Midterm Exam – Econ 5700
October 18, 2021
Department of Economics
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Student Name: SID Number:

Notes: Explain how you arrived at your answers, but be as brief and concise as you can.

1. The Solow Model [10 marks]

Consider a version of the Solow model. The total capital stock evolves according to

$$K_{t+1} = sY_t + (1-\delta)K_t,$$

where the notation is standard: K_t and Y_t denote the period-t levels of the total capital stock and total output, respectively; $s \in (0,1)$ is the saving rate; and $\delta \in (0,1)$ is the rate of capital depreciation. Output in period t is given by

$$Y_t = BK_t + K_t^{\alpha} L_t^{1-\alpha},$$

where $B > 0$ and $\alpha \in (0,1)$ are exogenous, and L_t denotes the total labor force in period t. We assume that L_t grows at rate $n > 0$, i.e., $L_{t+1} = (1+n)L_t$.

Let $k_t = K_t/L_t$ be output per worker.

(a) Find a difference equation $k_{t+1} = \phi(k_t)$. The expression for $\phi(k_t)$ should involve B, α, s, n, and δ (and the argument k_t). (*Hint:* setting $B = 0$ should bring you back to a case similar to what we considered in class.) [2 marks]

(b) Find a condition in terms of (some or all of) B, α, s, n, and δ under which k_t exhibits sustained growth (meaning $k_{t+1} > k_t$ holds as k_t goes to infinity). [2 marks]

(c) Assume that the condition for sustained growth derived under (b) holds. Illustrate the graph of $k_{t+1} = \phi(k_t)$ in a standard 45-degree diagram. Given some initial $k_0 > 0$, illustrate k_1 and k_2 on a suitable axis. Note that you do not need to be able to solve (b) to answer this question. [2 marks]

(d) Continue to assume that the sustained-growth condition under (b) holds. Find an expression for the long-run (asymptotic) growth rate of k_t. That is, find an expression for $(k_{t+1} - k_t)/k_t$ as k_t goes to infinity. Your answer should be in terms of (some or all of) B, α, s, n, and δ. Is the long-run growth rate of k_t increasing or decreasing in the population growth rate, n? [2 marks]

(e) Assume that the sustained-growth condition under (b) does *not* hold. Find an expression for the steady-state level of k_t, denoted $\overline{k}$, as given by $\overline{k} = \phi(\overline{k})$. Your answer should be in terms of (some or all of) B, α, s, n, and δ. Is $\overline{k}$ increasing or decreasing in the population growth rate, n? [2 marks]

2. Convergence and Divergence I [4 marks]

(a) Suppose that we have set up and solved a growth model that predicts that GDP per capita in country i, denoted $y_{i,t}$, evolves from one period to the next according to

$$y_{i,t+1} = By_{i,t}^{\gamma}\eta_{i,t}, \tag{1}$$

where $B > 0$ is a constant, γ can be positive or negative, and where $\ln(\eta_{i,t})$ has mean zero and variance σ_η^2 for all t. Express the growth rate of GDP per capita in country i from period t to $t+1$ as $\ln(y_{i,t+1}) - \ln(y_{i,t})$. For what values of γ and/or B does the model predict (beta) convergence? Explain how you arrived at your answer. [2 marks]

(b) Consider a world with many economies, whose levels of GDP per capita evolve according to (1). Assume that the values of γ and/or B are such that the economies all exhibit beta convergence. Must this imagined world also exhibit so-called sigma convergence? That is, does the variance in $\ln(y_{i,t})$ across countries have to be decreasing over time? Explain why or why not. [2 marks]

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3. Convergence and Divergence II [8 marks]

(a) According to Sokoloff and Engerman (2000), how did geography (or climate) influence historical inequality in the Carribean and US South? [2 marks]

(b) What is the difference between absolute and conditional convergence? Explain briefly. [2 marks]

(c) Consider the attached table from Kremer et al. (2021). Explain how the results in column (2) can tell us how the rate of (beta) convergence as changed over time since 1960. [2 marks]

(d) Mankiw (1995) talks about a "degrees-of-freedom" problem with cross-country growth regressions. Explain briefly what he means and how Gennaioli et al. (2014) try to solve this problem. [2 marks]

Tables

Table 1: Converging to convergence. Absolute convergence 1960-2017

	Average annual growth in next decade		
	(1)	(2)	(3)
log(GDPpc)	-0.270**	0.449**	
	[0.118]	[0.224]	
log(GDPpc) * (Year-1960)		-0.025***	
		[0.006]	
log(GDPpc) * 1960s			0.532***
			[0.191]
log(GDPpc) * 1970s			-0.075
			[0.293]
log(GDPpc) * 1980s			0.106
			[0.246]
log(GDPpc) * 1990s			-0.127
			[0.221]
log(GDPpc) * 2000s			-0.651***
			[0.168]
log(GDPpc) * 2007s			-0.764***
			[0.146]
Year FE	Y	Y	Y
Observations	863	863	863

