Midterm Exam – Econ 7110
22 March 2017
Department of Economics
York University

Instructions: Unless otherwise explicitly stated, for full mark you must show how you arrived at your answer to a problem. Answers must be given on the answer sheets provided. Ask for extra answer sheets if needed. Do not fold the answer sheets or write on the back.

Part A: do Problems 1 and 2 below

1. The Malthus model [10 marks]

Consider a standard Malthusian OLG model. Agents live for two periods, as dependent children and working adults. Adults active in period t earn y_t and consume c_t; they have n_t children and each child consumes/costs an exogenous $q > 0$ units of the consumption good. Utility is

$$U_t = (1 - \beta) \ln c_t + \beta \ln n_t,$$

where $\beta \in (0, 1)$ is an exogenous preference parameter. Total output (and income) in period t is

$$Y_t = (A_t L)^{\alpha} P_t^{1-\alpha},$$

where $\alpha \in (0, 1)$ is the exogenous land share of output, L is an exogenous (and constant) amount of land, P_t is the size of the adult population (and thus the workforce) in period t, and A_t is land productivity in period t. Per-adult income in period t is given by $y_t = Y_t/P_t$.
(a) Assume that land productivity grows at a constant and exogenous rate $g > 0$, meaning $A_{t+1} = (1 + g)A_t$. Show that income per adult (y_t) in steady state (on the balanced growth path) equals $q(1 + g)/\beta$. Show each step, including how to set up and solve the utility maximization problem. [3 marks]
(b) Under the same assumption about growth in land productivity as under (a), find a difference equation for y_t. Your answer should be an expression for y_{t+1} in terms of y_t and some, or all, of the exogenous variables β, q, g, L, and α. [3 marks]
(c) Now let $A_{t+1} = (1 + g_t)A_t$, where g_t equals some constant g_L for $t \in \{0, 1, ..., \widehat{t} - 1\}$, and then jumps to some higher level $g_H > g_L$ for $t \in \{\widehat{t}, \widehat{t} + 1, \widehat{t} + 2, ...\}$. Draw the time path of y_t. Use a diagram with y_t on the vertical axis and t on the horizontal axis. Assume that the economy is in steady state before the jump in g. Indicate $\widehat{t}$ on suitable axis. [2 marks]
(d) Under the same assumptions as under (c), draw the time path of log total adult population, $\ln(P_t)$. Use a diagram with $\ln(P_t)$ on the vertical axis and t on the horizontal axis. Indicate $\widehat{t}$ on suitable axis. [2 marks]
Note: for 1 (c)-(d) you do not need to explain how you arrived at your answer, just draw the diagrams correctly.

2. The Acemoglu and Robinson (2000) model [10 marks]
Let $\lambda > 1/2$ be the fraction of the population belonging to the non-elite, h^p the capital of a non-elite agent, and h^r the capital of an elite agent, where $h^r > h^p$. Let Ah^i be pre-tax income if working in market production, which is taxable, and Bh^i the income if working in non-taxable home production, where $A > B$ and $i = p, r$.
(a) Find an expression for the maximum tax rate, denoted $\widehat{\tau}$, consistent with agents choosing to work in the market sector. [2 marks]
(b) Let $H = \lambda h^p + (1-\lambda)h^r$ be total capital. Then τAH is the transfer per agent (both elite and non-elite) associated with any tax rate $\tau \leq \widehat{\tau}$. Find an expression for non-elite income, net of taxes and transfers, when $\tau \leq \widehat{\tau}$. Your answer should be in terms of some, or all, of τ, λ, h^r, h^p, A, and B. Also show that the expression you have derived is increasing in τ. [2 marks]
(c) Under democracy the tax rate is set to $\widehat{\tau}$. Let $V^p(D)$ be the value of democracy to the non-elite (i.e., the value of having democracy in the current period and onward). The non-elite's per-period payoff under democracy is the same as non-elite income, and the discount factor is denoted β. Find $V^p(D)$ in terms of some, or all, of β, λ, h^r, h^p, A, and B. [3 marks]
(d) Revolution means that the non-elite confiscate the elite's capital, and also that a fraction $1 - \mu^h$ of all capital (H) is permanently lost. Find the value to the non-elite of revolution, here denoted $V^p(R)$. You answer should be in terms of some, or all, of β, λ, μ^h, h^r, h^p, A, and B. [3 marks]

Part B: answer two of questions 3-5 below (not all three)

3. Migration [5 marks]
Explain how to compute an ancestry-adjusted measure of some cross-country variable, like state history, using the Putterman-Weil matrix. What are the elements of this matrix?

