Lecture Notes in Growth Theory – Part III

Growth in the very long run

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February 27, 2006

Population growth and technological progress

Recall: many endogenous growth models have “scale effects” – larger population means faster growth

Hardly true if we interpret nation states as closed economies: China not richer than Norway

But nation states are not closed economies! In one sense, Earth is one economy; but hard to come by intergalactic data to test for scale effects

Classic paper by Kremer (1993): look at times series data over very long time periods; compare previously separated regions when they were rediscovered by Europeans

Simple model: here discrete-time version; Kremer: continuous time

Production of food in period t:

$$Y_t = A_t L^{1-\alpha} P_t^{\alpha} \tag{1}$$

$A_t =$ technology

$L =$ land size (fixed)

$P_t =$ population

Technological progress:

$$A_{t+1} = B A_t^{1-\beta} P_t^{\beta} \tag{2}$$

More generally, we could write: $A_{t+1} = B A_t^{\delta} P_t^{\beta}$; here $\delta = 1 - \beta$. (Kremer focuses on the case where $\delta = 1$)

Assume that population adjusts in each period so that per-capita food production equals some exogenous subsistence requirement, $\overline{y}$; that is:

$$\frac{Y_t}{P_t} = A_t L^{1-\alpha} P_t^{\alpha-1} = \overline{y} \tag{3}$$

or:

$$P_t = \left(\frac{A_t}{\overline{y}}\right)^{\frac{1}{1-\alpha}} L \tag{4}$$

Note that population is proportional to land: population density reflects level of technology. This seems true for most of human existence; technological progress has generated larger populations but not higher living standards

(4) implies that population growth depends on technological progress:

$$\frac{P_{t+1}}{P_t} = \left(\frac{A_{t+1}}{A_t}\right)^{\frac{1}{1-\alpha}} \tag{5}$$

Use (2):

$$\frac{A_{t+1}}{A_t} = B\left(\frac{P_t}{A_t}\right)^{\beta} \tag{6}$$

And then (3):

$$\frac{P_t}{A_t} = \frac{L^{1-\alpha} P_t^{\alpha}}{\overline{y}} \tag{7}$$

Now (5), (6), and (7) give us population growth as a function of initial population:

$$\begin{aligned} \frac{P_{t+1}}{P_t} &= \left[B \left(\frac{P_t}{A_t} \right)^{\beta} \right]^{\frac{1}{1-\alpha}} \\ &= \left[B \left(\frac{L^{1-\alpha} P_t^{\alpha}}{\overline{y}} \right)^{\beta} \right]^{\frac{1}{1-\alpha}} \\ &= \underbrace{B^{\frac{1}{1-\alpha}} \overline{y}^{\frac{-\beta}{1-\alpha}}}_{\equiv Z} L^{\beta} \; P_t^{\frac{\alpha\beta}{1-\alpha}} \end{aligned} \tag{8}$$

Population growth increases in initial levels: the larger is population in period t, the faster is population growth from period t to $t+1$

Can be tested on time-series data:

Kremer looks at world population from 1 million years B.C.
prediction seems true for most of human history
the larger is initial population the faster is the growth rate to the next period

Next, think of several isolated regions of the world which are identical in all aspects except in (fertile) land size

Assume that population is initially spread evenly across lands, i.e., initial population density is the same across regions. Implies large land areas have larger initial population size

Rewrite (8), letting lower-case variables be logs:

$$p_{t+1} = (z + \beta l) + \gamma p_t \tag{9}$$

where

$$z = \ln(Z) = \ln\{B^{\frac{1}{1-\alpha}} \overline{y}^{\frac{-\beta}{1-\alpha}}\} \tag{10}$$

and

$$\gamma = 1 + \frac{\alpha\beta}{1-\alpha} = \frac{1-\alpha+\alpha\beta}{1-\alpha} > 1 \qquad (11)$$

Denote population density with a tilde; in logs: $\widetilde{p}_t = p_t - l$

$$\begin{aligned} \widetilde{p}_{t+1} &= z + (\beta + \gamma - 1)l + \gamma\widetilde{p}_t \\ &= z + \tfrac{\beta l}{1-\alpha} + \gamma\widetilde{p}_t \end{aligned} \qquad (12)$$

where we have used (11) to see that $(\beta + \gamma - 1) = \beta/(1-\alpha)$

Let initial population density be one (zero in logs): $\widetilde{p}_0 = p_0 - l = 0$.