Notes: This table reports absolute convergence regressions, Equation 1. The independent variable is the average annualized GDP growth (%) for the subsequent decade, in PPP (from the Penn World Tables v10.0), and the sample contains the data for the first year of each decade since 1960, with 2007 replacing 2010. We exclude countries with population $< 200{,}000$, and for which natural resources account for $> 75\%$ of GDP. Specification (1) pools the data since 1960. Specification (2) includes a time trend of absolute convergence β. Specification (3) estimates the absolute convergence β by decade. Year fixed effects are included in all three specifications. Standard errors, clustered at the country-level, are reported in the parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Sketches of solutions

1. (a)

$$\begin{aligned} k_{t+1} &= \frac{K_{t+1}}{L_{t+1}} = \frac{K_{t+1}}{L_t}\frac{L_t}{L_{t+1}} \\ &= \left[\frac{sBK_t + sK_t^\alpha L_t^{1-\alpha} + (1-\delta)K_t}{L_t}\right]\left(\frac{1}{1+n}\right) \\ &= \frac{sBk_t + sk_t^\alpha + (1-\delta)k_t}{1+n} = \phi(k_t) \end{aligned}$$

(b)

$$\lim_{k_t\to\infty}\frac{k_{t+1}}{k_t} = \lim_{k_t\to\infty}\frac{sB + sk_t^{\alpha-1} + 1-\delta}{1+n} = \frac{sB+1-\delta}{1+n} > 1$$

or

$$sB > n+\delta$$

(c) See attached; the asymptotic slope of $\phi(k_t)$ should exceed one, so that $\phi(k_t)$ does not intersect the 45-degree line.

(d)

$$\lim_{k_t\to\infty}\left(\frac{k_{t+1}-k_t}{k_t}\right) = \lim_{k_t\to\infty}\left(\frac{sB + sk_t^{\alpha-1} + 1-\delta}{1+n}\right) - 1 = \left(\frac{sB+1-\delta}{1+n}\right) - 1 = \frac{sB-(n+\delta)}{1+n},$$

which is decreasing in n.

(e) Setting $\overline{k} = \phi(\overline{k})$, and using the answer to (a), we get

$$\overline{k} = \frac{sB\overline{k} + s\overline{k}^\alpha + (1-\delta)\overline{k}}{1+n},$$

which can be solved for $\overline{k}$ to give

$$\overline{k} = \left(\frac{s}{n+\delta-sB}\right)^{\frac{1}{1-\alpha}},$$

which is decreasing in n.

2 (a) Logging (1) and subtracting $\ln(y_{i,t})$ gives an expression for the growth rate:

$$\ln(y_{i,t+1}) - \ln(y_{i,t}) = \ln(B) + (\gamma-1)\ln(y_{i,t}) + \ln(\eta_{i,t}).$$

Convergence implies a negative coefficient on initial log GDP/capita, i.e., $\gamma < 1$.

(b) The answer is no. (This is Galton's fallacy, discussed in class.) Logging (1) (again) we can write

