of capital per worker. [3 marks]

2. Inequality [10 marks]

Consider a two-country world, where the countries have populations P_1 and P_2 and per-capita GDP levels y_1 and y_2, as in the table below:

Country	**Population**	**GDP/capita**
1	P_1	y_1
2	P_2	y_2

Let $x = P_1/(P_1 + P_2)$ be the share of the world population living in country 1.

(a) Find the population-weighted cross-country mean of GDP/capita. Your answer should be in terms of some, or all, of x, y_1, and y_2. [2 marks]

(b) Find the population-weighted cross-country variance in GDP/capita. Your answer should be in terms of some, or all, of x and the factor $(y_1 - y_2)^2$. For what level of x is the variance maximized? [3 marks]

(c) Let the cumulative density functions of incomes be $F_1(y) = \min\{1, a_1 y\}$ and $F_2(y) = \min\{1, a_2 y\}$, for country 1 and country 2, respectively, where $0 < a_1 < a_2$. Find an expression for the world income distribution of individuals in terms of a_1, a_2, and x. (You do not need to graph it.) [3 marks]

(d) According to Sala-i-Martin, what has been the trend in total world income inequality from 1970 to 2000 as measured by the Gini coefficient? What has been the world trend if we exclude China? [1 mark]

(e) Aside from the Gini coefficient, mention two other measures of income inequality used by Sala-i-Martin. [1 mark]

3. Corruption [10 marks]

Consider the model by Murphy, Schleifer and Vishny (1993), but here with partly different notation. As in class, we let income per rent-seeker equal $R(n)$, where n is the ratio of rent-seekers over market producers. Similarly, income per producer equals $Y(n)$. Producers who work in the market place earn α, minus what is stolen by rent-seekers. Producers who work in home production (if any) earn income $\rho\alpha$, where $0 < \rho < 1$. Income from home production is safe from theft by rent-seekers. Finally, each rent-seeker can steal at most δ units.

(a) Write expressions for $R(n)$ and $Y(n)$. You do not need to motivate anything, just write the correct expressions, but be careful to define the threshold level of n above which some producers work in home production. (You can denote that level by n', as in class.) [3 marks]

(b) Assume $\delta > \alpha$. Using your answer to (a), find an expression for the equilibrium level of n. Show how it changes in response to an increase in α, and explain why in a few words. [3 marks]

(c) Name one of the country rankings of corruption discussed by Svensson (2005), and one of the 10% most corrupt countries on that list. [2 marks]

(d) How can the degree of political freedom affect corruption, according to Svensson? What is the cross-country relationship between press freedom and corruption? [2 marks]

4. Stata Coding [10 marks]

Consider the code below, with segments hidden by “[#]” in six places.

```
#delimit;
drop _all;
set obs 1000;
gen A=rnormal(0,5);
gen B=-5+.5*A+rnormal(0,1);
gen C=[1]-.5*A+rnormal(0,1);
label var A "Variable 3";
label var B "Variable [2]";
label var C "Variable 2";
twoway
(scatter B A, msymbol([3]) mcolor(red) )
(lfit B A, lcolor(red) )
(scatter C A, msymbol(o) mcolor(blue))
(lfit C [4], lcolor(blue) );
```

The code produces the attached figure. (You may not tell the colors, but that should not be needed to solve this problem.)

(a) Which are the hidden codes? Answer with the exact code and the numbered segment it replaces, [**1**] to [**4**]. [4 marks]

(b) Draw the graph produced by this code (following that above):

```
gen D=A-.5*A^2;
label var D "Nippe!!";
twoway(scatter D A, msymbol(+) );
```

[3 marks]

(c) Draw the graph produced by this code (following that above):

```
histogram A, bin(100)
normal normopts (lwidth(thick) lpattern(dash) )
legend(on position(11) ring(0))
title(Nippe!!);
```

[3 marks]

Note: For 4(b)-(c) you do not need to replicate the graphs exactly, just draw them as correctly and in as much detail as you can.

Answer sheet for Problem____Econ 5700, Midterm 16 November 2016

Student Name: SID Number:

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Write your answers below. Do **not** fold the answer sheets or write on the back.

Sketches of solutions to select problems

1.

(a) Substituting the budget constraint into the utility function gives

$$U_t = (1-\gamma)\ln(C_{1,t}) + \gamma \ln(w_t - C_{1,t}) + \gamma \ln(R_{t+1}).$$

The first-order condition for $C_{1,t}$ gives

$$C_{1,t} = (1-\gamma) w_t,$$

implying

$$w_t - C_{1,t} = w_t\left[1-(1-\gamma)\right] = \gamma w_t = \gamma\left[f(k_t) - f'(k_t)k_t\right].$$

Now $K_{t+1} = L_t(w_t - C_{1,t})$ and $L_{t+1} = (1+n)L_t$ give

$$k_{t+1} = \frac{K_{t+1}}{L_{t+1}} = \frac{(w_t - C_{1,t})L_t}{L_{t+1}} = \frac{(w_t - C_{1,t})}{L_{t+1}/L_t} = \frac{\gamma\left[f(k_t) - f'(k_t)k_t\right]}{1+n} = \phi(k_t).$$

Defining $S_t = w_t - C_{1,t}$ brings the problem closer to the notation used in class, and maybe easier to solve.

