Midterm Exam – Econ 5700
November 22, 2021
Department of Economics
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Student Name: SID Number:

Notes: Explain how you arrived at your answers, but be as brief and concise as you can.

1. The Diamond Model [6 marks]

We saw in class that the Diamond model with logarithmic utility, and with a general production function, gave us a difference equation for capital per worker on the form $k_{t+1} = \phi(k_t)$, where

$$\phi(k_t) = \frac{\beta\left[f(k_t) - f'(k_t)k_t\right]}{1+n}.$$

The notation is the same as in class: k_t is capital per worker in period t; $f(k_t)$ is the intensive-form production function; $f(k_t) - f'(k_t)k_t$ is the wage rate; β is a parameter from the utility function; and n is the rate of population growth. It is assumed that $f'(k_t) > 0$ and $f''(k_t) < 0$.

(a) Show that $\phi'(k_t) > 0$ always holds. [1 mark]

(b) Find an expression for $\phi''(k_t)$. It should involve the second and third derivatives of $f(k_t)$. [1 mark]

(c) Suppose that $\phi''(k_t) > 0$ for all $k_t \in [k^L, k^U]$, where $0 < k^L < k^U$, but $\phi''(k_t) \leq 0$ otherwise. Then the economy may exhibit multiple steady states. Illustrate how in a suitable 45-degree diagram. [2 marks]

(d) Suppose now that the production function is Cobb-Douglas: $f(k_t) = k_t^\alpha$, where $0 < \alpha < 1$. Show that $\phi''(k_t) < 0$ always holds with this production function. [2 marks]

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2. Inequality [6 marks]

Consider a model of the world where $N > 0$ different countries have different uniform income distributions. In country i the probability density function (pdf) is given by

$$f_i(y) = \begin{cases} a_i & \text{if } y \in [0, \frac{1}{a_i}], \\ 0 & \text{if } y > \frac{1}{a_i}, \end{cases}$$

where y denotes income, and a_i is a parameter that varies across countries.

(a) Find an expression for the cumulative density function (cdf) for one single country i, defined as $F_i(y) = \int_0^y f_i(\widetilde{y})d\widetilde{y}$, for all $y \geq 0$. Illustrate $F_i(y)$ in a diagram with y on the horizontal axis and $F_i(y)$ on the vertical. [2 marks]

(b) Suppose the $N = 2$ and that $a_1 < a_2$. Both countries have the same population size (i.e., the same number of individuals). Find an expression for the world cdf across all individuals and illustrate it in a diagram. [2 marks]

(c) Given the assumptions under (b), which of the two countries is more equal? (Hint: the less equal country is here the one with the higher top income, since the distributions are uniform.) [2 marks]

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3. Aid [6 marks]

(a) Consider the attached table from Burnside and Dollar (2000), and the regression results in the framed column [i.e., column (5), OLS]. The dependent variable is the growth rate. How can we see from the coefficient estimates in this table that aid had a more positive effect on growth in countries/periods with better policies? [2 marks]

(b) Why do Easterly, Levine, and Roodman (2004) get different regression results compared to Burnside and Dollar (2000)? Give a very brief and broad answer, without discussion how the results differ. [2 marks]

(c) Nunn and Qian (2014) estimate the effect of US food aid on conflict by constructing an instrument for the amount of food aid received by different countries in different periods. How do they construct that instrumental variable? [2 marks]

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4. Corruption [6 marks]

(a) Why are bribes not the same as taxes and fees according to Svensson (2005)? [2 marks]

(b) Miguel and Fisman (2007) study parking violations by UN diplomats in New York City. Give one example of a country among the top ten with the most violations in their data, and one example of a country with zero violations. [2 marks]

(c) Miguel and Fisman (2007) find that monthly parking violations dropped in November 2002. What caused that drop? [2 marks]

