Midterm Exam – Econ 5700
9 October 2019
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Notes: As a rule, you must explain how you arrived at your answers, unless otherwise stated.

1. The Diamond Model [10 marks]

Consider an otherwise standard version of the Diamond model, but here with working-age subsistence consumption. Agents live for two periods, working when young and living off savings when old. Utility is given by

$$U_t = (1-\beta)\ln(C_{1,t} - \widehat{C}) + \beta \ln(C_{2,t+1}),$$

where $\widehat{C} > 0$ is subsistence consumption in working age. The notation is otherwise the same as in class: $C_{1,t}$ is working-age consumption, $C_{2,t+1}$ is old-age consumption (both referring to an agent working in period t), and β is a preference parameter (such that $0 < \beta < 1$). Population is assumed to be constant, meaning each agent has one single offspring. Output in period t is given by

$$Y_t = AK_t^{\alpha}L^{1-\alpha},$$

where A and α are parameters (such that $A > 0$ and $0 < \alpha < 1$), and K_t denotes the total capital stock in period t, and L the total labor force in all periods (recall that population is constant).
The wage rate and the (net) interest rate faced by each agent are given by the marginal products to labor and capital, respectively, and denoted w_t and r_t. Capital depreciates fully in each period.

(a) Let $k_t = K_t/L$. Find and expression for w_t in terms of k_t, A, and α. [2 marks]

(b) Write the budget constraints for working and old age. [2 marks]

(c) Utility is not well defined for $w_t \leq \widehat{C}$, but we assume that saving in period t is zero when $w_t \leq \widehat{C}$, and set to maximize utility when $w_t > \widehat{C}$. Find an expression for saving in period t as a function of w_t, $\widehat{C}$, and β. A complete answer should involve a "curly bracket" and consider both the case when $w_t \leq \widehat{C}$, and the case when $w_t > \widehat{C}$. [3 marks]

(d) Use your answer under (c) to find a function ϕ, such that $k_{t+1} = \phi(k_t)$. Show the graph of $k_{t+1} = \phi(k_t)$, and *all* steady states, in a 45°-diagram. Indicate which steady states are (locally) stable and unstable. [3 marks]

2. The Solow model [10 marks]

Consider a simple Solow model with production given by

$$y_t = Zk_t^{\alpha},$$

where Z is a productivity factor, k_t is capital per worker in period t, y_t is output per worker in period t, and $\alpha \in (0, 1)$. Assuming full depreciation, capital accumulation can be written as

$$k_{t+1} = \frac{sy_t}{1+n},$$

where $s \in (0, 1)$ is the saving rate, and $n > 0$ is the population growth rate.

(a) The growth rate in output per worker from period t to $t + 1$ can be written as $g_t = \ln(y_{t+1}) - \ln(y_t)$. Find an expression for g_t in terms of $\ln(y_t)$ and some, or all, of the exogenous variables Z, s, n, and α. Show how you arrived at your answer. [3 marks]

(b) Suppose you run a cross-country regression with some measure of the growth rate in GDP per capita as the dependent variable, and log initial GDP per capita as the independent variable. That is, $g_t = \beta_0 + \beta_1 \ln(y_t) + \varepsilon_t$, where ε_t is an error term. (We suppress any subindex for the country.) What would your estimates of β_0 and β_1 correspond to in terms of the Solow model described in this problem? Your answer should be in terms of some, or all, of the exogenous variables Z, s, n, and α. Show how you arrived at your answer. [3 marks]

(c) Suppose now that the saving rate is not constant and exogenous, but instead a function of k_t, denoted $s(k_t)$. This function must be such that $0 \leq s(k_t) \leq 1$ for all $k_t \geq 0$. The production function is the same, so we can now write $k_{t+1} = s(k_t)Zk_t^{\alpha}/(1+n)$. Find one example of a parametric form for $s(k_t)$, such that the model can generate multiple steady states. (There are many such examples, but you only need to find one.) Derive explicit assumptions on the parameters of $s(k_t)$, and the other parameters of the model (Z, n, and α), that together imply that there are multiple steady states. The assumptions must also ensure that $0 \leq s(k_t) \leq 1$ holds. Show how you arrived at your answer; to that end, you may use a suitable diagram. [4 marks]

3. Divergence/convergence I [5 marks]

The questions below refer to the attached table.

(a) Using the information in the table, explain how to find the annual (per-year) growth rate of GDP per capita (in 1985\$) for United States between 1870 and 1960. You should not compute an exact number, only show how you would go about to compute it. [1 mark]

(b) Show how the number 4.5 is calculated from other numbers in the table. [2 marks]

(c) What is the title of the paper that this table is taken from? Who is the author? (Hint: the author's last name starts with a P.) [2 marks]

4. Divergence/convergence II [5 marks]

The questions below refer to Barro (1996) and Jones (2016).

(a) Explain the difference between conditional convergence and absolute convergence. Which one is more consistent with cross-country data? [3 marks]

(b) What does "Barro's iron law" mean? [2 marks]

Table 2
Estimates of the Divergence of Per Capita Incomes Since 1870

	1870	*1960*	*1990*
USA (*P*$)	2063	9895	18054
Poorest (*P*$)	250 (assumption)	257 (Ethiopia)	399 (Chad)
Ratio of GDP per capita of richest to poorest country	8.7	38.5	45.2
Average of seventeen "advanced capitalist" countries from Maddison (1995)	1757	6689	14845
Average LDCs from PWT5.6 for 1960, 1990 (imputed for 1870)	740	1579	3296
Average "advanced capitalist" to average of all other countries	2.4	4.2	4.5
Standard deviation of natural log of per capita incomes	.51	.88	1.06
Standard deviation of per capita incomes	*P*$459	*P*$2,112	*P*$3,988
Average absolute income deficit from the leader	*P*$1286	*P*$7650	*P*$12,662

Notes: The estimates in the columns for 1870 are based on backcasting GDP per capita for each country using the methods described in the text assuming a minimum of *P*$250. If instead of that method, incomes in 1870 are backcast with truncation at *P*$250, the 1870 standard deviation is .64 (as reported in Figure 1).