(b) The statement is false. Multiple steady states can occur if $\phi''(k_t) > 0$ over some interval, which cannot be ruled out even though $f'(k) > 0$ and $f''(k) < 0$. The reason is that the sign of $\phi''(k_t)$ depends on the sign of $f'''(k_t)$, about which we have not made any assumptions:

$$\begin{aligned}
\phi'(k_t) &= -\left(\frac{\gamma}{1+n}\right) f''(k_t)k_t > 0 \\
\phi''(k_t) &= -\left(\frac{\gamma}{1+n}\right) \left[f''(k_t) + f'''(k_t)k_t\right]
\end{aligned}$$

(c) With Cobb-Douglas production $f(k_t) - f'(k_t)k_t = (1-\alpha)Ak_t^{\alpha}$. Thus, the steady-state level of k_t, denoted $\overline{k}$, is given by

$$\overline{k} = \phi(\overline{k}) = \frac{\gamma(1-\alpha)A\overline{k}^{\alpha}}{1+n},$$

which can be solved to give

$$\overline{k} = \left[\frac{\gamma(1-\alpha)A}{1+n}\right]^{\frac{1}{1-\alpha}}.$$

2.

(a) Denoting the population-weighted mean by $\overline{y}$, we can write

$$\begin{aligned}
\overline{y} &= \left(\frac{P_1}{P_1+P_2}\right) y_1 + \left(\frac{P_2}{P_1+P_2}\right) y_2 \\
&= x y_1 + (1-x)\, y_2.
\end{aligned}$$

(b) Denoting the population-weighted variance by V, we can write

$$\begin{aligned}
V &= \left(\frac{P_1}{P_1+P_2}\right) (y_1 - \overline{y})^2 + \left(\frac{P_2}{P_1+P_2}\right) (y_2 - \overline{y})^2 \\
&= x(y_1 - \overline{y})^2 + (1-x)\,(y_2 - \overline{y})^2.
\end{aligned}$$

Next note from (a) that

$$\begin{aligned} y_1 - \overline{y} &= y_1 [1 - x] - (1 - x) y_2 = (1 - x)(y_1 - y_2) \\ y_2 - \overline{y} &= y_2 [1 - (1 - x)] - x y_1 = x (y_2 - y_1). \end{aligned}$$

Using the above, and $(y_1 - y_2)^2 = (y_2 - y_1)^2$, we get

$$\begin{aligned} V &= x \overbrace{(1-x)^2 (y_1 - y_2)^2}^{(y_1 - \overline{y})^2} + (1-x) \overbrace{x^2 (y_2 - y_1)^2}^{(y_2 - \overline{y})^2} \\ &= x(1-x)(y_1 - y_2)^2, \end{aligned}$$

which is maximized at $x = 1/2$.
(c) If we let F denote the cdf of the world income distribution, then

$$F(y) = xF_1(y) + (1-x)F_2(y).$$

That is, the fraction of the world's population with incomes below some level y equals the fraction living in country 1 times the fraction of that country's population with incomes below y, plus the fraction living in country 2 times the fraction of country 2's population with incomes below y.
We now see that:

- If $y < 1/a_2 < 1/a_1$, then $F_1(y) = a_1 y < 1$ and $F_2(y) = a_2 y < 1$, so $F(y) = [xa_1 + (1-x)a_2]\, y$.
- If $1/a_2 < y < 1/a_1$, then $F_1(y) = a_1 y < 1$ and $F_2(y) = 1$, so $F(y) = xa_1 y + (1-x)$.
- If $y > 1/a_1$, then $F_1(y) = F_2(y) = 1$ both fall below 1, so $F(y) = x + (1-x) = 1$.

Thus:

$$F(y) = \begin{cases} [xa_1 + (1-x)a_2]\, y & \text{if} \quad y < 1/a_2 \\ xa_1 y + (1-x) & \text{if} \quad 1/a_2 < y < 1/a_1 \\ 1 & \text{if} \quad y > 1/a_1 \end{cases}$$

(d) The Gini coefficinent has decreased. But if you exclude China it has increased.
(e) The Atkinson index, or the variance in log per-capita GDP, or the ratio of mean income among the richest 10% over the poorest 10%, or the same for the

3.
(a)

$$R(n) = \begin{cases} \delta & \text{if } n \leq n' \\ \frac{\alpha(1-\rho)}{n} & \text{if } n \geq n' \end{cases}$$

$$Y(n) = \begin{cases} \alpha - \delta n & \text{if } n \leq n' \\ \alpha\rho & \text{if } n \geq n' \end{cases}$$

where $n' = \alpha(1-\rho)/\delta$.
(b) Set $\alpha(1-\rho)/n = \alpha\rho$. This gives $n = (1-\rho)/\rho$, which does no depend on α. Intuitively, a higher α means that producers are more productive in market production, but this also

implies that there is more to steal for rent-seekers. Here an increase in α raises the payoffs to rent-seeking and production equally much, leaving equilibrium n unchanged.
(c) For example: the International Country Risk Guide (ICRG), the Corruption Perception Index (CPI), or Control of Corruption (CC). For example Nigeria is among the 10% most corrupt on all these lists.
(d) Svensson (2005, p. 26) suggests that a free press provides more information to voters on government and public sector misbehavior than a government-controlled press does, and that politicians who face elections have incentives not to be corrupt.
In the data, countries with more press freedom are less corrupt, even when controlling for GDP/capita and education.

4.
(a)
```
#delimit;
drop _all;
set obs 1000;
gen A=rnormal(0,5);
gen B=-5+.5*A+rnormal(0,1);
gen C=10-.5*A+rnormal(0,1);
label var A "Variable 3";
label var B "Variable 1";
label var C "Variable 2";
twoway
(scatter B A, msymbol(+) mcolor(red) )
(lfit B A, lcolor(red) )
(scatter C A, msymbol(o) mcolor(blue))
(lfit C A, lcolor(blue) );
```

(b)-(c) See attached, or run the codes to see the figures.