TABLE 4—GROWTH REGRESSIONS: USING ALL COUNTRIES AND THE POLICY INDEX

	(3)		(4)		(5)	
Estimation method	OLS	2SLS	OLS	2SLS	OLS	2SLS
Initial GDP	−0.61	−0.79	−0.56	−0.71	−0.60	−0.90
	(0.56)	(0.59)	(0.56)	(0.60)	(0.57)	(0.65)
Ethnic fractionalization	−0.54	−0.70	−0.42	−0.47	−0.42	−0.73
	(0.72)	(0.75)	(0.73)	(0.83)	(0.72)	(0.81)
Assassinations	−0.44*	−0.43	−0.45*	−0.44*	−0.45*	−0.41
	(0.26)	(0.27)	(0.26)	(0.26)	(0.26)	(0.27)
Ethnic fractionalization × assassinations	0.82*	0.78*	0.80*	0.75*	0.79*	0.71
	(0.44)	(0.44)	(0.44)	(0.45)	(0.44)	(0.45)
Institutional quality	0.64**	0.63**	0.67**	0.68**	0.69**	0.66**
	(0.17)	(0.17)	(0.17)	(0.19)	(0.17)	(0.18)
M2/GDP (lagged)	0.014	0.019	0.016	0.025	0.012	0.017
	(0.013)	(0.015)	(0.014)	(0.017)	(0.014)	(0.016)
Sub-Saharan Africa	−1.60**	−1.31*	−1.84**	−1.71**	−1.87**	−1.29
	(0.73)	(0.72)	(0.74)	(0.82)	(0.75)	(0.84)
East Asia	0.91*	0.81	1.20**	1.27**	1.31**	1.15**
	(0.54)	(0.53)	(0.58)	(0.63)	(0.58)	(0.56)
Policy index	1.00**	1.01**	0.78**	0.65**	0.71**	0.74**
	(0.14)	(0.14)	(0.20)	(0.30)	(0.19)	(0.20)
Aid/GDP	0.034	−0.12	0.49	−0.10	−0.021	−0.32
	(0.12)	(0.18)	(0.12)	(0.21)	(0.16)	(0.36)
(Aid/GDP) × policy	—	—	0.20**	0.37	0.19**	0.18*
			(0.09)	(0.33)	(0.07)	(0.10)
$(\text{Aid/GDP})^2$ × policy	—	—	−0.019**	−0.038	—	—
			(0.0084)	(0.038)		
Partial R^2 of first-stage regressions						
Aid/GDP	—	0.44	—	0.42	—	0.29
(Aid/GDP) × policy	—	—	—	0.16	—	0.60
$(\text{Aid/GDP})^2$ × policy	—	—	—	0.11	—	—
Test for exogeneity of the aid variables						
$\chi^2(j)$	—	1.10	—	0.85	—	1.51
		[0.29]		[0.84]		[0.47]
Other statistics						
Observations	275	275	275	275	270	270
$\bar{R}^2$	0.36	0.35	0.36	0.34	0.36	0.35

Notes: The variables are described in more detail in the text. The dependent variable is real per capita GDP growth. The excluded exogenous variables for 2SLS estimation are listed in Table 1. White heteroskedasticity consistent standard errors are in parentheses. p-values for the tests of exogeneity appear in brackets. The degrees of freedom parameter j is 1 in column (3), 3 in column (4), and 2 in column (5).

* Significant at the 10-percent level.

** Significant at the 5-percent level.

Sketches of solutions

1 (a)

$$\phi'(k_t) = \frac{-\beta}{1+n} f''(k_t) k_t > 0$$

(b)

$$\phi''(k_t) = \frac{-\beta}{1+n} \left[f'''(k_t) k_t + f''(k_t) \right]$$

(c) See attached

(d) With C-D production, we see that

$$\phi(k_t) = \frac{\beta \left[f(k_t) - f'(k_t) k_t \right]}{1+n} = \frac{\beta(1-\alpha) k_t^{\alpha}}{1+n}$$

$$\phi'(k_t) = \frac{-\beta}{1+n} f''(k_t) k_t = \frac{\beta\alpha(1-\alpha) k_t^{\alpha-1}}{1+n}$$

$$\phi''(k_t) = \frac{-\beta}{1+n} \left[f'''(k_t) k_t + f''(k_t) \right] = \frac{-\beta\alpha(1-\alpha)^2 k_t^{\alpha-2}}{1+n} < 0,$$

so $\phi''(k_t) < 0$ always holds.

2. (a)

$$F_i(y) = \min\{1, a_i y\} = \begin{cases} a_i y & \text{if } y \in [0, \frac{1}{a_i}], \\ 1 & \text{if } y > \frac{1}{a_i}. \end{cases} \tag{1}$$

(b) Since the two countries have the same populations, the world cdf can be written

$$F(y) = \frac{F_1(y) + F_2(y)}{2}$$

Now note that $1/a_1 > 1/a_2$ (since $a_2 > a_1$). It follows that:

- If $y > 1/a_1 > 1/a_2$, then $F_1(y) = F_2(y) = 1$. Thus, $F(y) = 1$ for all $y > 1/a_1$.
- If $y \geq 1/a_2$ and $y \leq 1/a_1$, then $F_1(y) = a_1 y \leq 1$ and $F_2(y) = 1$. Thus, $F(y) = (a_2 y + 1)/2$ for all $y \in [1/a_2, 1/a_1]$.
- If $y < 1/a_2 < 1/a_1$, then $F_1(y) = a_1 y < 1$ and $F_2(y) = a_2 y < 1$. Thus, $F(y) = (a_1 + a_2)y/2$ for all $y > 1/a_2$.