Solving the difference equations in (9) gives:

$$\begin{aligned} \widetilde{p}_t &= \{z + \tfrac{\beta l}{1-\alpha}\} \textstyle\sum_{i=0}^{t-1} \gamma^i \\ &= \{z + \tfrac{\beta l}{1-\alpha}\} \underbrace{\left(\frac{\gamma^{t-1} - 1}{\gamma - 1}\right)}_{>0} \end{aligned} \qquad (13)$$

$\widetilde{p}_t$ is increasing in l

Compare two economies with identical initial population density ($\widetilde{p}_0 = 0$) and identical parameters (α, β, $\overline{y}$, etc.), except for l; then the one with larger l will have higher population density (higher $\widetilde{p}_t$) in period $t \geq 1$. Note: not only *total population*, but *density*

Intuition: more land means larger initial population, since initial population proportional to land; larger initial population means faster population growth.

The other test of the model:

around 10,000 years B.C. previously connected regions of the world were disconnected due to rising sea levels as polar ice caps melted
those regions are: the "Old World" (Eurasia and Africa); the Americas; Australia; Tasmania; Flinder's Island
reconnected around A.D. 1500
use population estimates from A.D. 1500 and land size to compute population density
rank of land size correlates perfectly with rank of population density
also holds for technology: Australia had not even invented agriculture; the Americas had agriculture but technologically behind Europe

More anecdotal evidence:

as Britain got isolated from Europe 5,500 B.C. it fell technologically behind
similar story for Japan

Empirical work

Recall the big questions asked in beginning of the course:

In the world today, why are some countries poor and others rich?

How come today's rich are not the same ones as those which were rich 1,000 or 7,000 years ago?

Why did the whole world become colonized by a couple of Western European states?

What determined who colonized whom? Why did the Spaniards conquer the Aztec empire, not the other way around?

Some of the lands which were colonized grew rich (North America, Australia), and others not (Africa, Latin America). Why?

Historical perspective: two big human revolutions

Switch from hunting-gathering to agriculture (Agricultural Revolution), about 8,000 B.C. in Eurasia

Switch from agriculture to industrial production (Industrial Revolution), around A.D. 1500-1700 in Europe

When Europe had its Industrial Revolution some other parts of the world were still hunter-gatherers

Examples: Aborigines in Australia and native North American nations predominantly hunter-gatherer societies

Others had reached further: China, Mexico, India, Peru had advanced civilizations

Some hunter-gatherers had started the transition to agriculture when Europeans arrived (Australia, North America)

Seems like every society will go through these transitions (Agricultural and Industrial Revolutions) at some stage, but some societies move before others

Burkett, Humblet, and Putterman's hypothesis (1999): countries at earlier stages of development at the time when Europeans industrialized were not as "ready" to catch up with Europe as were countries at later stages

Potential reasons why this could be the case:

- agrarian economies had other work ethic
- agrarian skills more useful in industrial society than hunting-and-gathering skills
- agrarian societies had more experience with hierarchical state forms, commercial activity, and trade

Testing hypothesis: construct cross-country measure of Pre-Industrial Development (PID)

Focus on 3 measures: (1) population density, (2) land cultivated per farmer (cultivation intensity), (3) fraction of land being irrigated

Ideally one would like to measure growth from start of Industrial Revolution in Europe. However, little data before WWII; look at growth from 1960

(But why not levels?)

Regression: growth rate of GDP per capita 1960-1990 on LHS, and PID variables 1960 (plus other controls) on RHS

Hypothesis seems true: countries with favorable PID grew faster (at least controlling for other variables)

What drives the result? South-East Asia at a higher PID level than Africa and Latin America; also the region that has been taking off 1960-90

Bockstette, Chanda, and Putterman (2002) test similar theory

Use other measure of PID: the antiquity of the state

10,000 B.C. no states on the planet; today the whole world covered by states

China's state stretches 1000s of years back in time; Papua New Guinea's state very young

Construct cross-country measures of the age of the state; consider the one called *statehist5*

Basic idea: assign points to each half-century with state being present; discounted at some rate

Hypothesis seems true when looking at growth rates 1960-95 (thought of as a "catch up" phase): *statehist5* has positive effect on GDP per capita growth 1960-95

But some puzzle remains: North America and Oceania have young states but are very rich; Asia not as rich as Europe

Obviously, some countries have imported “states” or institutions from the colonizer (in particular from the UK)

Why did some countries import good institutions, and others not?

One (key?) answer: institutions follow settlements

Acemoglu et al. (2001):

Some colonies well suited for European settlement (US, Canada, Australia) – established "Neo-Europes"

Other colonies less suited for Europeans settlement (Africa, parts of Latin America and Asia). Europeans set up "extractive states," milking

Examples: the Spanish and Portuguese in the Americas

But what makes a colony well suited for settlement?