4. Reversals [5 marks]
Explain what the phenomenon *reversal of fortune* means, as used by Acemoglu, Johnson, and Robinson (2002). What do they argue caused the reversal of fortune?

5. Democracy [5 marks]
Describe very briefly what different cross-country measures of democracy were used in the papers discussed in class. How are these measures of democracy correlated with levels of (log) GDP/capita in the data? What does the *modernization hypothesis* say?

Answer sheet for Problem____Econ 7110, Midterm 22 March, 2017

Student Name: SID Number:

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Sketches of solutions to Problems 1 and 2

1.

(a) You should first find n_t that maximizes $U_t = (1-\beta)\ln(y_t - qn_t) + \beta\ln(n_t)$. You need to write the first-order condition with respect to n_t and show that it gives $n_t = (\beta/q)y_t$. Then you need to use that A_t and P_t grow at the same rate on a balanced growth path (where y_t is in steady state). Together with $P_{t+1} = n_t P_t$ and $A_{t+1} = (1+g)A_t$ this gives $n_t = (\beta/q)y_t = 1+g$, which can be solved for the steady-state level of y_t, which we may denote $\overline{y}$, i.e.,

$$\overline{y} = \frac{q(1+g)}{\beta}.$$

(b) You need to use $y_{t+1} = Y_{t+1}/P_{t+1}$, $P_{t+1} = n_t P_t$, $n_t = (\beta/q)y_t$ [from (a)], and the production function forwarded one period. The answer should be

$$y_{t+1} = y_t^{1-\alpha}\left(\frac{q\,[1+g]}{\beta}\right)^{\alpha}.$$

(c) y_t should be constant up to, and including, period $\widehat{t}$. Thereafter y_t increases and asymptotically converges to a higher (non-growing) level.

(d) P_t should initially grow at a constant (gross) rate of $1+g_L$, so the slope of the path of $\ln P_t$ should initially be $\ln(1+g_L) \approx g_L$, then exceeding the previous trend from $\widehat{t}+2$, and gradually converging to one with slope $1+g_H$. (Note that $\ln A_t$ exceeds its previous trend from $\widehat{t}+1$ and $\ln P_t$ from $\widehat{t}+2$.)

In the exam I accidentally asked for the time path of P_t, instead of $\ln P_t$. P_t should be growing at an exponential rate and exceed its previous (exponential) trend from $\widehat{t}+2$. You got full mark if you drew either of those paths, with deductions for anything inconsistent with both of them.

2.

(a)

$$\widehat{\tau} = \frac{A-B}{A}.$$

(b) Non-elite income can be written

$$\begin{aligned} & (1-\tau)Ah^p + \tau AH \\ = \; & (1-\tau)Ah^p + \tau A[\lambda h^p + (1-\lambda)h^r] \\ = \; & Ah^p + \tau\left\{A[\lambda h^p + (1-\lambda)h^r] - Ah^p\right\} \\ = \; & Ah^p + \tau A(1-\lambda)\left(h^r - h^p\right). \end{aligned}$$

which is increasing in τ.

(c) Non-elite income becomes

$$\begin{aligned} & Ah^p + \widehat{\tau}A(1-\lambda)\left(h^r - h^p\right) \\ = \; & Ah^p + (A-B)(1-\lambda)\left(h^r - h^p\right), \end{aligned}$$

which implies

$$V^p(D) = \frac{Ah^p + (A-B)(1-\lambda)\left(h^r - h^p\right)}{1-\beta}.$$

(d) The per-period income per non-elite agent under revolution is

$$\frac{\mu^h A\left[\lambda h^p + (1-\lambda)h^r\right]}{\lambda}$$

which implies

$$V^p(R) = \frac{\mu^h A\left[\lambda h^p + (1-\lambda)h^r\right]}{(1-\beta)\lambda}$$