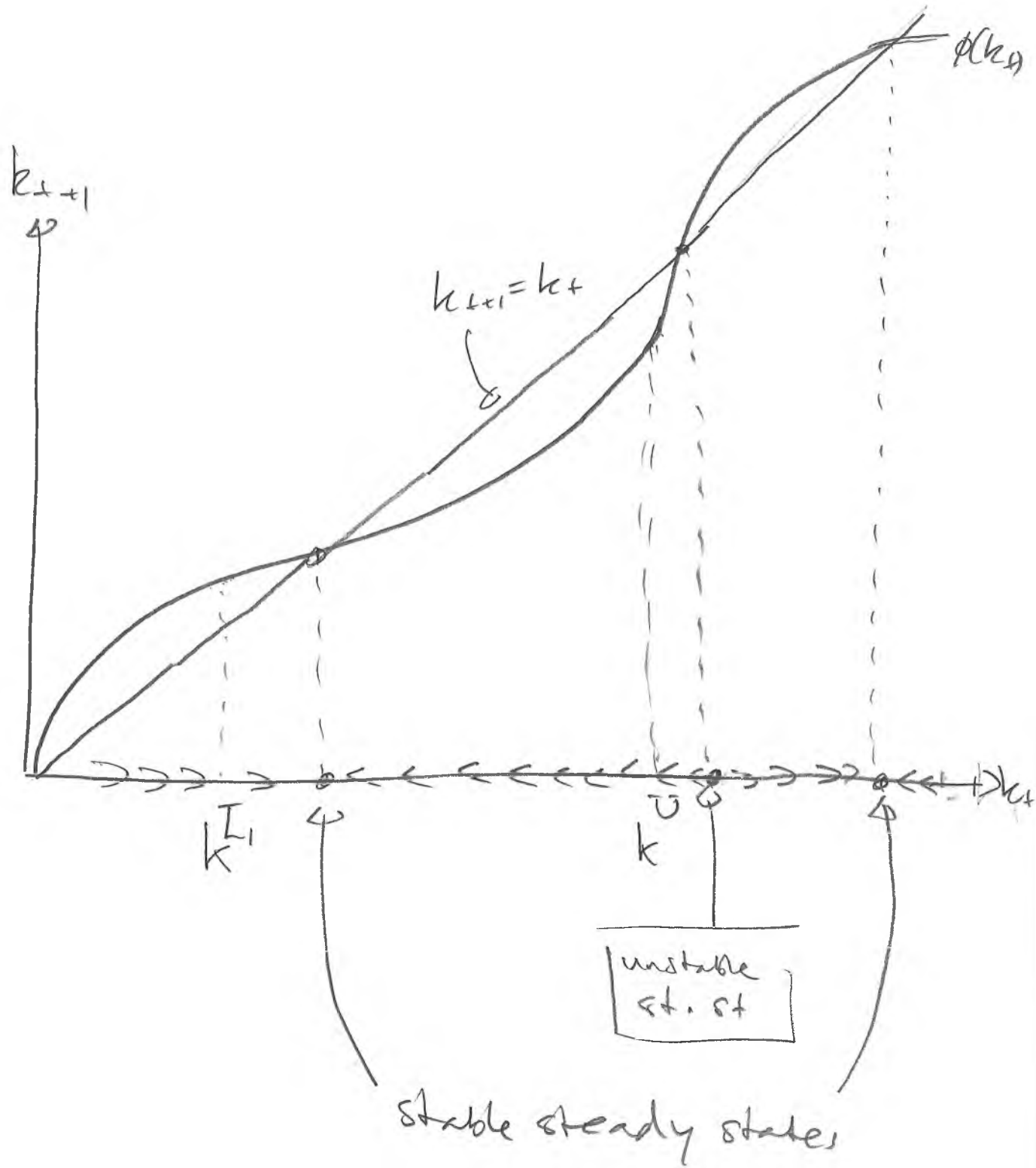
In sum:

$$F(y) = \begin{cases} \left(\frac{a_1+a_2}{2}\right) y & y \in [0, \frac{1}{a_2}] \\ \left(\frac{a_2}{2}\right) y + \frac{1}{2} & y \in [\frac{1}{a_2}, \frac{1}{a_1}] \\ 1 & y > \frac{1}{a_1} \end{cases} \tag{2}$$

The graph of $F(y)$ should be piece-wise linear, with two kinks, and positive slopes in each of the two segments where $y < 1/a_1$. The slope should be flatter for higher levels of y.
(c) Country 2 is more equal (meaning country 1 is less equal), since the top income in country 1 is higher ($1/a_1 > 1/a_2$).

(3) (a) This follows from the positive estimate of the coefficient on the interaction term between policy and aid.
(b) They look at newer data, adding more years and countries.
(c) Their IV is the product of the US wheat harvest in the previous year and the average amount of food aid received by a country over the sample period.

(4) (a) Revenue from bribes is not used for public goods. Bribes create transaction costs, just like fees and taxes, but those transaction costs are higher for bribes than for taxes/fees, because there are no written or enforceable contracts associated with bribes. Officials can renege on promises made when taking bribes.
(b) The top ten were Kuwait, Egypt, Chad, Sudan, Bulgaria, Mozambique, Albania, Angola, Senegal, and Pakistan. Among countries with zero violations are Japan, Latvia, Norway, Oman, Panama, Sweden, Turkey, Ireland, Israel, and Jamaica.
(c) Starting in November 2002, the city was allowed to revoke diplomatic license plates after more than three parking violations.


$\phi(k_t)$
k_{t+1}
$k_{t+1} = k_t$
k_t
k^L
k^U
unstable st. st
stable steady states