Acemoglu et al. (2001)'s answer: *mortality rates among settlers*

Big killers of Europeans: malaria, yellow fever

Locals more resistant; supported by e.g. data over death rates among British soldiers vs. local recruits in India

Related story by Acelmoglu et al. (2002)

Regions having reached more advanced stages of development by 1500 AD (e.g. Mexico, India, Peru) were endowed with worse institutions by European colonizers

Measures of early development: population density, urbanization

Called *reversal of fortune*

Some questions unanswered by Acemoglu et al. (2001, 2002):

What determined death rates?

Why did Europeans colonize the rest of the world, not the other way around?

Why did growth take off in Europe, and no where else?

Some answers given in celebrated best-seller: Diamond's (1999) "Guns, Germs, and Steel"

(Question in the prologue posed by local New Guinean politician, Yali; Diamond calls it "Yali's question")

Immediate answer: by A.D. 1500 (as colonization was about to set in) Europe was the technological leader

In turn the result of early timing of previous transitions, like Agricultural Revolution

But then why early transitions? Look for *ultimate* explanation, rather than *proximate*

Key story: sequences of causalities, starting with a purely exogenous factor: geography

Some facts about Eurasia

- Continent larger than other continents (cf. Kremer 1993)

- Stretched out East-West, rather than North-South (like the Americas); not cut off by deserts (like Africa)

- More plants and animals suited for domestication

Diamond argues that how these ultimate factors explain the rest:

- East-West stretch facilitates spread of plants and expanding settlements, due to same climactic zone; the Incas and Aztecs never met, but Europe imported technology from e.g. China (gun powder), India (the alphabet) and the Arabic world (numbers)

- More plants to domesticate makes early Agricultural Revolution likely

- More animals to domesticate means more *livestock*; more livestock means more *diseases* (which we get from interacting closely with animals); diseases killed more Native Americans than guns

- More livestock and agriculture means *denser population*; more complex, hierarchical social structures; invention of *writing*

- Writing is hard to invent (happened only a few times in the world); usually imported; East-West stretch of Eurasian continent helped

- Denser populations again means more diseases (cf. Acemoglu *et al.*; regions where Europeans death rates were high were often densely populated)

Diamond's main point: differences in historical paths is not due to differences between people

Ambition to kill common myth: "Europeans smarter, but the truth is politically incorrect"

Genetical differences exist but are the *result* of different histories, not the *cause*

Example:

Blood groups differ between Europeans (where groups B and O dominate) and Native Americans (mostly A)
Why? Because B and O makes you more resistant to smallpox
In Europe smallpox has been around longer
Natural selection favored B and O in Europe
Europeans not smarter, just different blood groups
Similar stories: resistance to alcohol, lactose intolerance, obesity/diabetes (the "thrifty gene" hypothesis)
Geography is the *ultimate* cause of all the above *proximate* factors

Causality:
Diseases helped defeat Incas
Diseases the result of livestock
Livestock result of geographical coincidences

Attempt to give an overview of the literature

- Diamond and Putterman + co-authors suggest early development is “good for growth”

- Acemoglu et al. suggest (almost) the opposite: per-capita income levels today lower in previously more developed areas, where Europeans did not migrate and/or set up bad institutions

More recent work by Chanda and Putterman (2005) seeks a more unified view; three-stage process:

- Up until 1500 AD: areas like China and India were the economic leaders; estimates of GDP/capita by 1500 AD are positively related to measures of "early development"
  - Early development = variables measuring how early was the transition to agriculture, and state history by 1500 (cf Burkett et al. 1999)
  - GDP/capita by 1500 AD available for small set of West European countries from Angus Maddison; here estimated from urbanization and population density data to get broader data set (see paper)
- Then colonization: growth from 1500 to 1960 negatively correlated with same measures of early development (Acemoglu-style reversal of fortune)

- Finally resurgence of early developers: early development has positive effect on growth from 1960 to 1998

Gaps in levels today reflect recent colonial history

Long-run perspective: colonial phase was the historical exception; now we see a rapid undoing of the reversal

Land and property rights

Lucas (2000, Ch. 5); here stick to Section 3 and 4

Model with endogenous population dynamics and land – same components as in other papers (e.g. Kremer 1993)

Now also explicit *property rights* to land

Three settings:

1. With no property rights to land and homogenous population (no classes)

2. With property rights to land and homogenous population

3. With property rights to land and heterogenous population: a landowning class and a working class

Preferences

Dynastic (Ramsey) setting throughout

Here: logarithmic; Lucas' book: more general

$$u_t = (1 - \beta) \ln c_t + \beta [\gamma \ln n_t + u_{t+1}] \qquad (14)$$

Lucas' notation: $\eta = \beta\gamma$

Assume $\gamma > 1$ ($\eta > \beta$); explained soon

No property rights

Interpreted as a hunter-gatherer society

Budget constraint

$$f(x_t) = c_t + kn_t \tag{15}$$

$k =$ cost per child (here only goods cost)