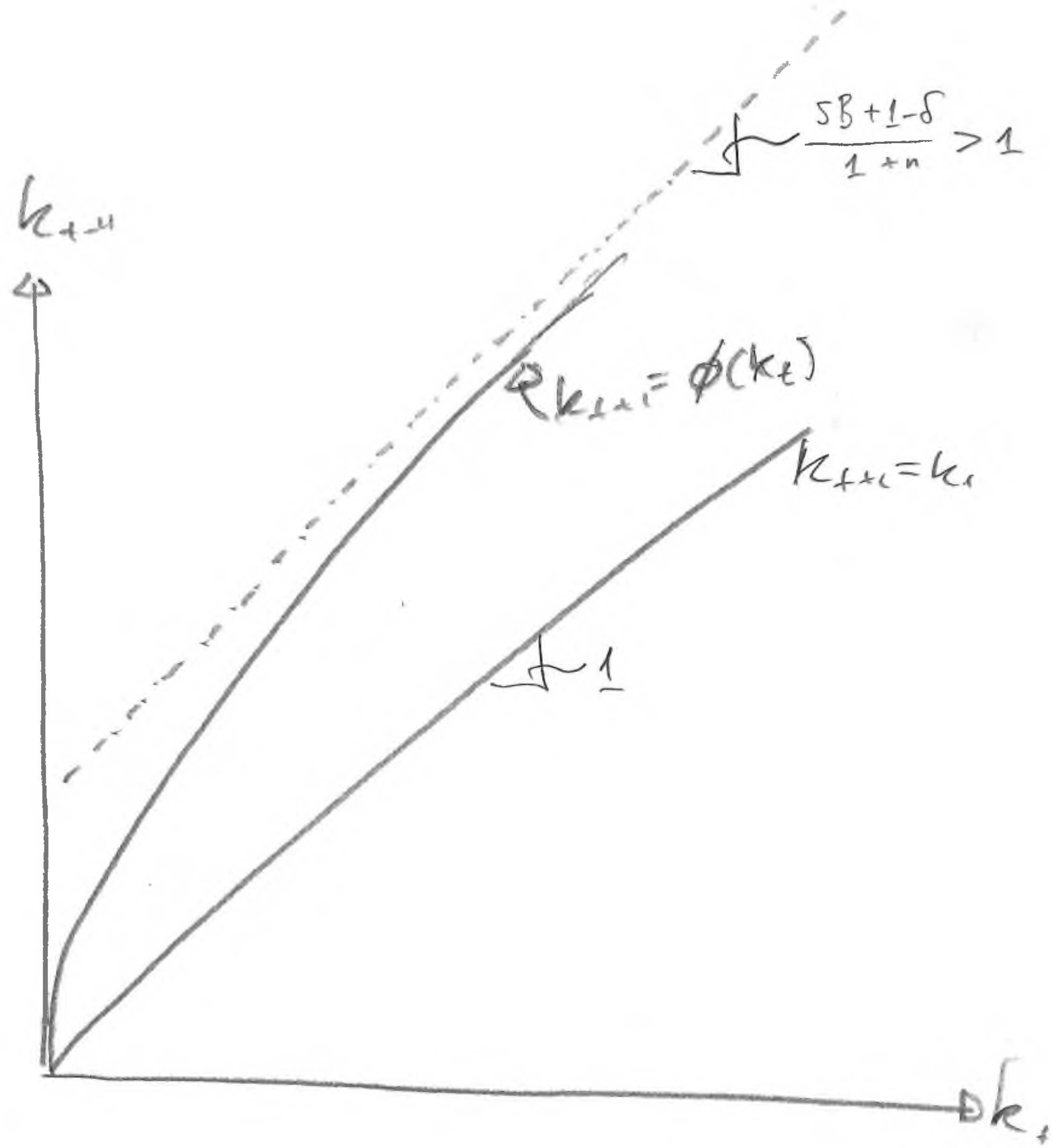
$$\ln(y_{i,t+1}) = \ln(B) + \gamma\ln(y_{i,t}) + \ln(\eta_{i,t}).$$

Denote the variance in $\ln(y_{i,t})$ by V_t, and assume that $\gamma < 1$, implying beta convergence; see (a). Taking the variance of both sides of the above expression gives

$$V_{t+1} = \gamma^2 V_t + \sigma_\eta^2,$$

where we recall that $\ln(\eta_{i,t})$ has variance σ_η^2. It can now be seen that V_t converges to a steady-state level equal to $\sigma_\eta^2/(1-\gamma^2)$ (note that $\gamma^2 < 1$ and use a 45-degree diagram). If initial V_t is below that steady-state level, i.e., if $V_0 < \sigma_\eta^2/(1-\gamma^2)$, then V_t is increasing over time. We can thus have increasing variance in $\ln(y_{i,t})$ over time (the opposite of sigma convergence), even though there is beta convergence ($\gamma < 1$).

3. (a) Soils and climates in the Carribean and US South allowed for growing staple crops (e.g., sugar and coffee). These crops were best suited to grow on large-scale slave plantations. Slavery gave rise to (or implied) high levels of inequality.
(b) Conditional convergence means that per-capita incomes in poorer countries grow faster than in rich countries when including certain control variables in the growth regressions, while absolute convergence means that the same holds without such controls. One can also say that conditional convergence means that countries converge to their own steady states, while under absolute convergence they converge to the same steady state.
(c) The estimated coefficient on the interaction term between initial log GDP/capita and "Year−1960" is negative. Because "Year−1960" is increasing with time, the effect on growth of initial log GDP/capita is more negative for later decades. Thus, the rate of (beta) convergence increases (becomes faster) over time since 1960.
(d) The degrees-of-freedom problem refers to the fact that there are only about one hundred countries in the world. This limits the number of observations, and thus the number of coefficients that can be precisely estimated. Gennaioli et al. (2014) increase the number of observations by using subnational data, allowing multiple observations per country (e.g., provinces and states within countries).



Answer to 1(c)