Sketches of solutions

1.
(a) $w_t = (1-\alpha)Ak_t^\alpha$
(b) Letting S_t be saving, the budget contraints state that

$$C_{1,t} = w_t - S_t, \tag{1}$$

and

$$C_{2,t+1} = S_t(1 + r_{t+1}). \tag{2}$$

(c)

$$S_t = \begin{cases} 0 & \text{if} \quad w_t \leq \widehat{C}, \\ \beta\left(w_t - \widehat{C}\right) & \text{if} \quad w_t > \widehat{C}, \end{cases} \tag{3}$$

(d) Use $K_{t+1} = LS_t$, which gives $k_{t+1} = K_{t+1}/L = S_t$. Then define the level of k_t at which $w_t = \widehat{C}$ as $\widehat{k}$. Using the answer under (a) gives

$$\widehat{k} = \left[\frac{\widehat{C}}{(1-\alpha)A}\right]^{\frac{1}{\alpha}}. \tag{4}$$

Correct answers under (a) and (c) should now give

$$k_{t+1} = \left\{\begin{matrix} 0 & \text{if} \quad k_t \leq \widehat{k}, \\ \beta\left[(1-\alpha)Ak_t^\alpha - \widehat{C}\right] & \text{if} \quad k_t > \widehat{k}, \end{matrix}\right\} = \phi(k_t). \tag{5}$$

The graph of $\phi(k_t)$ has a stable steady state (a poverty trap) at $k_t = 0$, another stable steady state where $k_t > 0$, and an unstable (threshold) steady state between the two.

2.
(a)
Note that

$$y_{t+1} = Zk_{t+1}^\alpha = Z\left(\frac{sy_t}{1+n}\right)^\alpha,$$

which gives

$$\frac{y_{t+1}}{y_t} = Z\left(\frac{s}{1+n}\right)^\alpha y_t^{\alpha-1}$$

and

$$g_t = \ln\left(\frac{y_{t+1}}{y_t}\right) = \ln(y_{t+1}) - \ln(y_t) = \ln\left[Z\left(\frac{s}{1+n}\right)^\alpha\right] - (1-\alpha)\ln(y_t)$$

(b) From the answer under (a) we see the additive term and the coefficient on $\ln(y_t)$, which gives

$$\begin{aligned} \beta_0 &= \ln\left[Z\left(\frac{s}{1+n}\right)^\alpha\right] \\ \beta_1 &= 1-\alpha \end{aligned}$$

(c) It is given in the question that

$$k_{t+1} = \frac{s(k_t) Z k_t^{\alpha}}{1+n}.$$

One example of parametric form for $s(k_t)$ that can generate multiple steady states is this:

$$s(k_t) = \begin{cases} \underline{s} & \text{if} \quad k_t < \widehat{k}, \\ \\ \overline{s} & \text{if} \quad k_t \geq \widehat{k}, \end{cases}$$

where $\widehat{k} > 0$, and where

$$0 < \underline{s} < \overline{s} < 1,$$

which ensures that $0 < s(k_t) < 1$ for all k_t.
Now it follows that

$$k_{t+1} = \left\{ \begin{array}{ll} \frac{\underline{s} Z k_t^{\alpha}}{1+n} & \text{if} \quad k_t < \widehat{k}, \\ \\ \frac{\overline{s} Z k_t^{\alpha}}{1+n} & \text{if} \quad k_t \geq \widehat{k}, \end{array} \right\} \equiv \phi(k_t)$$

You can illustrate $\phi(k_t)$ in a 45-degree diagram, where the graph makes a jump at $k_t = \widehat{k}$. Parametric assumptions that generate multiple steady are:

$$\left(\frac{\underline{s} Z}{1+n} \right)^{\frac{1}{1-\alpha}} < \widehat{k} < \left(\frac{\overline{s} Z}{1+n} \right)^{\frac{1}{1-\alpha}}.$$

The expressions greater and less than $\widehat{k}$ are the two steady states, both of which are stable.

3.
(a) In the 90 years from 1870 to 1960 U.S. GDP per capita grew from \$2063 to \$9895. The annual growth rate, g, is given by

$$g = \left(\frac{9895}{2063} \right)^{\frac{1}{90}} - 1.$$

(b)

$$\frac{\text{Average of 17 advanced capitalist countries}}{\text{Same average among LDCs}} = \frac{14845}{3296} \approx 4.5.$$

(c) Lant Pritchett; Divergence, Big Time

4.
(a) Many formulations are okay, but the general message should be that absolute convergence means that all countries are fundamentally identical and thus converge to the same steady state, while conditional convergence means countries are different and thus converge to their own steady states.
One can also express this in terms of how we test these hypotheses against data: absolute convergence implies a negative coefficient on initial (log) GDP per capita, without controlling for anything; conditional convergence implies a negative coefficient on initial (log) GDP per

capita when entering suitable controls for the determinants of the steady state towards which the economies converge.
Conditional convergence is more consistent with data.
(b) Barro's "iron law" states that the rate at which countries converge to their own steady state (the speed of convergence) is around 2% per year.