$f(x_t) =$ per-agent output

$x_t = L/N_t =$ land per agent

$L =$ fixed amount of land

$N_t =$ total population, where $N_{t+1} = n_t N_t$

$$x_{t+1} = \frac{L}{N_{t+1}} = \underbrace{\left(\frac{L}{N_t}\right)}_{=x_t} \underbrace{\left(\frac{N_t}{N_{t+1}}\right)}_{1/n_t} = \frac{x_t}{n_t} \tag{16}$$

Value function:

$$W(x_t) = \max_{n_t} \left\{ \begin{array}{c} (1-\beta)\ln\left[f(x_t) - kn_t\right] \\ +\beta\gamma \ln n_t + \beta W(x_{t+1}) \end{array} \right\} \quad (17)$$

No property rights: agents take land per agent in the next period, x_{t+1}, as given

FOC for n_t

$$(1-\beta)\frac{k}{c_t} = \beta\gamma\frac{1}{n_t} \quad (18)$$

Dynamics: use $c_t = f(x_t) - kn_t$; substitute into the FOC; this gives:

$$n_t = \left(\frac{\beta\gamma}{1-\beta+\beta\gamma}\right)\frac{f(x_t)}{k} \quad (19)$$

Substitute into $x_{t+1} = x_t/n_t$; let output be Cobb-Douglas: $f(x_t) = Ax_t^{\alpha}$; this gives:

$$x_{t+1} = \left[\frac{1+\beta(\gamma-1)}{\beta\gamma}\right]\left(\frac{k}{A}\right)x_t^{1-\alpha} \quad (20)$$

Economy converges to steady state with constant land per agent, implying constant population: $N_{t+1} = N_t$, $n_t = 1$; and constant consumption per agent

Steady-state consumption per agent:

$$c_m = \left(\frac{1-\beta}{\beta\gamma}\right) k \tag{21}$$

m as in Malthusian

Contrast to e.g. Becker, Murphy and Tamura (1990); Barro and Becker (1989): here stable population in *levels*; not growth rates. Sustained growth ruled out because of fixed supply of land and diminishing marginal product to land ($\alpha < 1$)

Property rights

Implies agents take into account that $x_{t+1} = x_t/n_t$ when choosing n_t:

$$W(x_t) = \max_{n_t} \left\{ \begin{array}{c} (1-\beta)\ln\left[Ax_t^{\alpha} - kn_t\right] \\ +\beta\gamma \ln n_t + \beta W(\frac{x_t}{n_t}) \end{array} \right\} \tag{22}$$

FOC for n_t

$$(1-\beta)\frac{k}{c_t} = \beta\gamma\frac{1}{n_t} + \beta W'(\frac{x_t}{n_t})\frac{-x_t}{n_t^2} \tag{23}$$

Envelope:

$$W'(x_t) = (1-\beta)\frac{A\alpha x_t^{\alpha-1}}{c_t} + \beta W'(\frac{x_t}{n_t})\left(\frac{1}{n_t}\right) + 0 \tag{24}$$

Dynamics: see problem set 3

Steady state: $x_t = x$, $n_t = 1$; $c_t = c$

$$\begin{array}{c} (1-\beta)W'(x) = (1-\beta)\frac{A\alpha x^{\alpha-1}}{c} \\ W'(x) = \frac{A\alpha x^{\alpha-1}}{c} \end{array} \tag{25}$$

Use FOC:

$$(1-\beta)\frac{k}{c} = \beta\gamma - \beta\left[\frac{A\alpha x^{\alpha-1}}{c}\right]x \tag{26}$$

$$(1-\beta)k = \beta\gamma c - \beta A\alpha x^{\alpha} \tag{27}$$

Use steady-state budget constraint, $Ax^{\alpha} = c + k$. This gives steady state consumption:

$$c_e = \left(\frac{1-\beta+\alpha\beta}{\beta\left(\gamma-\alpha\right)}\right)k \tag{28}$$

e as in egalitarian; all agents the same

Note: $c_e > c_m$; property rights in land provides incentives to keep fertility down; land dilution effect internalized

Landowners and landless

Landless workers earn

$$w_t = f(z_t) - f'(z_t)z_t \tag{29}$$

where

$$z_t = \frac{L}{N_{w,t}} \tag{30}$$

is land per worker; $N_{w,t}$ = number of workers

Landowners earn

$$r_t x_t = f'(z_t)x_t \tag{31}$$

where

$$x_t = \frac{L}{N_{l,t}}$$

is land per landowner; $N_{l,t}$ = number of landowners

First: workers' max problem

Budget constraint:

$$c_t = w_t - kn_t \tag{32}$$

Cannot influence offspring's welfare; same FOC as is in the case with no property rights; gives:

$$c = c_m = \left(\frac{1-\beta}{\beta\gamma}\right) k \tag{33}$$

Use budget constraint in (32), evaluated in steady-state ($c_m = w - k$); and the steady-state wage equation in (29); this gives:

$$c_m + k = w = k\left(1 + \frac{1-\beta}{\beta\gamma}\right) = f(z) - f'(z)z \tag{34}$$

Defines z in terms of exogenous parameters

Next: landowners max problem

Budget constraint:

$$c_t = r_t x_t - k n_t \tag{35}$$

Value function:

$$W(x_t) = \max_{n_t} \left\{ \begin{array}{c} (1-\beta) \ln [r_t x_t - k n_t] \\ \\ +\beta\gamma \ln n_t + \beta W(\frac{x_t}{n_t}) \end{array} \right\} \tag{36}$$

FOC for n_t; same as in (23):

$$(1-\beta)\frac{k}{c_t} = \beta\gamma\frac{1}{n_t} + \beta W'(\frac{x_t}{n_t})\left(\frac{-x_t}{n_t^2}\right) \tag{37}$$

Envelope:

$$W'(x_t) = (1-\beta)\frac{r_t}{c_t} + \beta W'(\frac{x_t}{n_t})\left(\frac{1}{n_t}\right) + 0 \tag{38}$$

Dynamics: complicated

Steady state: $n_t = 1$; FOC for n_t:

$$(1-\beta)\frac{k}{c} = \beta\gamma - \beta W'(x)x \tag{39}$$

Envelope:

$$\begin{gathered}(1-\beta)W'(x) = (1-\beta)\tfrac{r}{c} \\ W'(x) = \tfrac{r}{c}\end{gathered} \tag{40}$$

Insert into steady-state FOC in (39):

$$(1-\beta)\frac{k}{c} = \beta\gamma - \beta\frac{rx}{c} \tag{41}$$

Use steady-state budget constraint ($rx = c + k$); this gives:

$$c = c_l = \frac{k}{\beta(\gamma - 1)} \tag{42}$$

Note: $c_l > c_m$, since

$$c_m = \frac{k}{\beta}\left(\frac{1-\beta}{\gamma}\right) < \frac{k}{\beta}\left(\frac{1}{\gamma-1}\right) = c_l \tag{43}$$

Intuition:

Incentives not to dilute land induces lower fertility among landowners, captured by the term $-\beta W'(x)x$
Thus higher steady-state consumption
Note: higher incomes induce higher fertility too; net effect in steady state such that $n = 1$

Examine the relative size of the classes

Assume Cobb-Douglas output: $f(z) = Az^{\alpha}$

Total total steady-state output = output per worker, times number of workers: $N_w A z^{\alpha}$

So total income of landowning class $= N_w \alpha A z^{\alpha}$

Must equal total spending by landowners:

$$N_w \alpha A z^{\alpha} = N_l [c_l + k] = N_l \left(\frac{\beta(\gamma - 1) + 1}{\beta(\gamma - 1)} \right) k \quad (44)$$

where we have used (42)

Recall that (34) stated that $w = k\left(1+\frac{1-\beta}{\beta\gamma}\right)$, so with Cobb-Douglas production:

$$w = (1-\alpha)Az^{\alpha} = k\left(1+\frac{1-\beta}{\beta\gamma}\right) \qquad (45)$$

Or:

$$\alpha Az^{\alpha} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\beta(\gamma-1)+1}{\beta\gamma}\right)k \qquad (46)$$

Substituting (46) into (44) gives:

$$\frac{N_l}{N_w} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\gamma-1}{\gamma}\right) \qquad (47)$$

Landowning class larger relative to landless if land share of output large, or weight on quantity high

Natural resources

Models of population and land so far: technological progress increases the productivity of land

Example: invention of agriculture

Brander and Taylor (1998): endogenous reource dynamics; limit on how much can be harvested without depleting the resource

Examples: fish, wild animals, forest, soil

Central assumption: no (or incomplete) property rights to the natural resource; tragedy of the commons

Story told in context of the history of *Easter Island*

Ancient civilization that went under due to environmental degradation

When the first people arrived A.D. 400: plenty of forests all over

Trees used to make canoes for fishing

Rapid population expansion; ample time and resources to use for other things than food production: making of big statues

Eventually: forest depletion, deteriorating diet, violent conflicts over resources

Population peak around A.D. 1400; then decline

Here: discrete-time simplified setting

Resource stock at time t: S_t

Dynamic equation for S_t:

$$S_{t+1} - S_t = G(S_t) - H_t \tag{48}$$

where H_t is the harvest, and $G(S_t)$ is the natural growth in the resource stock

Functional form:

$$G(S_t) = rS_t \left[1 - \frac{S_t}{K}\right] \tag{49}$$

Note:
$G(0) = 0$; once the resource is gone, it's gone
$G(K) = 0$; $K =$ carrying capacity $=$ maximum steady-state level of S_t in the absence of harvesting

Each agent harvests (consumes) a fraction α of the resource; population size $= L_t$; so total amount harvested becomes:

$$H_t = \alpha S_t L_t \tag{50}$$

Population dynamics depend on per-capita income (harvest) relative to subsistence consumption, $\overline{y}$

$$\frac{L_{t+1}}{L_t} = \frac{\alpha S_t}{\overline{y}} \tag{51}$$

Phase diagram

Population growing (falling) when $S_t > (<) S^*$, where

$$S^* = \frac{\overline{y}}{\alpha} \tag{52}$$

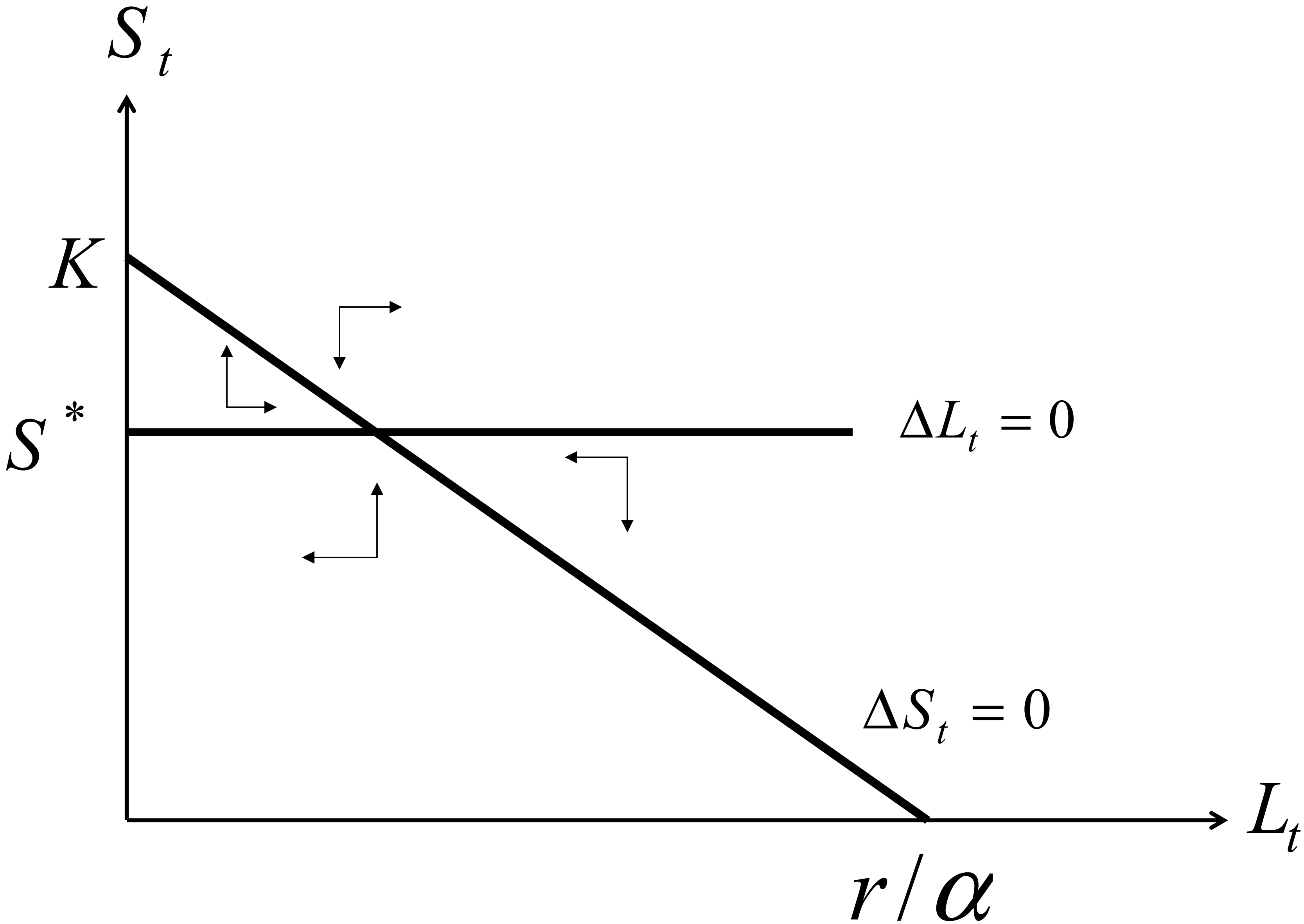
Assume: $K > S^*$

Resources are growing (falling) when

$$G(S_t) > (<) H_t \tag{53}$$

or, using (49) and (50):

$$S_t > (<) K \left[1 - \frac{\alpha L_t}{r} \right] \tag{54}$$



S_t
K
S^*
$\Delta L_t = 0$
$\Delta S_t = 0$
L_t
r/α

Three types of steady state:

1. $L_t = 0$, $S_t = K$; no human population, natural resource stock at its carrying capacity

2. $L_t = S_t = 0$; no humans, resource stock depleted

3. $L_t = L^* = r\left(\frac{\alpha K - \overline{y}}{\alpha^2}\right)$, $S_t = S^*$; constant non-zero population and resource stock

Steady state 3 has interesting osciallatory properties

Called *spiral node* – path around steady state is circular

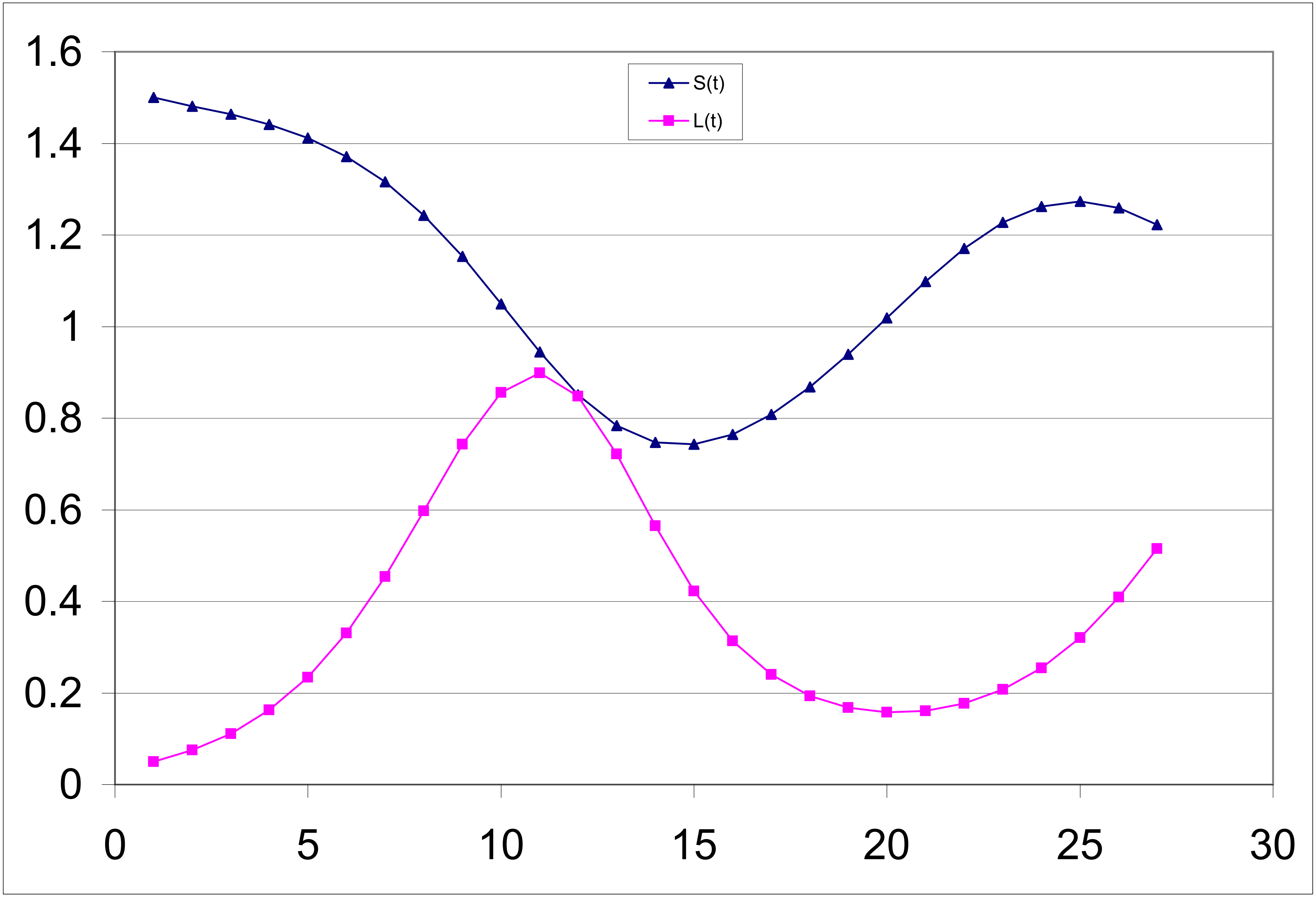
Numerical illustration:

Initially un-populated ("virgin") land colonized by small groupof settlers; e.g. Easter Island

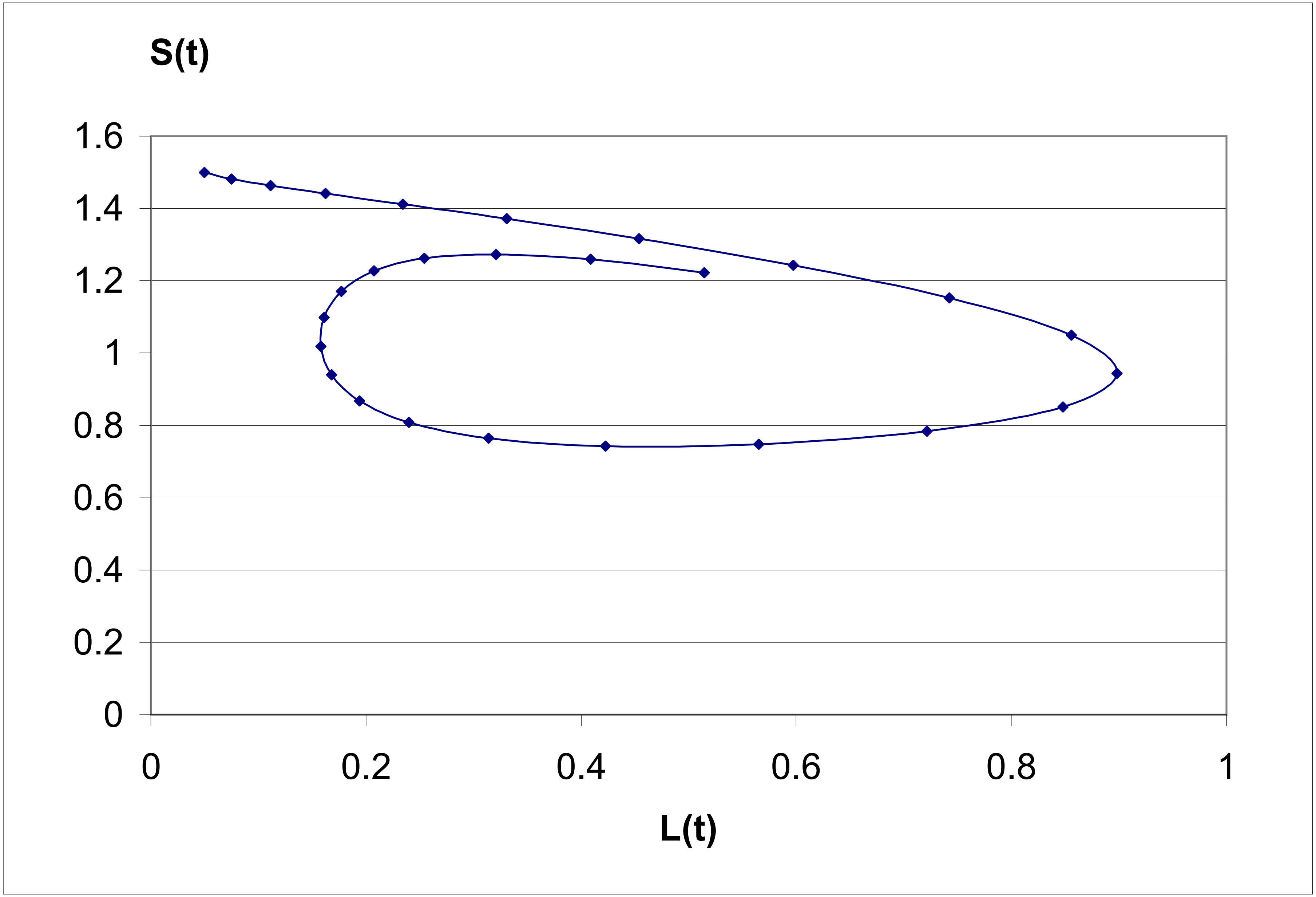
Start with initial resource stock at carrying capacity: $S_0 = K$; settler population exogenous: L_0
Simulate for some (here completely arbitrary) parameter values: $S_0 = K = 1.5$, $\overline{y} = \alpha = .25$, $r = 0.36$, $L_0 = .05$; implies $L^* = .48$ and $S^* = 1$. See problem set 3.In paper: values chosen more realistically

Result: cycles in population and resource stock

The time paths: in the S_t-L_t plane



S(t)
L(t)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30


S(t)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
L(t)